KUNCI JAWABAN

PENDAHULUAN FISIKA KUANTUM

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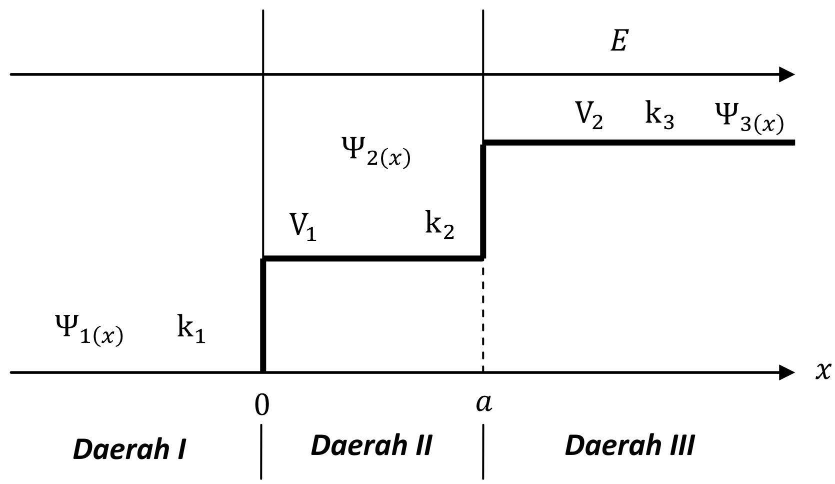
SOAL 1



Persamaan Schrodinger :

Daerah I

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{1(x)} = E\ \Psi_{1(x)}$$

$$\frac{d^2}{dx^2}\Psi_{1(x)} = -\frac{2mE}{\hbar^2}\Psi_{1(x)}$$

$$\frac{d^2}{dx^2}\Psi_{1(x)} = -k_1^2\ \Psi_{1(x)}$$

Solusinya : $\boldsymbol{\Psi_{1(x)} = A\ e^{ik_1 x} + B\ e^{-ik_1 x}}$

Dimana : $k_1^2 = \frac{2mE}{\hbar^2} \rightarrow k_1 = \frac{\sqrt{2mE}}{\hbar}$

Daerah II

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{2(x)} + V_{(x)}\Psi_{2(x)} = E\ \Psi_{2(x)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{2(x)} + V_1\Psi_{2(x)} = E\ \Psi_{2(x)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{2(x)} = (E - V_1)\Psi_{2(x)}$$

$$\frac{d^2}{dx^2}\Psi_{2(x)} = -\frac{2m}{\hbar^2}(E - V_1)\Psi_{2(x)}$$

$$\frac{d^2}{dx^2}\Psi_{2(x)} = -k_2^2\ \Psi_{2(x)}$$

Solusinya : $\boldsymbol{\Psi_{2(x)} = C\ e^{ik_2 x} + D\ e^{-ik_2 x}}$

Dimana : $k_2^2 = \frac{2m}{\hbar^2}(E - V_1) \quad \rightarrow k_2 = \frac{\sqrt{2m(E-V_1)}}{\hbar}$

Daerah III

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{3(x)} + V_{(x)}\Psi_{3(x)} = E\ \Psi_{3(x)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{3(x)} + V_2\Psi_{3(x)} = E\ \Psi_{3(x)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi_{3(x)} = (E - V_2)\Psi_{3(x)}$$

$$\frac{d^2}{dx^2}\Psi_{3(x)} = -\frac{2m}{\hbar^2}(E - V_2)\Psi_{3(x)}$$

$$\frac{d^2}{dx^2}\Psi_{3(x)} = -k_3^2\ \Psi_{3(x)}$$

$$\boldsymbol{\Psi_{3(x)} = E\ e^{-ik_3 x} + F\ e^{ik_3 x}}$$

Untuk solusi : $\Psi_{3(x)} = E\ \mathrm{e}^{-\mathrm{i}k_3\,x} + F\ \mathrm{e}^{\mathrm{i}k_3\,x}$ Karena tidak rintangan pada x = ~ sehingga berlaku : $\Psi_{3(x)} = F\ \mathrm{e}^{\mathrm{i}k_3\,x}$

Dimana : $k_3^2 = \frac{2m}{\hbar^2}(E - \mathrm{V}_2) \rightarrow \mathrm{k}_3 = \frac{\sqrt{2m(E-\mathrm{V}_2)}}{\hbar}$

Susun kembali semua solusi

$$\Psi_{1(x)} = A\ \mathrm{e}^{\mathrm{i}k_1\,x} + B\ \mathrm{e}^{-\mathrm{i}k_1\,x} \qquad (\mathbf{1})$$

$$\Psi_{2(x)} = C\ \mathrm{e}^{\mathrm{i}k_2\,x} + D\ \mathrm{e}^{-\mathrm{i}k_2\,x} \qquad (\mathbf{2})$$

$$\Psi_{3(x)} = F\ \mathrm{e}^{\mathrm{i}k_3\,x} \qquad (\mathbf{3})$$

Syarat kontinuitas :

$$\left.\begin{matrix}\Psi_1 = \Psi_2 \\ \Psi_1{}' = \Psi_2{}'\end{matrix}\right\} \text{ Pada x } = 0$$

i) $\Psi_1 = \Psi_2 \rightarrow x = 0$

$$A\ \mathrm{e}^{\mathrm{i}k_1\,x} + B\ \mathrm{e}^{-\mathrm{i}k_1\,x} = C\ \mathrm{e}^{\mathrm{i}k_2\,x} + D\ \mathrm{e}^{-\mathrm{i}k_2\,x}$$

$$A + B = C + D \qquad (\mathbf{4})$$

ii) $\Psi_1{}' = \Psi_2{}' \rightarrow x = 0$

$$\mathrm{i}k_1\left[A\ \mathrm{e}^{\mathrm{i}k_1\,x} - B\ \mathrm{e}^{-\mathrm{i}k_1\,x}\right] = \mathrm{i}k_2\left[C\ \mathrm{e}^{\mathrm{i}k_2\,x} - D\ \mathrm{e}^{-\mathrm{i}k_2\,x}\right]$$

$$\mathrm{i}k_1\,[A - B] = \mathrm{i}k_2\,[C - D\,] \qquad (\mathbf{5})$$

$$\left.\begin{matrix}\Psi_2 = \Psi_3 \\ \Psi_2{}' = \Psi_3{}'\end{matrix}\right\} \text{ Pada x } = \text{a}$$

i) $C\ \mathrm{e}^{\mathrm{i}k_2\,a} + D\ \mathrm{e}^{-\mathrm{i}k_2\,x} = F\ \mathrm{e}^{\mathrm{i}k_3\,a}$ $(\mathbf{6})$

ii) $\mathrm{i}k_2\left[C\ \mathrm{e}^{\mathrm{i}k_2\,a} - D\ \mathrm{e}^{-\mathrm{i}k_2\,a}\right] = i\mathrm{k}_3 F\ \mathrm{e}^{\mathrm{i}k_3\,a}$ $(\mathbf{7})$

Lakukan eliminasi pers. (6) dan pers. (7) guna memperoleh konstanta C dan D dalam bentuk konstanta F.

pers. (6) x ik_2 , hasilnya tambahkan dengan pers. (7) guna mereduksi D.

i

$$k_2 \left[C\, e^{ik_2\, a} + D\, e^{-ik_2\, a}\right] = ik_2\, F\, e^{ik_3\, a}$$

$$ik_2 \left[C\, e^{ik_2\, a} - D\, e^{-ik_2\, a}\right] = ik_3\, F\, e^{ik_3\, a}$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \; +$$

$$2ik_2\, C\, e^{ik_2\, a} = i\,(k_2 + k_3)\, F\, e^{ik_3\, a}$$

$$\mathbf{C} = \frac{(k_2 + k_3)}{2k_2} F\, e^{ik_3\, a - ik_2\, a} \qquad \textbf{(8)}$$

Terlihat C = f (F)

Kemudian pers. (8) dan (7) untuk memperoleh **D = f(F)**

$$ik_2 \left[C\, e^{ik_2\, a} - D\, e^{-ik_2\, a}\right] = ik_3\, F\, e^{ik_3\, a}$$

$$ik_2 \left[\frac{(k_2 + k_3)}{2k_2} F\, e^{ik_3\, a} - D\, e^{-ik_2\, a}\right] = ik_3\, F\, e^{ik_3\, a}$$

$$k_2 \left[\frac{(k_2 + k_3)}{2k_2} F\, e^{ik_3\, a}\right] - k_3\, F\, e^{ik_3\, a} = k_2 D\, e^{-ik_2\, a}$$

$$\frac{(k_2 + k_3)}{2} F\, e^{ik_3\, a} - k_3\, F\, e^{ik_3\, a} = k_2 D\, e^{-ik_2\, a}$$

$$F\, e^{ik_3\, a} \left[\frac{(k_2 + k_3)}{2} - k_3\right] = k_2 D\, e^{-ik_2\, a}$$

Jadi, $$\boldsymbol{D} = \left[\frac{(k_2 + k_3)}{2k_2} - \frac{k_3}{k_2}\right] \boldsymbol{F}\, e^{ik_3\, a + ik_2\, a} \qquad (9)$$

Terlihat **D = f(F)**

Selanjutnya :

i) $\left.\begin{matrix} Pers.8 \\ Pers.9 \end{matrix}\right\} ke\ 4$ → menjadi pers. (10)

ii) $\left.\begin{matrix} Pers.8 \\ Pers.9 \end{matrix}\right\} ke\ 5$ → menjadi pers. (11)

i) $A + B = C + D$

$$A + B = \frac{(k_2+k_3)}{2k_2} F\ e^{ik_3\, a - ik_2\, a} + \left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] F\ e^{ik_3\, a + ik_2\, a} \quad \textbf{(10)}$$

ii) $ik_1\,[A - B] = ik_2\,[C - D]$

$$ik_1\,[A - B] = ik_2 \left\{\frac{(k_2+k_3)}{2k_2} F\ e^{ik_3\, a - ik_2\, a} - \left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] F\ e^{ik_3\, a + ik_2\, a}\right\} \quad \textbf{(11)}$$

Untuk mereduksi B, kalikan pers. (10) dengan ik_1, lalu selesaikan dengan pers. (11) hingga diperoleh pers. 12 dalam bentuk B untuk A= f(F)

$$ik_1\,[A+B] = ik_1\frac{(k_2+k_3)}{2k_2} F\ e^{ik_3\, a - ik_2\, a} + ik_1\left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] F\ e^{ik_3\, a + ik_2\, a}$$

$$ik_1\,[A-B] = ik_2\left\{\frac{(k_2+k_3)}{2k_2} F\ e^{ik_3\, a - ik_2\, a} - \left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] F\ e^{ik_3\, a + ik_2\, a}\right\} \quad +$$

$$2\ ik_1 A = F\ e^{ik_3\, a - ik_2\, a}\left\{ik_1\frac{(k_2+k_3)}{2k_2} + ik_2\frac{(k_2+k_3)}{2k_2}\right\} + F\ e^{ik_3\, a + ik_2\, a}\left\{ik_1\left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right] - ik_2\left[\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right]\right\}$$

Lalu kelompokan menjadi sederhana :

$$2\ ik_1\ A = F\ e^{ik_3\, a}\left\{\left[\frac{(k_2+k_3)}{2k_2}\ (ik_1 + ik_2)\right] e^{-ik_2\, a} + \left[\left(\frac{(k_2+k_3)}{2k_2} - \frac{k_3}{k_2}\right)(ik_1 - ik_2)\right] e^{ik_2\, a}\right\}$$

$$2\ ik_1\ A = F\ e^{ik_3\, a}\left\{\left[\frac{(k_2+k_3)}{2k_2}\ (k_1 + k_2)\ i\right] e^{-ik_2\, a} + \left[\left(\frac{(k_2+k_3)}{2k_2} - \frac{2k_3}{2k_2}\right)(k_1 - k_2)\ i\right] e^{ik_2\, a}\right\}$$

$$2\ ik_1 A = F\ e^{ik_3 a}\left\{\left[\frac{[i\ k_1(k_2 + k_3\ ) + k_2(k_2 + k_3\ )]}{2k_2}\right] e^{-ik_2 a} + \left[i\ k_1\left(\frac{(k_2 - k_3\ )}{2k_2}\right) - i\ k_2\ \left(\frac{(k_2 - k_3\ )}{2k_2}\right)\right] e^{\ ik_2 a}\right\}$$

$$2\ k_1 A = F\ e^{ik_3 a}\left\{\left[\frac{k_1k_2}{2k_2} + \frac{k_1k_3}{2k_2} + \frac{k_2k_2}{2k_2} + \frac{k_2k_3}{2k_2}\right] e^{-ik_2 a} + \left[\frac{k_1k_2}{2k_2} - \frac{k_1k_3}{2k_2} - \frac{k_2k_2}{2k_2} + \frac{k_2k_3}{2k_2}\right] e^{\ ik_2 a}\right\}$$

$$4\ k_1k_2 A = F\ e^{ik_3 a}\{[\ k_1k_2 + k_1k_3 + k_2k_2 + k_2k_3]\ e^{-ik_2 a} + [\ k_1k_2 - k_1k_3 - k_2k_2 + k_2k_3] e^{\ ik_2 a}\}$$

$$4\ k_1k_2 A = F\ e^{ik_3 a}\{\ k_1k_2\left(e^{\ ik_2 a} + e^{-ik_2 a}\right) + k_2k_3\left(e^{\ ik_2 a} + e^{-ik_2 a}\right) - k_2k_2\left(e^{\ ik_2 a} - e^{-ik_2 a}\right) - k_1k_3\left(e^{\ ik_2 a} - e^{-ik_2 a}\right)\}$$

$$\frac{1}{T} = \left|\frac{A^2}{F^2}\right|$$

$$\frac{1}{T} = \frac{[(\ k_1k_2 + k_2k_3)^2 - (\ k_1k_2 + k_2k_3)^2\ \sin^2 k_2 a] + (k_2k_2 + k_1k_3)^2 \sin^2 k_2 a}{4\ {k_1}^2{k_2}^2}$$

$$T = \frac{4k_1k_3}{(\ k_1 + k_3)^2 + \left({k_3}^2 - {k_2}^2\right)^2\left({k_1}^2 - {k_2}^2\right)\sin^2 k_2 a}$$

$$\boldsymbol{T} = \frac{\mathbf{k_2}}{\mathbf{k_1}}\left|\frac{\boldsymbol{A}}{\boldsymbol{F}}\right|^{\mathbf{2}}$$

SOAL 2

Buktikan : $[X^3, \mathrm{P}_x] = 3x^2\ i\hbar$

Jawab :

$[X^3, \mathrm{P}_x] = [X^2X, \mathrm{P}_x]$

$$[A\ \ B, C] = [A, C]B + A[B, C]$$

Jadi, $[X^2X, \mathrm{P}_x] = [X^2, \mathrm{P}_x]X + X^2[X, \mathrm{P}_x]$

$$= [X^2, \mathrm{P}_x]X + X^2 i\hbar \qquad (1)$$

- $[X^2, \mathrm{P}_x]X = [XX, \mathrm{P}_x]X$

$$= \{[X, \mathrm{P}_x]X + X[X, \mathrm{P}_x]\}\ X$$

$$= \{i\hbar\ X + X\ i\hbar\ \}\ X$$

$$= X^2 i\hbar + X^2 i\hbar \qquad (2)$$

Substitusi Pers. (2) ke (1), sehingga diperoleh :

$[X^2X, \mathrm{P}_x] = X^2 i\hbar + X^2 i\hbar + X^2 i\hbar$

$[X^2X, \mathrm{P}_x] = 3X^2 i\hbar$

$[X^2X, \mathrm{P}_x]\Psi_{(\mathrm{x})} = 3X^2 i\hbar$