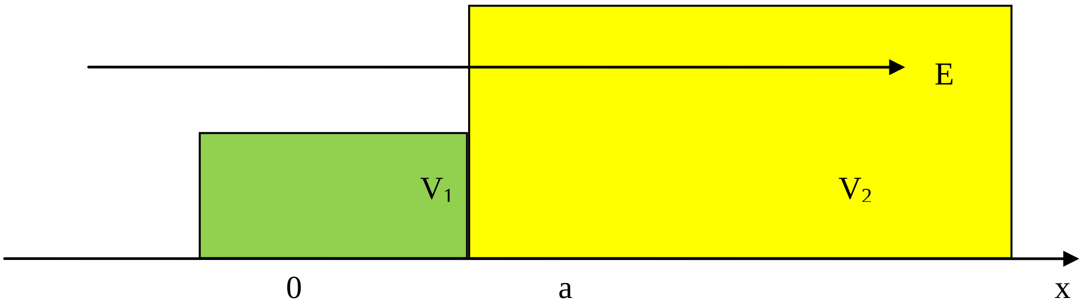
TES UNIT I PENDAHULUAN FISIKA KUANTUM

1. Diketahui $\psi(x) = Ce^{-\frac{(16x_0^2 + x^2 - 8xx_0)}{8a^2}}$

   Dimana C, a, dan x_0 merupakan konstanta positif

   Tentukan :
   a. Konstanta Normalisasi C
   b. Ketidakpastian posisi Δx
   c. Ketidakpastian momentum Δp
   d. Ekspektasi $< p >$ dalam ruang momentum

2. Sebuah partikel bergerak dengan energi E yang melalui potensial V_1 dan menerobos pada potensial V_2 seperti pada gambar di bawah ini.



Dapatkan a. solusi masing-masing fungsi gelombangnya mulai dari berangkat hingga berhenti

Sederhanakan b. fungsi gelombang yang terakhir ke dalam bentuk fungsi konstanta tertentu

Dapatkan c. koefisien transisi (T)

Solusi I

a) Menentukan konstanta normalisasi C

$$\psi(x) = Ce^{-\frac{(16x_0^2 + x^2 - 8xx_0)}{8a^2}} = Ce^{-\frac{(x-4x_0)^2}{8a^2}}$$

$$misal\ \eta = \frac{x - 4x_0}{a} \rightarrow \eta^2 = \frac{(x - 4x_0)^2}{a^2}$$

$$jadi\ \frac{\eta^2}{8} = \frac{(x - 4x_0)^2}{8a^2}$$

$$\eta = \frac{x - 4x_0}{a} = \frac{x}{a} - \frac{4x_0}{a} = \frac{x}{a} - \eta_0$$

$$atau\ x = a\eta + a\eta_0\ ;\ \eta_0 = \frac{4x_0}{a}$$

$$x = a(\eta + \eta_0) \rightarrow dx = a\, d\eta$$

$$\int_{-\sim}^{\sim} Ce^{-\frac{(x-4x_0)^2}{a^2}} \times Ce^{-\frac{(x-4x_0)^2}{a^2}} dx = 1$$

$$aC^2 \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}} d\eta = 1$$

$$aC^2\, 2\sqrt{\pi} = 1 \rightarrow C^2 = \frac{1}{2a\sqrt{\pi}}$$

b) Menentukan Δx

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\langle x^2 \rangle = \int_{-\sim}^{\sim} \psi(x)^* \hat{x}^2\, \psi(x) dx$$

$$= \int_{-\sim}^{\sim} \psi(x)^* x^2\, \psi(x) dx$$

$$= aC^2 \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}} [a\eta + a\eta_0]^2\, d\eta$$

$$= aC^2 \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}} [a^2\eta^2 + 2a^2\eta\eta_0 + a^2\eta_0^2] d\eta$$

$$= a^3C^2 \int_{-\sim}^{\sim} \eta^2 e^{-\frac{\eta^2}{4}} d\eta + 2a^3C^2 \int_{-\sim}^{\sim} \eta\eta_0 e^{-\frac{\eta^2}{4}} d\eta + a^3C^2 \int_{-\sim}^{\sim} \eta_0^2 e^{-\frac{\eta^2}{4}} d\eta$$

$$= a^3C^2\, 2\sqrt{\pi} + 0 + a^3C^2\eta_0^2\, 2\sqrt{\pi}$$

$$= a^3 \frac{1}{2a\sqrt{\pi}}\, 2\sqrt{\pi} + 0 + a^3 \frac{1}{2a\sqrt{\pi}} \eta_0^2\, 2\sqrt{\pi}$$

$$= a^2 + a^2\eta_0^2$$

$$\langle x^2 \rangle = a^2 + 16x_0^2$$

$$\langle x \rangle = \int_{-\sim}^{\sim} \psi(x)^* \, \hat{x} \, \psi(x) dx$$

$$= aC^2 \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}} \, a(\eta + \eta_0) d\eta$$

$$= a^2C^2 \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}} \, (\eta + \eta_0) d\eta$$

$$= a^2C^2 \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}} \, \eta d\eta + a^2C^2 \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}} \, \eta_0 d\eta$$

$$= 0 + a^2C^2\eta_0 \, 2\sqrt{\pi}$$

$$= a^2 \frac{1}{2a\sqrt{\pi}} \eta_0 \, 2\sqrt{\pi} = a\eta_0 = 4x_0$$

$$jadi \; \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$= \sqrt{a^2 + 16x_0^2 - 16x_0^2}$$

$$= \sqrt{a^2}$$

$$\Delta x = a$$

c) Menentukan Δp

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

$$\langle p^2 \rangle = \int_{-\sim}^{\sim} \psi(x)^* \, \hat{p}^2 \, \psi(x) dx$$

$$= \int_{-\sim}^{\sim} \psi(x)^* \, (-i\hbar \frac{\partial}{\partial x})^2 \, \psi(x) dx$$

$$= -\int_{-\sim}^{\sim} Ce^{-\frac{(x-4x_0)^2}{8a^2}} (\hbar \frac{\partial}{\partial x})^2 Ce^{-\frac{(x-4x_0)^2}{8a^2}} \, dx$$

$$catatan:$$

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x} e^{-\frac{(x-4x_0)^2}{8a^2}} = \frac{\partial}{\partial x} e^{-\frac{(x-4x_0)^2}{8a^2}} \frac{d}{dx}(-\frac{(x-4x_0)^2}{8a^2})$$

$$\frac{\partial}{\partial x}[(e^{-\frac{(x-4x_0)^2}{8a^2}})(-2)\frac{(x-4x_0)}{8a^2}]$$

$$= 4e^{-\frac{(x-4x_0)^2}{8a^2}} \frac{(x-4x_0)}{8a^2} - \frac{1}{4a^2} e^{-\frac{(x-4x_0)^2}{8a^2}}$$

$$jadi$$

$$\langle p^2\rangle = -\hbar^2 C^2 \int_{-\sim}^{\sim} e^{-\frac{(x-4x_0)^2}{8a^2}} \left(4e^{-\frac{(x-4x_0)^2}{8a^2}} \frac{(x-4x_0)}{8a^2} - \frac{1}{4a^2} e^{-\frac{(x-4x_0)^2}{8a^2}}\right) dx$$

$$\langle p^2\rangle = \frac{1}{4a}\hbar^2 C^2 \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}}\, d\eta - \frac{\hbar^2 C^2}{2} \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}}\, \eta d\eta$$

$$\langle p^2\rangle = \frac{1}{4a}\hbar^2 \frac{1}{2a\sqrt{\pi}} 2\sqrt{\pi} - \frac{\hbar^2}{2}\frac{1}{2a\sqrt{\pi}}.0$$

$$\langle p^2\rangle = \frac{\hbar^2}{4a^2}$$

$$\langle p\rangle = \int_{-\sim}^{\sim} \psi(x)^* \left(-i\hbar\frac{\partial}{\partial x}\right) \psi(x) dx$$

$$= -i\hbar C^2 \int_{-\sim}^{\sim} e^{-\frac{(x-4x_0)^2}{8a^2}} \frac{\partial}{\partial x} e^{-\frac{(x-4x_0)^2}{8a^2}}\, dx$$

$$= -i\hbar C^2 \int_{-\sim}^{\sim} e^{-\frac{(x-4x_0)^2}{4a^2}} (-2) \frac{(x-4x_0)}{8a^2}\, dx$$

$$= \frac{i\hbar C^2}{4} \int_{-\sim}^{\sim} e^{-\frac{\eta^2}{4}}\, \eta\, d\eta = 0$$

$$jadi\ \Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

$$= \sqrt{\frac{\hbar^2}{4a^2} - 0}$$

$$\Delta p = \frac{\hbar}{2a}$$

$$\Delta x . \Delta p = a . \frac{\hbar}{2a} = \frac{\hbar}{2}$$

Solusi II

a. Solusi masing-masing fungsi gelombang

Pers. Schrodinger

Daerah I

$$-\frac{\hbar^2}{2m}\frac{d\psi_1(x)}{dx} = E\psi_1(x)$$

$$\frac{d\psi_1(x)}{dx} = -\frac{2mE}{\hbar^2}\psi_1(x)$$

$$\frac{d\psi_1(x)}{dx} = -{k_1}^2\psi_1(x)$$

Solusi

$$\psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$$

Dengan

$${k_1}^2 = \frac{2mE}{\hbar^2} \rightarrow k_1 = \frac{\sqrt{2mE}}{\hbar}$$

Daerah II

$$-\frac{\hbar^2}{2m}\frac{d\psi_2(x)}{dx} + V_1\psi_2(x) = E\psi_2(x)$$

$$\frac{d\psi_2(x)}{dx} = -\frac{2m}{\hbar^2}(E - V_1)\psi_2(x)$$

$$\frac{d\psi_2(x)}{dx} = -{k_2}^2\psi_2(x)$$

Solusi

$$\psi_2(x) = Ce^{ik_2x} + De^{-ik_2x}$$

Dengan

$${k_2}^2 = \frac{2m}{\hbar^2}(E - V_1) \rightarrow k_2 = \frac{\sqrt{2m(E - V_1)}}{\hbar}$$

Daerah III

$$-\frac{\hbar^2}{2m}\frac{d\psi_3(x)}{dx} + V_2\psi_3(x) = E\psi_3(x)$$

$$\frac{d\psi_3(x)}{dx} = +\frac{2m}{\hbar^2}(V_2 - E)\psi_3(x)$$

$$\frac{d\psi_3(x)}{dx} = {k_3}^2\psi_3(x)$$

Solusi

$$\psi_3(x) = Ee^{k_3x} + Fe^{-k_3x}$$

Dengan

$$k_3{}^2 = \frac{2m}{\hbar^2}(V_2 - E) \to k_3 = \frac{\sqrt{2m(V_2 - E)}}{\hbar}$$

Yang memenuhi syarat (fisis)

$$\psi_3(x) = Fe^{-k_3x}$$

Jadi

$$\psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x} \qquad \text{......... (1)}$$

$$\psi_2(x) = Ce^{ik_2x} + De^{-ik_2x} \qquad \text{......... (2)}$$

$$\psi_3(x) = Fe^{-k_3x} \qquad \text{......... (3)}$$

b. Fungsi gelombang terakhir dalam bentuk fungsi konstanta tertentu

Syarat kontinuitas I (pada x = 0)

$$\psi_1(x) = \psi_2(x)$$

$$\psi'_1(x) = \psi'_2(x)$$

i) $\psi_1(x) = \psi_2(x) \to x = 0$

$$Ae^{ik_10} + Be^{-ik_10} = Ce^{ik_20} + De^{-ik_20}$$

$$A + B = C + D \qquad \text{......... (4)}$$

ii) $\psi'_1(x) = \psi'_2(x) \to x = 0$

$$ik_1\left[Ae^{ik_1x} - Be^{-ik_1x}\right] = ik_2\left[Ce^{ik_2x} - De^{-ik_2x}\right]$$

$$ik_1\left[Ae^{ik_10} - Be^{-ik_10}\right] = ik_2\left[Ce^{ik_20} - De^{-ik_20}\right]$$

$$ik_1[A - B] = ik_2[C - D] \qquad \text{......... (5)}$$

Syarat kontinuitas II (pada x = a)

$$\psi_2(x) = \psi_3(x)$$

$$\psi'_2(x) = \psi'_3(x)$$

i) $\psi_1(x) = \psi_2(x) \to x = 0$

$$Ce^{ik_2x} + De^{-ik_2x} = Fe^{-k_3x}$$

$$Ce^{ik_2a} + De^{-ik_2a} = Fe^{-k_3a} \qquad \text{......... (6)}$$

ii) $\psi'_1(x) = \psi'_2(x) \to x = 0$

$$ik_2\left[Ce^{ik_2x} - De^{-ik_2x}\right] = -k_3Fe^{-k_3x}$$

$$ik_2\left[Ce^{ik_2a} - De^{-ik_2a}\right] = -k_3Fe^{-k} \qquad \text{......... (7)}$$

Kalikan persamaan 6 dengan ik_2 dan jumlahkan dengan persamaan 7

$$ik_2[Ce^{ik_2a} + De^{-ik_2a}] = ik_2Fe^{-k_3a}$$

$$ik_2\left[Ce^{ik_2a} - De^{-ik_2a}\right] = -k_3Fe^{-k_3a} \quad +$$

$$2ik_2Ce^{ik_2a} = (ik_2 - k_3)Fe^{-k_3a}$$

$$C = \frac{(ik_2 - k_3)}{2ik_2}Fe^{-(k_3a+ik_2a)} \qquad \dots\dots\dots (8)$$

Substitusi persamaan 8 ke persamaan 7

$$ik_2\left[\frac{(ik_2 - k_3)}{2ik_2}Fe^{-(k_3a+ik_2a)}e^{ik_2a} - De^{-ik_2a}\right] = -k_3Fe^{-k_3a}$$

$$ik_2\left[\frac{(ik_2 - k_3)}{2ik_2}Fe^{-k_3a} - De^{-ik_2a}\right] = -k_3Fe^{-k_3a}$$

$$ik_2\left[\frac{(ik_2 - k_3)}{2ik_2}Fe^{-k_3a}\right] + k_3Fe^{-k_3a} = ik_2De^{-ik_2a}$$

$$\left[\frac{(ik_2 - k_3)}{2}Fe^{-k_3a}\right] + k_3Fe^{-k_3a} = ik_2De^{-ik_2a}$$

$$\left[\frac{(ik_2 + k_3)}{2}Fe^{-k_3a}\right] = ik_2De^{-ik_2a}$$

$$D = \left[\frac{(ik_2 + k_3)}{2ik_2}\right]Fe^{-k_3a+ik_2a} \qquad \dots\dots\dots (9)$$

Substitusi persamaan 8 & 9 ke persamaan 4

$$A + B = \frac{(ik_2 - k_3)}{2ik_2}Fe^{-(k_3a+ik_2a)} + \left[\frac{(ik_2 + k_3)}{2ik_2}\right]Fe^{-k_3a+ik_2a} \quad \dots\dots (10)$$

Substitusi persamaan 8 & 9 ke persamaan 5

$$ik_1[A - B] = ik_2\left[\frac{(ik_2 - k_3)}{2ik_2}Fe^{-(k_3a+ik_2a)} - \left[\frac{(ik_2 + k_3)}{2ik_2}\right]Fe^{-k_3a+ik_2a}\right] \quad \dots (11)$$

Kalikan persamaan 10 dengan ik_1 kemudian jumlahkan dengan persamaan 11

$$ik_1[A + B] = ik_1\left[\frac{(ik_2 - k_3)}{2ik_2}Fe^{-(k_3a+ik_2a)} + \left[\frac{(ik_2 + k_3)}{2ik_2}\right]Fe^{-k_3a+ik_2a}\right]$$

$$ik_1[A - B] = ik_2\left[\frac{(ik_2 - k_3)}{2ik_2}Fe^{-(k_3a+ik_2a)} - \left[\frac{(ik_2 + k_3)}{2ik_2}\right]Fe^{-k_3a+ik_2a}\right] \quad +$$

$$2ik_1A = \left[\frac{(ik_2 - k_3)(ik_1 + ik_2)}{2ik_2}Fe^{-(k_3a+ik_2a)} + \left[\frac{(ik_2 + k_3)(ik_1 - ik_2)}{2ik_2}\right]Fe^{-k_3a+ik_2a}\right]$$

$$2ik_1A = \left[\frac{(ik_2 - k_3)(k_1 + k_2)}{2k_2} Fe^{-(k_3a + ik_2a)} + \left[\frac{(ik_2 + k_3)(k_1 - k_2)}{2k_2}\right] Fe^{-k_3a + ik_2a}\right]$$

$$4ik_1A = \left[\frac{(ik_2 - k_3)(k_1 + k_2)}{k_2} Fe^{-(k_3a + ik_2a)} + \left[\frac{(ik_2 + k_3)(k_1 - k_2)}{k_2}\right] Fe^{-k_3a + ik_2a}\right]$$

$$4ik_1A = Fe^{-k_3a}\left[\frac{ik_2k_1 - k_3k_1 + ik_2k_2 - k_3k_2}{k_2} e^{-ik_2a} + \left[\frac{ik_2k_1 + k_3k_1 - ik_2k_2 - k_3k_2}{k_2}\right] e^{ik_2a}\right]$$

$$4ik_1k_2A = Fe^{-k_3a}\left[(ik_2k_1 - k_3k_1 + ik_2k_2 - k_3k_2)e^{-ik_2a} + (ik_2k_1 + k_3k_1 - ik_2k_2 - k_3k_2)e^{ik_2a}\right]$$

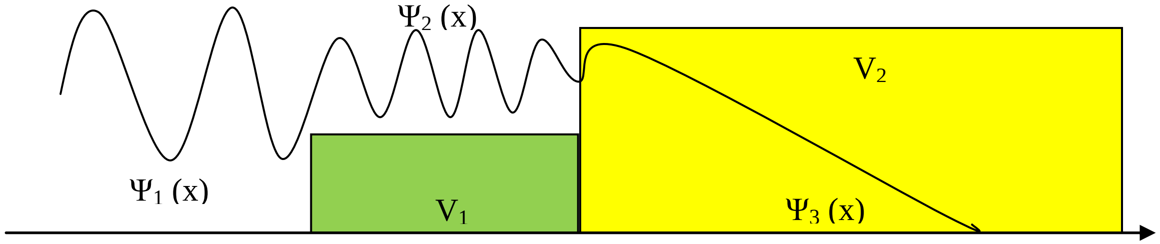
c. Koefisien transmitansi

$$T = \left|\frac{F}{A}\right|^2 \frac{k_3}{k_1}$$

$$T = \frac{16{k_1}^2}{e^{-ik_3a}\left[\left[\frac{ik_2k_1 - k_3k_1 + ik_2k_2 - k_3k_2}{ik_2}\right]\left[\frac{ik_2k_1 + k_3k_1 + ik_2k_2 + k_3k_2}{ik_2}\right] + \left[\frac{-k_2k_1 - ik_3k_1 + k_2k_2 - ik_3k_2}{k_2}\right]\left[\frac{-k_2k_1 + ik_3k_1 + k_2k_2 + ik_3k_2}{k_2}\right]\right]}\frac{k_3}{k_1}$$

$$T = \frac{16k_1k_3}{e^{-ik_3a}\left[\left[\frac{ik_2k_1 - k_3k_1 + ik_2k_2 - k_3k_2}{ik_2}\right]\left[\frac{ik_2k_1 + k_3k_1 + ik_2k_2 + k_3k_2}{ik_2}\right] + \left[\frac{-k_2k_1 - ik_3k_1 + k_2k_2 - ik_3k_2}{k_2}\right]\left[\frac{-k_2k_1 + ik_3k_1 + k_2k_2 + ik_3k_2}{k_2}\right]\right]}$$

d. Sketsa rapat probabilitas terhadap variabel x



$$4k_1k_2A = Fe^{ik_3a}[(k_2k_1 + k_3k_2)2\cos k_2a - 2i(k_3k_1 + k_2k_2)\sin k_2a]$$

$$2k_1k_2A = Fe^{ik_3a}[(k_2k_1 + k_3k_2)\cos k_2a - i(k_3k_1 + k_2k_2)\sin k_2a]$$

$$4{k_1}^2{k_2}^2A^2 = F^2[(k_2k_1 + k_3k_2)^2\cos^2 k_2a + (k_3k_1 + k_2k_2)^2\sin^2 k_2a]$$

$$4{k_1}^2{k_2}^2A^2 = F^2[(k_2k_1 + k_3k_2)^2\,(1 - \sin^2 k_2a) + (k_3k_1 + k_2k_2)^2\sin^2 k_2a]$$

$$4{k_1}^2{k_2}^2A^2 = F^2[(k_2k_1 + k_3k_2)^2\;1 - (k_2k_1 + k_3k_2)^2\sin^2 k_2a + (k_3k_1 + k_2k_2)^2\sin^2 k_2a]$$