KUNCI JAWABAN TU II

PENDAHULUAN FISIKA KUANTUM

Dosen : Yuyu Rachmat

SOAL 1

Jika fungsi radial dari atom hydrogen diketahui seperti :

$$R_{31} = \frac{4}{81\sqrt{6}\ a_0{}^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0}\, e^{-\frac{r}{3a_0}}$$

Tentukkan : harga ekspetasi <r> dari solusi radial tersebut serta dapatkan pula rapat peluangnya !

SOAL 2

Solusi total atom hydrogen dinyatakan dengan persamaan :

$$\Psi_{200}(\vec{r}, \theta, \varphi) = \frac{4}{4\sqrt{2\pi}\, a_0{}^{3/2}}\left(2 - \frac{r}{a_0}\right) e^{-\frac{r}{2a_0}}$$

Dapatkan energy kinetic rata-rata sebuah electron untuk keadaan tersebut !

SOAL 3

Jika fungsi gelombang $\Psi_{(r)}$, dinyatakan dengan :

$$\Psi_{(r)} = \frac{1}{5} + \left\{3\ \Psi_{200\,(\vec{r})} + 2\Psi_{100\,(\vec{r})} + \sqrt{5}\ \Psi_{21,-1\,(\vec{r})} - \Psi_{210\,(\vec{r})}\right\}$$

Dengan energy bergantung pada bilangan kuantum, yaitu : $E_n = \frac{E_n}{n^2}$, n : bilangan kuantum.

Tentukan :

i) harga ekspetasi energy <E>

ii) harga ekspetasi L^2

iii) harga ekspetasi L_z

SOAL 4

Jika suatu system terdiri dari lima electron. Jika electron pertama dan kedua menempati sub orbital s, electron ketiga dan keempat menempati sub orbital p, dan electron kelima menempati sub orbital d.

Tentukanlah :

i) bilangan kuantum orbital (l) yang diizinkan

ii) jumlah keadaan eigen bersama

iii) menulis seluruh fungsi eigen bersama

SOLUSI

SOAL 1

Karena solusinya keadaan stasioner : $\Psi_{nlm} = R_{nl}\Psi_{lm}\, e^{-iE_{nl}\, t/\hbar}$

Mengingat syarat Normalisasi : $\Psi_{nlm} \int |\Psi_{nlm}|^2 d\tau = 1$

maka: $1 = \int_0^{\sim} dr\, r^2 \left|R_{nl(r)}\right|^2 \Psi_{nlm} \int d\Omega\ \ |\Psi_{nlm}|^2$

Dimana : $d\tau = r^2 dr\ \sin\theta\, d\theta\, d\varphi$

$d\tau = r^2 dr\ d\Omega$

$$1 = \int_0^{\sim} dr\, r^2 \left|R_{nl(r)}\right|^2 \Psi_{nlm} \int d\Omega\ \ |\Psi_{nlm}|^2 \quad ;\ |\Psi_{nlm}|^2 = 1$$

$$1 = \int_0^{\sim} \left|R_{nl(r)}\right|^2 r^2 dr$$

$$1 = \int_0^{\sim} P_{nl(r)} \quad ;\ dimana\ P_{nl(r)}\ berupa\ peluang$$

Sedangkan ekspetasi <r> :

$$\langle r\rangle = \int_0^{\sim} r\, P_{nl(r)}\ dr$$

$$\langle r\rangle = \int_0^{\sim} r\,.r^2 \left|R_{nl(r)}\right|^2 dr$$

$$\langle r\rangle = \int_0^{\sim} r^3 \left|R_{nl(r)}\right|^2 dr$$

Diketahui : $R_{31} = \frac{4}{81\sqrt{6}\ a_0{}^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0} e^{-\frac{r}{3a_0}}$ maka :

$$\langle r\rangle = \int_0^{\sim} r^3 \left|\frac{4}{81\sqrt{6}\ a_0{}^{3/2}}\left(6\frac{r}{a_0} e^{-\frac{r}{3a_0}} - \frac{r^2}{a_0{}^2}\right)\right|^2 dr$$

$$= \int_0^{\sim} r^3\ \frac{16}{81^2\ 6\ a_0{}^3}\left(36\frac{r^2}{a_0{}^2} e^{-\frac{2r}{3a_0}} - 12\frac{r^3}{a_0{}^3} e^{-\frac{2r}{3a_0}} + \frac{r^4}{a_0{}^4} e^{-\frac{2r}{3a_0}}\right) dr$$

$$= \frac{16\,.36}{81^2\ 6\ a_0{}^5}\int_0^{\sim} r^5 e^{-\frac{2r}{3a_0}}\ dr - \frac{16\ .\ 12}{81^2\ 6\ a_0{}^6}\int_0^{\sim} r^6 e^{-\frac{2r}{3a_0}}\ dr + \frac{16}{81^2\ 6\ a_0{}^7}\int_0^{\sim} r^7 e^{-\frac{2r}{3a_0}}\ dr$$

$$\boldsymbol{\langle r\rangle = 2\ a_0}$$

SOAL 2

$$\langle E_k\rangle = \int_{-\infty}^{\infty} \Psi_{200}\left(\frac{p^2}{2m}\right)\Psi_{200}\, dV$$

Dengan $\quad \Psi_{200} = \frac{4}{4\sqrt{2\pi}\, a_0{}^{3/2}}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}$

$$\langle E_k\rangle = \frac{1}{2m}\int_{-\infty}^{\infty} \Psi_{200}\left(-i\hbar\nabla^2\right)\Psi_{200}\, dV$$

$$= -\frac{\hbar^2}{2m}\int_{-\infty}^{\infty} \frac{1}{32\pi\, a_0{}^3}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}\left\{\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)\right\}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}\, 4\pi r^2$$

$$= -\frac{\hbar^2}{16m\, a_0{}^3}\int_{-\infty}^{\infty}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}\left\{\frac{d}{dr^2}+\frac{2}{r}\frac{d}{dr}\right\}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}\; r^2 dr$$

$$= -\frac{\hbar^2}{16m\, a_0{}^3}\int_{-\infty}^{\infty}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}\left\{\frac{d}{dr^2}+\frac{2}{r}\frac{d}{dr}\right\}\left(2e^{-\frac{r}{2a_0}}-\frac{r}{a_0}e^{-\frac{r}{2a_0}}\right)\; r^2 dr \;\; \ldots.(1)$$

Masing-masing turunkan, sehingga :

1. $\frac{d^2}{dr^2}\left(2e^{-\frac{r}{2a_0}}\right) = \frac{1}{2a_0{}^2}e^{-\frac{r}{2a_0}}$
2. $\frac{d^2}{dr^2}\left(-\frac{r}{a_0}e^{-\frac{r}{2a_0}}\right) = -\frac{1}{a_0}e^{-\frac{r}{2a_0}} + \frac{r}{2a_0{}^2}e^{-\frac{r}{2a_0}}$
3. $\frac{2}{r}\frac{d}{dr}\left(-\frac{r}{a_0}e^{-\frac{r}{2a_0}}\right) = -\frac{2}{ra_0}e^{-\frac{r}{2a_0}} + \frac{1}{a_0{}^2}e^{-\frac{r}{2a_0}}$
4. $\frac{2}{r}\frac{d}{dr}\left(2e^{-\frac{r}{2a_0}}\right) = -\frac{2}{ra_0}e^{-\frac{r}{2a_0}}$

Kelompokkan hasil turunan tersebut, sehingga :

$$\frac{3}{2a_0{}^2}e^{-\frac{r}{2a_0}} - \frac{4}{ra_0}e^{-\frac{r}{2a_0}} + \frac{r}{2a_0{}^2}e^{-\frac{r}{2a_0}} - \frac{1}{a_0}e^{-\frac{r}{2a_0}} \quad \ldots..(2)$$

Kemudian pers. (2) substitusikan ke pers. (1), sehingga :

$$= -\frac{2\hbar^2}{16m\, a_0{}^3}\int_0^{\infty}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}\left[\frac{3}{2a_0{}^2}e^{-\frac{r}{2a_0}} - \frac{4}{ra_0}e^{-\frac{r}{2a_0}} + \frac{r}{2a_0{}^2}e^{-\frac{r}{2a_0}} - \frac{1}{a_0}e^{-\frac{r}{2a_0}}\right] r^2 dr$$

$$= -\frac{2\hbar^2}{16m\,a_0{}^3}\Big[\int_0^\infty \frac{3}{a_0{}^2} e^{-\frac{r}{a_0}}\ r^2 dr - \int_0^\infty \frac{8}{a_0} e^{-\frac{r}{a_0}}\ r dr + \int_0^\infty \frac{1}{a_0{}^2} e^{-\frac{r}{a_0}}\ r^3 dr -$$
$$\int_0^\infty \frac{2}{a_0} e^{-\frac{r}{a_0}}\ r^2 dr - \int_0^\infty \frac{3}{2a_0{}^3} e^{-\frac{r}{a_0}}\ r^3 dr + \int_0^\infty \frac{4}{a_0{}^2} e^{-\frac{r}{a_0}}\ r^2 dr -$$
$$\int_0^\infty \frac{1}{2a_0{}^3} e^{-\frac{r}{a_0}}\ r^4 dr + \int_0^\infty \frac{1}{a_0{}^2} e^{-\frac{r}{a_0}}\ r^3 dr\Big]$$

$$= -\frac{6\hbar^2}{16m\,a_0{}^5}\int_0^\infty e^{-\frac{r}{a_0}}\ r^2 dr \quad \ldots.(a)$$
$$= \frac{\hbar^2}{m a_0{}^4}\int_0^\infty e^{-\frac{r}{a_0}}\ r dr \quad \ldots.(b)$$
$$= -\frac{2\hbar^2}{16m\,a_0{}^5}\int_0^\infty e^{-\frac{r}{a_0}}\ r^3 dr \quad \ldots.(c)$$
$$= \frac{4\hbar^2}{16m a_0{}^4}\int_0^\infty e^{-\frac{r}{a_0}}\ r^2 dr \quad \ldots.(d)$$
$$= \frac{3\hbar^2}{16m\,a_0{}^6}\int_0^\infty e^{-\frac{r}{a_0}}\ r^3 dr \quad \ldots.(e)$$
$$= -\frac{8\hbar^2}{16m\,a_0{}^5}\int_0^\infty e^{-\frac{r}{a_0}}\ r^2 dr \quad \ldots.(f)$$
$$= \frac{\hbar^2}{16m\,a_0{}^6}\int_0^\infty e^{-\frac{r}{a_0}}\ r^4 dr \quad \ldots.(g)$$
$$= -\frac{2\hbar^2}{16m\,a_0{}^6}\int_0^\infty e^{-\frac{r}{a_0}}\ r^3 dr \quad \ldots.(h)$$

Pers. (3)

Selesaikan masing-masing integral, sehingga :

a. $$-\frac{6\hbar^2}{16m\,a_0{}^5}\int_0^\infty e^{-\frac{r}{a_0}}\ r^2 dr = -\frac{6\hbar^2}{16m\,a_0{}^5}\frac{\Gamma(3)}{\left(\frac{1}{a_0}\right)^3}$$
$$= -\frac{6\hbar^2}{16m\,a_0{}^5}\frac{2}{\frac{1}{a_0{}^3}}$$
$$= -\frac{12\,\hbar^2}{16m\,a_0{}^2}$$

b. $$\frac{\hbar^2}{m a_0{}^4}\int_0^\infty e^{-\frac{r}{a_0}}\ r dr = \frac{\hbar^2}{m a_0{}^4}\frac{\Gamma(2)}{\left(\frac{1}{a_0}\right)^2}$$
$$= \frac{\hbar^2}{m a_0{}^4}\frac{1}{\frac{1}{a_0{}^2}}$$
$$= \frac{\hbar^2}{m a_0{}^2}$$

c. $$-\frac{2\hbar^2}{16m\,a_0{}^5}\int_0^\infty e^{-\frac{r}{a_0}}\ r^3 dr = -\frac{2\hbar^2}{16m\,a_0{}^5}\frac{\Gamma(4)}{\left(\frac{1}{a_0}\right)^4}$$
$$= -\frac{6\,\hbar^2}{8\,m a_0}$$

d. $\frac{4\hbar^2}{16ma_0{}^4}\int_0^\infty e^{-\frac{r}{a_0}}\ r^2dr \ = \frac{4\hbar^2}{16ma_0{}^4}\frac{\Gamma(3)}{\left(\frac{1}{a_0}\right)^3}$

$$= \frac{8\,\hbar^2}{16ma_0}$$

e. $\frac{3\hbar^2}{16ma_0{}^6}\int_0^\infty e^{-\frac{r}{a_0}}\ r^3dr \ = \frac{3\hbar^2}{16ma_0{}^6}\frac{\Gamma(4)}{\left(\frac{1}{a_0}\right)^4}$

$$= \frac{36\hbar^2}{16ma_0{}^2}$$

f. $-\frac{8\hbar^2}{16m\,a_0{}^5}\int_0^\infty e^{-\frac{r}{a_0}}\ r^2dr \ = -\frac{8\hbar^2}{16m\,a_0{}^5}\frac{\Gamma(3)}{\left(\frac{1}{a_0}\right)^3}$

$$= -\frac{\hbar^2}{ma_0{}^2}$$

g. $\frac{\hbar^2}{16ma_0{}^6}\int_0^\infty e^{-\frac{r}{a_0}}\ r^4dr \ = \frac{\hbar^2}{16ma_0{}^6}\frac{\Gamma(5)}{\left(\frac{1}{a_0}\right)^5}$

$$= \frac{24\hbar^2}{16ma_0}$$

h. $-\frac{2\hbar^2}{16m\,a_0{}^6}\int_0^\infty e^{-\frac{r}{a_0}}\ r^3dr \ = -\frac{2\hbar^2}{16m\,a_0{}^6}\frac{\Gamma(4)}{\left(\frac{1}{a_0}\right)^4}$

$$= -\frac{6\,\hbar^2}{8\,ma_0{}^2}$$

Kelompokkan dan sederhanakan, sehingga :

$$\langle E_k\rangle = -\frac{12\hbar^2}{16ma_0{}^2} + \frac{\hbar^2}{ma_0{}^2} + \frac{36\hbar^2}{16m\,a_0{}^2} - \frac{\hbar^2}{ma_0{}^2} - \frac{6\,\hbar^2}{8\,ma_0{}^2} - \frac{6\,\hbar^2}{8\,ma_0} + \frac{8\,\hbar^2}{16ma_0} + \frac{24\,\hbar^2}{16ma_0}$$

$$\langle E_k\rangle = \frac{\hbar^2}{ma_0}\left[\frac{24}{16} + \frac{1}{2} - \frac{6}{8}\right] + \frac{\hbar^2}{ma_0{}^2}\left[\frac{36}{32} + 1 - 1 - \frac{12}{16} - \frac{6}{8}\right]$$

$$= \frac{\hbar^2}{ma_0}\left(\frac{1}{4}\right) + \frac{\hbar^2}{ma_0{}^2}\left(-\frac{3}{8}\right)$$

$$\langle E_k\rangle = \frac{1}{4}\frac{\hbar^2}{ma_0} - \frac{3}{8}\frac{\hbar^2}{ma_0{}^2}$$

SOAL 3

a. Harga ekspetasi energy $\langle E \rangle$

Gunakan persamaan harga eigen

$$\hat{H}\Psi_{nlm(\vec{r})} = E_n \Psi_{nlm(\vec{r})}$$; dengan sifat "orthonormalitas"

$$\langle E \rangle = \int \Psi^* \mathrm{H}\Psi \, \mathrm{dV}$$

$$= \int \Psi^* \mathrm{H} \, \frac{1}{5}\left\{3\Psi_{200(\vec{r})} + 2\Psi_{100(\vec{r})} + \sqrt{5}\Psi_{21,-1(\vec{r})} - \Psi_{210(\vec{r})}\right\} \mathrm{dV}$$

$$= \int \Psi^* \, \frac{1}{5}\left\{3E_2\Psi_{200(\vec{r})} + 2E_1\Psi_{100(\vec{r})} + \sqrt{5}E_2\Psi_{21,-1(\vec{r})} - E_2\Psi_{210(\vec{r})}\right\} \mathrm{dV}$$

$$= \frac{1}{25}\int\left\{9\Psi^*{}_{200}E_2\Psi_{200} + 4\Psi^*{}_{100}E_1\Psi_{100} + 5\Psi^*_{21,-1}E_2\,\Psi_{21,-1} + \Psi^*_{210}E_2\Psi_{210}\right\} \mathrm{dV}$$

$$= \frac{1}{25}\{9\langle E_2\rangle + 4\langle E_1\rangle + 5\langle E_2\rangle + \langle E_2\rangle\}$$

$$= \frac{1}{25}\{4\langle E_1\rangle + 15\langle E_2\rangle\}$$

$$= \frac{1}{25}\left\{4\langle E_1\rangle + \frac{15}{4}\langle E_1\rangle\right\}$$

$$\langle \boldsymbol{E} \rangle = \frac{\mathbf{31}}{\mathbf{100}}\langle \boldsymbol{E_1} \rangle$$

Catatan : $E_n = -\dfrac{m_e \; e^4}{32\,\pi\,\varepsilon_o^2\hbar^2}\dfrac{1}{n^2}$

$$E_2 = \frac{E_1}{4}$$

b. $\langle L^2 \rangle = ?$

Dengan menggunakan : $L^2\Psi_{lm} = l\,(l+1)\hbar^2\Psi_{lm}$

i) $L^2\Psi_{200} = 0$

ii) $L^2\Psi_{100} = 0$

iii) $L^2\Psi_{21,-1} = 2\hbar^2\Psi_{21,-1(\vec{r})}$

iv) $L^2\Psi_{210} = 2\hbar^2\Psi_{210(\vec{r})}$

$$\langle L^2\rangle = \int \Psi^* L^2 \Psi \, dV$$

$$= \int \Psi^*_{(\vec{r})} \frac{\hbar^2}{5}\left\{0 + 0 + \sqrt{5}\, 2\Psi_{21,-1(\vec{r})} - 2\Psi_{210\,(\vec{r})}\right\} \, dV$$

$$= \frac{\hbar^2}{25}\int\left\{0 + 0 + 5\,\Psi^*_{21,-1}\, 2\Psi_{21,-1} + 2\Psi^*_{210}\,\Psi_{210\,(\vec{r})}\right\} \, dV$$

$$\langle \boldsymbol{L}^2\rangle = \frac{\mathbf{12}\hbar^2}{\mathbf{25}}$$

c. $\langle L_z\rangle = ?$

Dengan menggunakan $L_z\Psi_{nlm(\theta,\varphi)} = m\hbar\Psi_{lm(\theta,\varphi)}$

i) $L_z\Psi_{200(\vec{r})} = 0$

ii) $L_z\Psi_{100(\vec{r})} = 0$

iii) $L_z\Psi_{21,-1(\vec{r})} = -\hbar^2\Psi_{21,-1(\vec{r})}$

iv) $L_z\Psi_{210(\vec{r})} = 0$

$$\langle L_z\rangle = \int \Psi^* L_z \Psi \, dV$$

$$= \frac{\hbar}{5}\int \Psi^*\left\{0 + 0 - \sqrt{5}\,\Psi_{21,-1(\vec{r})} + 0\right\} dV$$

$$= \frac{\hbar}{25}\int\left\{5\,\Psi_{21,-1}\,.\,\Psi_{21,-1}\right\} dV$$

$$\langle L_z\rangle = \frac{5\,\hbar}{25}$$

$$\langle \boldsymbol{L_z}\rangle = \frac{\hbar}{\mathbf{5}}$$

SOAL 4

a. Menentukkan l yang diizinkan

Diketahui : $l_1 = 0$; $l_2 = 0$; $l_3 = 1$; $l_4 = 1$; $l_5 = 2$

Dengan cara system penjumlahan dua electron

$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \vec{L}_4 + \vec{L}_5$

Atau ditulis :

$l = l_1 + l_2 + l_3 + l_4 + l_5$

- $l = l^{'} + l_3 + l_4 + l_5$ dengan $l^{'} = l_1 + l_2 \rightarrow l^{'} = |l_1 + l_2|, \ldots, |l_1 - l_2|$

  sehingga $l^{'} = 0$

- $l = l^{''} + l_4 + l_5$ dengan $l^{''} = l^{'} + l_3 \rightarrow l^{''} = |l^{'} + l_3|, \ldots, |l^{'} - l_3|$

  Sehingga $l^{''} = 1$

- $l = l^{'''} + l_5$ dengan $l^{'''} = l^{''} + l_4 \rightarrow l^{'''} = |l^{''} + l_4|, \ldots, |l^{''} - l_4|$

  sehingga $l^{'''} = 2, 1, 0$

Maka untuk :

$l^{'''} = 2 \rightarrow l = |l^{'''} + l_5|, \ldots, |l^{'''} - l_5|$ sehingga $l = 4, 3, 2, 1, 0$

$l^{'''} = 1 \rightarrow l = |l^{'''} + l_5|, \ldots, |l^{'''} - l_5|$ sehingga $l = 3, 2, 1, 0$

$l^{'''} = 0 \rightarrow l = |l^{'''} + l_5|, \ldots, |l^{'''} - l_5|$ sehingga $l = 2, 1$

Cara langsung dengan penjumlahan biasa untuk menentukkan $l,\ N = 5$

$\Lambda = \sum_{i=1}^{N-1} l_i = l_1 + l_2 + l_3 + l_4$

$\Lambda = 0 + 0 + 1 + 1 \rightarrow \Lambda = 2$

Dengan $L_N = L_5 = 2$

Karena $(L_5 - \Lambda) > 0 \rightarrow l^{min} = (L_N - \Lambda)$

Sehingga $l^{min} = 2 - 2 \rightarrow l^{min} = 0$

$$l^{max} = \sum_{i=1}^{N-1} l_i = l_1 + l_2 + l_3 + l_4$$

$l^{max} = 0 + 0 + 1 + 1 + 2 \quad \rightarrow \quad l^{max} = 4$

Jadi, $l = |l^{max}|, |l^{max} - 1|, \ldots, l^{min}$

$l = 4, 3, 2, 1, 0$

b. Untuk menentukan jumlah eigen bersama

$N = (2l_1 + 1)(2l_2 + 1)(2\,l_3 + 1)(2l_4 + 1)(2l_5 + 1)$

$= (2\,.0 + 1)(2\,.0 + 1)(2\,.1 + 1)(2\,.1 + 1)(2\,.2 + 1)$

$= 1.\,1.\,3.\,3.\,5$

$= 45$

jadi, jumlah keadaan eigen bersama adalah 45

c. Untuk menuliskan fungsi-fungsi eigen bersama dengan $l_5 = 2$ maka gunakan :

$|\, l_m \ l''' \ l_5 \,\rangle$ dengan $l''' = 2, 1, 0$

- $l''' = 2$

  $l = 4, 3, 2, 1, 0$

  $l = 4 \ \rightarrow \ m = 4, 3, 2, 1, 0, -1, -2, -3, -4 \ \rightarrow \ m = 9$

  $l = 3 \ \rightarrow \ m = 3, 2, 1, 0, -1, -2, -3 \ \rightarrow \ m = 7$

  $l = 2 \ \rightarrow \ m = 2, 1, 0, -1, -2 \ \rightarrow \ m = 5$

  $l = 1 \ \rightarrow \ m = 1, 0, -1 \ \rightarrow \ m = 3$

  $l = 0 \ \rightarrow \ m = 0 \ \rightarrow \ m = 1$

$|4\ 4\ 2\ 2\rangle \qquad |4\ -1\ 2\ 2\rangle$

$|4\ 3\ 2\ 2\rangle \qquad |4\ -2\ 2\ 2\rangle$

Maka : $|4\ 2\ 2\ 2\rangle$ $|4\ -3\ 2\ 2\rangle$ **9 Keadaan**

$|4\ 1\ 2\ 2\rangle$ $|4\ -4\ 2\ 2\rangle$

$|4\ 0\ 2\ 2\rangle$

$|3\ 3\ 2\ 2\rangle$ $|3\ -1\ 2\ 2\rangle$

$|3\ 2\ 2\ 2\rangle$ $|3\ -2\ 2\ 2\rangle$ **7 Keadaan**

$|3\ 1\ 2\ 2\rangle$ $|3\ -3\ 2\ 2\rangle$

$|3\ 0\ 2\ 2\rangle$

$|2\ 2\ 2\ 2\rangle$

$|2\ 1\ 2\ 2\rangle$

$|2\ 0\ 2\ 2\rangle$ **5 Keadaan**

$|2\ -1\ 2\ 2\rangle$

$|2\ -2\ 2\ 2\rangle$

$|1\ 1\ 2\ 2\rangle$

$|1\ 0\ 2\ 2\rangle$ **3 Keadaan**

$|1\ -1\ 2\ 2\rangle$

$|0\ 0\ 2\ 2\rangle$ **1 Keadaan**

- $l''' = 1$

  $l = 3, 2, 1$

  $l = 3 \rightarrow m = 3, 2, 1, 0, -1, -2, -3 \rightarrow m = 7$

  $l = 2 \rightarrow m = 2, 1, 0, -1, -2 \rightarrow m = 5$

$l = 1 \quad \rightarrow \quad m = 1, 0, -1 \qquad\qquad \rightarrow \quad m = 3$

Maka :

$|3\ 3\ 1\ 2\rangle$ $|3\ -1\ 1\ 2\rangle$

$|3\ 2\ 1\ 2\rangle$ $|3\ -2\ 1\ 2\rangle$ **7 Keadaan**

$|3\ 1\ 1\ 2\rangle$ $|3\ -3\ 1\ 2\rangle$

$|3\ 0\ 1\ 2\rangle$

$|2\ 2\ 1\ 2\rangle$

$|2\ 1\ 1\ 2\rangle$

$|2\ 0\ 1\ 2\rangle$ **5 Keadaan**

$|2\ -1\ 1\ 2\rangle$

$|2\ -2\ 1\ 2\rangle$

$|1\ 1\ 1\ 2\rangle$

$|1\ 0\ 1\ 2\rangle$ **3 Keadaan**

$|1\ -1\ 1\ 2\rangle$

- $l''' = 0$

  $l = 2 \quad \rightarrow \quad m = 2,1,0,-1,-2$

  Maka :

$|2\ 2\ 0\ 2\rangle$

$|2\ 1\ 0\ 2\rangle$

$|2\ 0\ 0\ 2\rangle$ **5 Keadaan**

$|2\ -1\ 0\ 2\rangle$

$|2\ -2\ 0\ 2\rangle$