KUNCI JAWABAN TU

PENDAHULUAN FISIKA KUANTUM

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<u>Soal 1</u>

Pada t = 0 partikel berada dalam keadaan Ψ , yaitu :

$$\Psi_{(r,0)} = \pi^{-5/2} \tan 2x \; e^{i(4y+z)}$$

Jika energy partikel diukur pada t = 0 , tentukan energy partikel tersebut ?

Jawab :

Gunakan persamaan Schrodinger, dengan V = 0

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Psi_{(r,0)} = E\,\Psi_{(r,0)}$$

1. $\frac{\partial \Psi}{\partial x} = 2\sec^2 2x \; \pi^{-5/2} \; e^{i(4y+z)}$

   $\frac{\partial^2 \Psi}{\partial x^2} = 8\sec^2 2x \; \tan 2x \; \pi^{-5/2} \; e^{i(4y+z)}$

2. $\frac{\partial \Psi}{\partial y} = 4i \; e^{i(4y+z)} \; \pi^{-5/2} \; \tan 2x$

   $\frac{\partial^2 \Psi}{\partial z^2} = -16 \; e^{i(4y+z)} \; \pi^{-5/2} \; \tan 2x$

   $\frac{\partial \Psi}{\partial z} = i \; e^{i(4y+z)} \; \pi^{-5/2} \; \tan 2x$

   $\frac{\partial^2 \Psi}{\partial z^2} = - \; e^{i(4y+z)} \; \pi^{-5/2} \; \tan 2x$

$$-\frac{\hbar^2}{2m}\Big[8\sec^2 2x \; \tan 2x \; \pi^{-5/2} \; e^{i(4y+z)} - 16 \; e^{i(4y+z)} \; \pi^{-5/2} \; \tan 2x - e^{i(4y+z)} \; \pi^{-5/2} \; \tan 2x\Big] = E\,\Psi_{(r,0)}$$

$$-\frac{\hbar^2}{2m}[8\sec^2 2x - 17]\Psi = E\,\Psi$$

Jadi,

$$\mathbf{E} = \frac{\hbar^2}{2m}[17 - 8\sec^2 2x]$$

SOAL 2

i*) Menentukan masing-masing konstanta A, B, dan C. Gunakan syarat Normalisasi.

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} \Psi^*{}_{1(x)}\,\Psi_{(x)}\,dx = 1 \quad ; \text{ dengan } \Psi_{1(x)} = A\cos\frac{\pi x}{L}$$

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} A\cos\left(\frac{\pi x}{L}\right)\, A\cos\left(\frac{\pi x}{L}\right) dx = 1$$

$$A^2\int_{-\frac{L}{2}}^{\frac{L}{2}} \cos^2\left(\frac{\pi x}{L}\right)\, dx = 1 \quad ; \cos 2A = \cos^2 A - 1$$

$$A^2\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{2}\left[\cos\frac{2\pi x}{L} + 1\right] dx = 1 \quad ; \cos 2A = \cos^2 A - 1$$

$$\frac{A^2}{2}\left\{\int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\cos\frac{2\pi x}{L} + 1\right] dx + \int_{-\frac{L}{2}}^{\frac{L}{2}} dx\right\} = 1$$

$$\frac{A^2}{2}\left\{\left[\frac{L}{2\pi}\sin\frac{2\pi x}{L}\right]_{-\frac{L}{2}}^{\frac{L}{2}} + [x]_{-\frac{L}{2}}^{\frac{L}{2}}\right\} = 1 \quad ; \sin\frac{2\pi x}{L} = 0$$

$$\frac{A^2}{2}L = 1 \quad \rightarrow \quad A = \sqrt{\frac{2}{L}}$$

Jadi,

$$\boldsymbol{\Psi_{1(x)} = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)}$$

i**) $\Psi_{2(x)} = B\sin\frac{2\pi x}{L}$

Gunakan syarat Normalisasi

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} \Psi^*{}_{2(x)}\, \Psi_{2(x)}\, dx = 1$$

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} B \sin\left(\frac{2\pi x}{L}\right)\, B \sin\left(\frac{2\pi x}{L}\right)\, dx = 1$$

$$B^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} \sin^2\left(\frac{2\pi x}{L}\right)\, dx = 1$$

$$B^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{2}\left(1 - \cos\left(\frac{4\pi x}{L}\right)\right)\, dx = 1$$

$$\frac{B^2}{2}\left\{[x]_{-\frac{L}{2}}^{\frac{L}{2}} - \left[\frac{L}{4\pi}\sin\left(\frac{4\pi x}{L}\right)\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right\} = 1 \quad ;\ \sin\left(\frac{4\pi x}{L}\right) = 0$$

$$\frac{B^2}{2} L = 1 \quad \rightarrow \quad B = \sqrt{\frac{2}{L}}$$

Jadi,

$$\boldsymbol{\Psi_{2(x)} = \sqrt{\frac{2}{L}}\, \sin\left(\frac{2\pi x}{L}\right)}$$

i***) $\Psi_{3(x)} = C\ \cos\left(\frac{3\pi x}{L}\right)$

Gunakan syarat Normalisasi. Dengan cara yang sama, maka diperoleh harga :

$$C = \sqrt{\frac{2}{L}}$$

Jadi, $$\boldsymbol{\Psi_{3(x)} = \sqrt{\frac{2}{L}}\, \cos\left(\frac{3\pi x}{L}\right)}$$

ii) Menentukan energy dari $\Psi_{1(x)}, \Psi_{2(x)},$ dan $\Psi_{3(x)}$

a. $E_1 = \int_{-\frac{L}{2}}^{\frac{L}{2}} \Psi^*{}_{1(x)} \left[\frac{(i\hbar)^2}{2m}\frac{\partial^2}{\partial x^2}\right] \Psi_{1(x)}\, dx$

$$E_1 = -\frac{\hbar^2}{2m}\int_{-\frac{L}{2}}^{\frac{L}{2}} A\cos\left(\frac{\pi x}{L}\right) \frac{\partial^2}{\partial x^2}\left[A\cos\frac{\pi x}{L}\right] dx$$

$$= -\frac{\hbar^2}{2m} A^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} \cos\left(\frac{\pi x}{L}\right) \left[-\frac{\pi^2}{L^2}\cos\frac{\pi x}{L}\right] dx$$

$$= \frac{\hbar^2}{2m} A^2 \frac{\pi^2}{L^2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \cos^2\left(\frac{\pi x}{L}\right) dx$$

$$= \frac{\hbar^2}{2m}\frac{A^2}{2}\frac{\pi^2}{L^2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \left[1+\cos\frac{2\pi x}{L}\right] dx$$

$$= \frac{\hbar^2}{2m}\frac{A^2}{2}\frac{\pi^2}{L^2}\left\{[x]_{-\frac{L}{2}}^{\frac{L}{2}} + \left[\frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right\} \quad ;\ \sin\left(\frac{2\pi x}{L}\right) = 0$$

$$= \frac{\hbar^2}{2m}\frac{A^2}{2}\frac{\pi^2}{L^2}(L) \quad ; \quad A = \sqrt{\frac{2}{L}}$$

Jadi,

$$\mathbf{E_1 = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}}$$

b. $E_2 = \int_{-\frac{L}{2}}^{\frac{L}{2}} \Psi^*{}_{2(x)} \left[\frac{(i\hbar)^2}{2m}\frac{\partial^2}{\partial x^2}\right] \Psi_{2(x)}\, dx$

$$= -\frac{\hbar^2}{2m}\int_{-\frac{L}{2}}^{\frac{L}{2}} B\sin\left(\frac{2\pi x}{L}\right) \frac{\partial^2}{\partial x^2}\left[B\sin\left(\frac{2\pi x}{L}\right)\right] dx$$

$$= -\frac{\hbar^2}{2m} B^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} \sin\left(\frac{2\pi x}{L}\right) \left[-\frac{4\pi^2}{L^2}\sin\left(\frac{2\pi x}{L}\right)\right] dx$$

$$= \frac{\hbar^2}{2m} B^2 \frac{4\pi^2}{L^2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sin^2\left(\frac{\pi x}{L}\right) dx$$

$$= \frac{\hbar^2}{2m}\frac{B^2}{2}\frac{4\pi^2}{L^2}\int_{-\frac{L}{2}}^{\frac{L}{2}} \left[1-\cos\frac{4\pi x}{L}\right] dx$$

$$= \frac{\hbar^2}{2m}\frac{B^2}{2}\frac{4\pi^2}{L^2}\left\{[x]_{-\frac{L}{2}}^{\frac{L}{2}} - \left[\frac{L}{4\pi}\sin\left(\frac{4\pi x}{L}\right)\right]_{-\frac{L}{2}}^{\frac{L}{2}}\right\} \quad ;\ \sin\left(\frac{4\pi x}{L}\right) = 0$$

$$= \frac{\hbar^2}{2m}\frac{B^2}{2}\frac{4\pi^2}{L^2}\ (L)\ \ ;\ \ B = \sqrt{\frac{2}{L}}$$

Jadi,

$$\mathbf{E_2 = \frac{\hbar^2}{2m}\frac{4\pi^2}{L^2}}$$

Dengan cara yang sama, maka diperoleh harga energy untuk $\Psi_{3(x)}$

$$\mathbf{E_3 = 9\frac{\hbar^2}{2m}\frac{\pi^2}{L^2}}$$

Perbandingan $E_2 = 4\ E_1$ dan $E_3 = 9\ E_1$. Untuk sketnya lihat lampiran

SOAL 3

$$\Psi_{(x,y)} = N\left(7\Psi_{1(x)}e^{-\,i\,\mathbf{E_1}t/\hbar} + 6\Psi_{2(x)}\,e^{-\,i\,\mathbf{E_2}t/\hbar} + 5\Psi_{3(x)}\,e^{-\,i\,\mathbf{E_3}t/\hbar}\right)$$

a) Menentukan konstanta Normalisasi

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} \Psi^*_{(x,y)}\,\Psi_{(x,y)}\,dx = 1$$

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} N\left(7\Psi_{1(x)}e^{\,i\,\mathbf{E_1}t/\hbar} + 6\Psi_{2(x)}\,e^{\,i\,\mathbf{E_2}t/\hbar} + 5\Psi_{3(x)}\,e^{\,i\,\mathbf{E_3}t/\hbar}\right).N\left(7\Psi_{1(x)}e^{-\,i\,\mathbf{E_1}t/\hbar} + 6\Psi_{2(x)}\,e^{-\,i\,\mathbf{E_2}t/\hbar} + 5\,\Psi_{3(x)}\,e^{-\,i\,\mathbf{E_3}t/\hbar}\right)\,dx = 1$$

Gunakan : $\int \Psi^*\,\Psi = \delta_{ij}$

$i = j \ \rightarrow\ \delta_{ij} = 1$

$i \neq j \ \rightarrow\ \delta_{ij} = 0$

maka :

$$N^2\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(49\Psi^2_{1(x)} + 36\,\Psi^2_{2(x)} + 25\,\Psi^2_{3(x)}\right)\,dx = 1$$

karena $\int_{-\frac{L}{2}}^{\frac{L}{2}} \Psi_{1(x)}^2\, dx = 1$ maka : $N^2[49 + 36 + 25] = 1$

$$\mathbf{N = \sqrt{\frac{1}{110}}}$$

b) Menentukan ketidakpastian energy ΔE

$$\Delta E = \sqrt{\langle E^2\rangle - \langle E\rangle^2}$$

$$\langle E\rangle = \int \Psi^*{}_{(x,t)}\, i\hbar \frac{\partial}{\partial t} \Psi_{(x,t)}\, dx$$

$$\langle E\rangle = N^2 \int\Big\{\left[7\Psi_{1(x)} e^{\,i E_1 t/\hbar} + 6\Psi_{2(x)}\, e^{\,i E_2 t/\hbar} + 5\,\Psi_{3(x)}\, e^{\,i E_3 t/\hbar}\right] .$$
$$\Big[(i\hbar)7\Psi_{1(x)}\left(-\frac{i E_1}{\hbar}\right) e^{-\,i E_1 t/\hbar} + (i\hbar)\, 6\Psi_{2(x)}\left(-\frac{i E_2}{\hbar}\right)\, e^{-\,i E_2 t/\hbar} +$$
$$(i\hbar)\, 5\,\Psi_{3(x)}\left(-\frac{i E_3}{\hbar}\right)\, e^{-\,i E_3 t/\hbar}\Big]\Big\}\, dx$$

$$= \frac{1}{110}\left\{49\ E_1 \int \left|\Psi_{1(x)}\right|^2 dx + 36\ E_2 \int \left|\Psi_{2(x)}\right|^2 dx + 25\ E_3 \int \left|\Psi_{3(x)}\right|^2 dx\right\}$$

$$= \frac{1}{110}\,[49\ E_1 + 36\ E_2 + 25\ E_3]$$

$$= \frac{1}{110}\,[49\ E_1 + 36.\ 4E_1 + 25.\ 9E_1]$$

$$\langle \mathbf{E}\rangle = \mathbf{3,8\ E_1}$$

$$\langle E^2\rangle = \int \Psi^*{}_{(x,t)}\left(i\hbar \frac{\partial}{\partial t}\right)^2 \Psi_{(x,t)}\, dx$$

$$= \frac{1}{110}\int\Big\{\left[7\Psi_{1(x)} e^{\,i E_1 t/\hbar} + 6\Psi_{2(x)}\, e^{\,i E_2 t/\hbar} + 5\,\Psi_{3(x)}\, e^{\,i E_3 t/\hbar}\right] .$$
$$\Big[(i\hbar)^2 7\Psi_{1(x)}\left(-\frac{i E_1}{\hbar}\right)^2 e^{-\,i E_1 t/\hbar} + (i\hbar)^2\, 6\Psi_{2(x)}\left(-\frac{i E_2}{\hbar}\right)^2\, e^{-\,i E_2 t/\hbar} +$$
$$(i\hbar)^2\, 5\,\Psi_{3(x)}\left(-\frac{i E_3}{\hbar}\right)^2\, e^{-\,i E_3 t/\hbar}\Big]\Big\}\, dx$$

$$= \frac{1}{110}\left\{49\ {E_1}^2 \int \left|\Psi_{1(x)}\right|^2 dx + 36\ {E_2}^2 \int \left|\Psi_{2(x)}\right|^2 dx + 25\ {E_3}^2 \int \left|\Psi_{3(x)}\right|^2 dx\right\}$$

$$= \frac{1}{110}\,[49\ {E_1}^2 + 36.16\ {E_1}^2 + 25.81\ {E_1}^2]$$

$$= \frac{1}{110}(2650\ E_1{}^2)$$

$$\langle E^2 \rangle = 24{,}09\ E_1{}^2$$

Jadi,

$$\Delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$$

$$= \sqrt{24{,}09\ E_1{}^2 - (3{,}8\ \ E_1)^2}$$

$$\boldsymbol{\Delta E = 3,106\ \ E_1}$$

c) Kesimpulan : fungsi gelombang yang direpresentasikan oleh berbagai kombinasi linier dari $\Psi_{1(x)}, \Psi_{2(x)},$ dan $\Psi_{3(x)}$ dapat dinyatakan tidak stasioner / karena $\Delta E \neq 0$