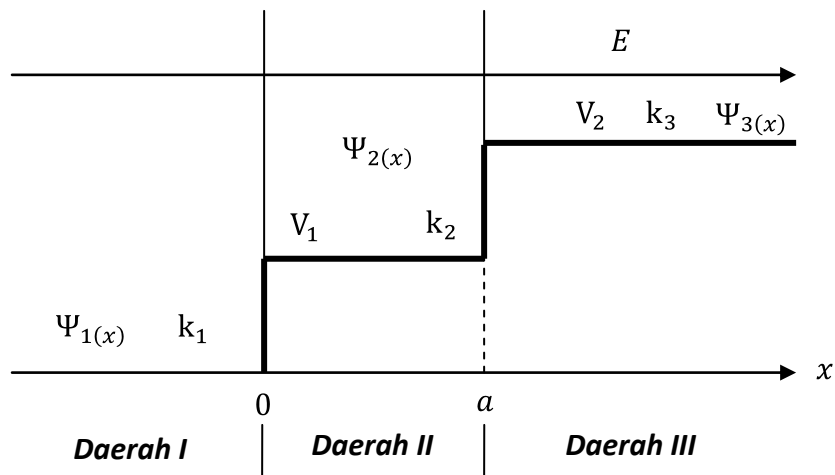KUNCI JAWABAN

PENDAHULUAN FISIKA KUANTUM

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SOAL 1

Persamaan Schrodinger :

Daerah I

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{1(x)} = E\ \Psi_{1(x)}$$

$$\frac{d^2}{dx^2}\Psi_{1(x)} = -\frac{2mE}{\hbar^2}\Psi_{1(x)}$$

$$\frac{d^2}{dx^2}\Psi_{1(x)} = -k_1^2\ \Psi_{1(x)}$$

Solusinya : $\Psi_{1(x)} = A\ e^{ik_1 x} + B\ e^{-ik_1 x}$

Dimana : $k_1^2 = \frac{2mE}{\hbar^2}\ \rightarrow\ k_1 = \frac{\sqrt{2mE}}{\hbar}$

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Daerah II

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{2(x)} + V_{(x)}\Psi_{2(x)} = E\,\Psi_{2(x)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{2(x)} + V_1\Psi_{2(x)} = E\,\Psi_{2(x)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{2(x)} = (E - V_1)\Psi_{2(x)}$$

$$\frac{d^2}{dx^2}\Psi_{2(x)} = -\frac{2m}{\hbar^2}(E - V_1)\Psi_{2(x)}$$

$$\frac{d^2}{dx^2}\Psi_{2(x)} = -k_2^2\,\Psi_{2(x)}$$

Solusinya : $$\Psi_{2(x)} = C\,e^{ik_2\,x} + D\,e^{-ik_2\,x}$$

Dimana : $$k_2^2 = \frac{2m}{\hbar^2}(E - V_1) \quad \rightarrow k_2 = \frac{\sqrt{2m(E - V_1)}}{\hbar}$$

Daerah III

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{3(x)} + V_{(x)}\Psi_{3(x)} = E\,\Psi_{3(x)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{3(x)} + V_2\Psi_{3(x)} = E\,\Psi_{3(x)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{3(x)} = (E - V_2)\Psi_{3(x)}$$

$$\frac{d^2}{dx^2}\Psi_{3(x)} = -\frac{2m}{\hbar^2}(E - V_2)\Psi_{3(x)}$$

$$\frac{d^2}{dx^2}\Psi_{3(x)} = -k_3^2\,\Psi_{3(x)}$$

$$\Psi_{3(x)} = E\,e^{-ik_3\,x} + F\,e^{ik_3\,x}$$

Untuk solusi : $\Psi_{3(x)} = E \, e^{-ik_3 \, x} + F \, e^{ik_3 \, x}$ Karena tidak rintangan pada

x = ~   sehingga berlaku : $\Psi_{3(x)} = F \, e^{ik_3 \, x}$

Dimana : $k_3^2 = \frac{2m}{\hbar^2}(E - V_2)$ $\rightarrow k_3 = \frac{\sqrt{2m(E-V_2)}}{\hbar}$

Susun kembali semua solusi

$\Psi_{1(x)} = A \, e^{ik_1 \, x} + B \, e^{-ik_1 \, x}$ (**1**)

$\Psi_{2(x)} = C \, e^{ik_2 \, x} + D \, e^{-ik_2 \, x}$ (**2**)

$\Psi_{3(x)} = F \, e^{ik_3 \, x}$ (**3**)

Syarat kontinuitas :

$\left.\begin{array}{l}\Psi_1 = \Psi_2 \\ \Psi_1' = \Psi_2'\end{array}\right\}$ Pada x = 0

i) $\Psi_1 = \Psi_2$ $\rightarrow$ $x = 0$

$A \, e^{ik_1 \, x} + B \, e^{-ik_1 \, x} = C \, e^{ik_2 \, x} + D \, e^{-ik_2 \, x}$

$A + B = C + D$ (**4**)

ii) $\Psi_1' = \Psi_2'$ $\rightarrow$ $x = 0$

$ik_1 \left[A \, e^{ik_1 \, x} - B \, e^{-ik_1 \, x} \right] = ik_2 \left[C \, e^{ik_2 \, x} - D \, e^{-ik_2 \, x} \right]$

$ik_1 \left[A - B \right] = ik_2 \left[C - D \right]$ (**5**)

$\left.\begin{array}{l}\Psi_2 = \Psi_3 \\ \Psi_2' = \Psi_3'\end{array}\right\}$ Pada x = a

i) $C \, e^{ik_2 \, a} + D \, e^{-ik_2 \, x} = F \, e^{ik_3 \, a}$ (**6**)

ii) $ik_2 \left[C \, e^{ik_2 \, a} - D \, e^{-ik_2 \, a} \right] = ik_3 F \, e^{ik_3 \, a}$ (**7**)

Lakukan eliminasi pers. (6) dan pers. (7) guna memperoleh konstanta C dan D dalam bentuk konstanta F.

pers. (6) x ik_2 , hasilnya tambahkan dengan pers. (7) guna mereduksi D.

i

$$k_2 \left[C\, e^{ik_2\,a} + D\, e^{-ik_2\,a} \right] = ik_2\, F\, e^{ik_3\,a}$$

$$\underline{ik_2 \left[C\, e^{ik_2\,a} - D\, e^{-ik_2\,a} \right] = ik_3\, F\, e^{ik_3\,a}} \quad +$$

$$2ik_2\, C\, e^{ik_2\,a} = i\,(k_2 + k_3)\, F\, e^{ik_3\,a}$$

$$C = \frac{(k_2 + k_3)}{2k_2} F\, e^{ik_3\,a - ik_2\,a} \tag{8}$$

Terlihat C = f (F)

Kemudian pers. (8) dan (7) untuk memperoleh **D = f(F)**

$$ik_2 \left[C\, e^{ik_2\,a} - D\, e^{-ik_2\,a} \right] = ik_3\, F\, e^{ik_3\,a}$$

$$ik_2 \left[\frac{(k_2 + k_3)}{2k_2} F\, e^{ik_3\,a} - D\, e^{-ik_2\,a} \right] = ik_3\, F\, e^{ik_3\,a}$$

$$k_2 \left[\frac{(k_2 + k_3)}{2k_2} F\, e^{ik_3\,a} \right] - k_3\, F\, e^{ik_3\,a} = k_2 D\, e^{-ik_2\,a}$$

$$\frac{(k_2 + k_3)}{2} F\, e^{ik_3\,a} - k_3\, F\, e^{ik_3\,a} = k_2 D\, e^{-ik_2\,a}$$

$$F\, e^{ik_3\,a} \left[\frac{(k_2 + k_3)}{2} - k_3 \right] = k_2 D\, e^{-ik_2\,a}$$

Jadi, $$\boldsymbol{D} = \left[\frac{(k_2 + k_3)}{2k_2} - \frac{k_3}{k_2} \right] \boldsymbol{F}\, e^{ik_3\,a\,+\,ik_2\,a} \tag{9}$$

Terlihat   **D = f(F)**

Selanjutnya :

i) $\left.\begin{matrix} Pers.8 \\ Pers.9 \end{matrix}\right\} ke\ 4 \quad \rightarrow \quad$ menjadi pers. (10)

ii) $\left.\begin{matrix} Pers.8 \\ Pers.9 \end{matrix}\right\} ke\ 5 \quad \rightarrow \quad$ menjadi pers. (11)

i) $A + B = C + D$

$$A + B = \frac{(k_2+k_3)}{2k_2} F\ e^{ik_3\,a - ik_2\,a} + \left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] F\ e^{ik_3\,a\,+\,ik_2\,a} \qquad \textbf{(10)}$$

ii) $ik_1\,[A - B] = ik_2\,[C - D]$

$$ik_1\,[A - B] = ik_2\left\{\frac{(k_2+k_3)}{2k_2} F\ e^{ik_3\,a - ik_2\,a} - \left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] F\ e^{ik_3\,a\,+\,ik_2\,a}\right\} \qquad \textbf{(11)}$$

Untuk mereduksi B, kalikan pers. (10) dengan ik_1, lalu selesaikan dengan pers. (11) hingga diperoleh pers. 12 dalam bentuk B untuk A= f(F)

$$ik_1\,[A+B] = ik_1\frac{(k_2+k_3)}{2k_2}F\ e^{ik_3\,a - ik_2\,a} + ik_1\left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] F\ e^{ik_3\,a\,+\,ik_2\,a}$$

$$\underline{ik_1\,[A-B] = ik_2\left\{\frac{(k_2+k_3)}{2k_2}F\ e^{ik_3\,a - ik_2\,a} - \left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] F\ e^{ik_3\,a\,+\,ik_2\,a}\right\}} +$$

$$2\,ik_1 A = F\ e^{ik_3\,a - ik_2\,a}\left\{ik_1\frac{(k_2+k_3)}{2k_2} + ik_2\frac{(k_2+k_3)}{2k_2}\right\} +$$

$$F\ e^{ik_3\,a\,+\,ik_2\,a}\left\{ik_1\left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] - ik_2\left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right]\right\}$$

Lalu kelompokan menjadi sederhana :

$$2\,ik_1\,A = F\ e^{ik_3\,a}\left\{\left[\frac{(k_2+k_3)}{2k_2}\,(ik_1 + ik_2)\right] e^{-ik_2\,a} + \left[\left(\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right)(ik_1 - ik_2)\right] e^{ik_2\,a}\right\}$$

$$2\,ik_1\,A = F\ e^{ik_3\,a}\left\{\left[\frac{(k_2+k_3)}{2k_2}\,(k_1 + k_2)\,i\right] e^{-ik_2\,a} + \left[\left(\frac{(k_2+k_3)}{2k_2} - \frac{2k_3}{2k_2}\right)(k_1 - k_2)\,i\right] e^{ik_2\,a}\right\}$$

$$2\,\mathrm{i}k_1 A = F\,\mathrm{e}^{\mathrm{i}k_3\,a}\left\{\left[\frac{[\mathrm{i}\,k_1(k_2+k_3)+k_2(k_2+k_3)]}{2k_2}\right]\mathrm{e}^{-\mathrm{i}k_2\,a}+\right.$$

$$\left.\left[i\,k_1\left(\frac{(k_2-k_3)}{2k_2}\right)-i\,k_2\left(\frac{(k_2-k_3)}{2k_2}\right)\right]\mathrm{e}^{\mathrm{i}k_2\,a}\right\}$$

$$2\,k_1 A = F\,\mathrm{e}^{\mathrm{i}k_3\,a}\left\{\left[\frac{k_1k_2}{2k_2}+\frac{k_1k_3}{2k_2}+\frac{k_2k_2}{2k_2}+\frac{k_2k_3}{2k_2}\right]\mathrm{e}^{-\mathrm{i}k_2\,a}+\right.$$

$$\left.\left[\frac{k_1k_2}{2k_2}-\frac{k_1k_3}{2k_2}-\frac{k_2k_2}{2k_2}+\frac{k_2k_3}{2k_2}\right]\mathrm{e}^{\mathrm{i}k_2\,a}\right\}$$

$$4\,k_1k_2 A = F\,\mathrm{e}^{\mathrm{i}k_3\,a}\left\{[\,k_1k_2+k_1k_3+k_2k_2+k_2k_3]\,\mathrm{e}^{-\mathrm{i}k_2\,a}+\right.$$

$$\left.[\,k_1k_2-k_1k_3-k_2k_2+k_2k_3]\mathrm{e}^{\mathrm{i}k_2\,a}\right\}$$

$$4\,k_1k_2 A = F\,\mathrm{e}^{\mathrm{i}k_3\,a}\left\{k_1k_2\left(\mathrm{e}^{\mathrm{i}k_2\,a}+\mathrm{e}^{-\mathrm{i}k_2\,a}\right)+k_2k_3\left(\mathrm{e}^{\mathrm{i}k_2\,a}+\mathrm{e}^{-\mathrm{i}k_2\,a}\right)-\right.$$

$$\left.k_2k_2\left(\mathrm{e}^{\mathrm{i}k_2\,a}-\mathrm{e}^{-\mathrm{i}k_2\,a}\right)-k_1k_3\left(\mathrm{e}^{\mathrm{i}k_2\,a}-\mathrm{e}^{-\mathrm{i}k_2\,a}\right)\right\}$$

$$\frac{1}{T}=\left|\frac{A^2}{F^2}\right|$$

$$\frac{1}{T}=\frac{[(k_1k_2+k_2k_3)^2-(k_1k_2+k_2k_3)^2\,\sin^2 k_2 a]+(k_2k_2+k_1k_3)^2\sin^2 k_2 a}{4\,k_1{}^2 k_2{}^2}$$

$$T=\frac{4k_1k_3}{(k_1+k_3)^2+\left(k_3{}^2-k_2{}^2\right)^2\left(k_1{}^2-k_2{}^2\right)\sin^2 k_2 a}$$

$$T=\frac{k_2}{k_1}\left|\frac{A}{F}\right|^2$$

SOAL 2

Jawab :

$$[X^3, \mathrm{P}_x] = [X^2X, \mathrm{P}_x]$$

$$[A\ B, C] = [A, C]B + A[B, C]$$

Jadi, $[X^2X, \mathrm{P}_x] = [X^2, \mathrm{P}_x]X + X^2[X, \mathrm{P}_x]$

$$= [X^2, \mathrm{P}_x]X + X^2 i\hbar \qquad\qquad (1)$$

➢ $[X^2, \mathrm{P}_x]X = [XX, \mathrm{P}_x]X$

$$= \{[X, \mathrm{P}_x]X + X[X, \mathrm{P}_x]\}\, X$$

$$= \{i\hbar\, X + X\, i\hbar\,\}\, X$$

$$= X^2 i\hbar + X^2 i\hbar \qquad\qquad (2)$$

Substitusi Pers. (2) ke (1), sehingga diperoleh :

$$[X^2X, \mathrm{P}_x] = X^2 i\hbar + X^2 i\hbar + X^2 i\hbar$$

$$[X^2X, \mathrm{P}_x] = 3X^2 i\hbar$$

$$[X^2X, \mathrm{P}_x]\Psi_{(x)} = 3X^2 i\hbar$$