1. Pembuktian Rumus

$V_{coulomb} = \frac{-\alpha e^2}{4\pi\varepsilon_0 r}$ secara pendekatan : $V_{rep} = \frac{B}{r^n}$

Sehingga energy potensial total :

$V = V_{coulomb} + V_{rep}$

$V = \frac{-\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^n}$ .... (1)

Jarak kesetimbangan ion-ion terjadi bila V mempunyai harga minimum

V_0 pada $r = r_0$

$$\frac{dV}{dr} = 0$$

$$\frac{d}{dr}\left(\frac{-\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^n}\right) = 0$$

$$\frac{-\alpha e^2}{4\pi\varepsilon_0 r}(-1)r^{-2} + B(-n)r^{(-n-1)} = 0, \quad r = r_0$$

$$\frac{-\alpha e^2}{4\pi\varepsilon_0 r_0^2} - \frac{B}{r^{n+1}} = 0$$

$B = \frac{\alpha e^2}{4\pi\varepsilon_0 r_0^2}\frac{r^{n+1}}{n} = \frac{\alpha e^2 r^{n-1}}{4\pi\varepsilon_0 n}$ .... (2)

Subtitusi persamaan 2 ke 1

$$V_0 = \frac{-\alpha e^2}{4\pi\varepsilon_0 r} + \frac{1}{r^n}\left(\frac{\alpha e^2}{4\pi\varepsilon_0 n} r^{n-1}\right)$$

$$= \frac{-\alpha e^2}{4\pi\varepsilon_0 r} + \frac{\alpha e^2}{4\pi\varepsilon_0 r n} = \frac{\alpha e^2}{4\pi\varepsilon_0 r}\left(1 - \frac{1}{n}\right)$$

2. Kapasitas Panas Model Einstein

$$f(v) = \frac{1}{e^{hv/kT} - 1}$$

Energi rata-rata nya:

$$\overline{E} = hv \,.\, f(v)$$

$$\bar{E} = \frac{hv}{e^{hv/kT} - 1}$$

Untuk 1 kilo mol maka energy dalamnya:

$$U = 3N_0\bar{E}$$

$$U = 3N_0 \frac{hv}{e^{hv/kT} - 1}$$

↘ Untuk temperature tinggi T >> → $\frac{hv}{kT}$ << 1

Ingat : $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ ....$

$e^{hv/kT} = 1 + \frac{hv}{kT} + \ ....$

Maka :

$$U = 3N_0 \frac{hv}{e^{hv/kT} - 1} = 3N_0 \frac{hv}{\left(1 + \frac{hv}{kT}\right) - 1} = \frac{3N_0 hv}{\frac{hv}{kT}}$$

$$U = 3N_0 kT \quad \rightarrow dimana\ N_0 k = R$$

$$U = 3RT$$

$$Cv = \frac{\partial u}{\partial T} = \frac{\partial}{\partial T}\ (3RT)$$

Maka : $Cv = 3R$ → kapasitas panas model Einstein untuk T >>

(sesuai dengan eksperimen Dulog dan Petit)

↘ Untuk suhu rendah T << → $\frac{h\upsilon}{kT}$ >> 1

$$U = 3N_0 \; \frac{h\upsilon}{e^{h\upsilon/kT} - 1}$$

$$Cv = \frac{\partial U}{\partial T} = \frac{\partial}{\partial T}\left(3N_0 \; \frac{h\upsilon}{e^{h\upsilon/kT} - 1}\right)$$

$$= 3N_0\, h\upsilon \left(-1\left(e^{h\upsilon/kT} - 1\right)^{-2}\right)\left(\frac{-h\upsilon}{kT}\, e^{h\upsilon/kT}\right)$$

$$Cv = 3N_0\, h\upsilon\, \frac{h\upsilon}{kT^2} \frac{e^{h\upsilon/kT}}{\left(e^{h\upsilon/kT} - 1\right)^2}$$

$$Cv = 3N_0\, k \left(\frac{h\upsilon}{kT}\right)^2 \frac{e^{h\upsilon/kT}}{\left(e^{h\upsilon/kT} - 1\right)^2}$$

$$Cv = 3R \left(\frac{h\upsilon}{kT}\right)^2 \frac{e^{h\upsilon/kT}}{\left(e^{h\upsilon/kT} - 1\right)^2}$$

Temperatur khusus : suhu Einstein $\theta_E = \frac{h\upsilon}{k}$

Maka : $Cv = 3R \left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{\left(e^{\theta_E/T - 1}\right)^2}$ → kapasitas panas model Einstein untuk T <<

Kapasitas panas Model Debye

Rapat keadaan

Di definisikan : $D(W) = \frac{Jumlah\ keadaan\ (dN)}{Rentang\ energi\ (dW)}$

Maka jumlah keadaan : $dN = D(W)\, dW$

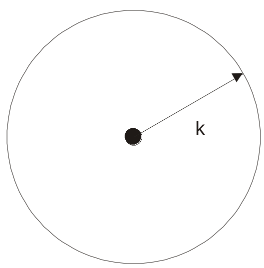
Energi Total

$$U = \Sigma_k \left\{ \Sigma_p \frac{\hbar w_{kp}}{e^{\frac{\hbar w_{kp}}{k_B T}} - 1} \right\}$$

Debye menganggap energinya bersifat continue, maka :

$$U = \Sigma_p \int \frac{\hbar w_{kp}}{e^{\frac{\hbar w_{kp}}{k_B T}} - 1} D(W)\, dW \quad \text{...... (1)}$$

Volume Ruang



$$N = \frac{\frac{4}{3}\pi k^3}{\left(\frac{2\pi}{L}\right)^3}$$

$$N = \frac{4}{3}\pi k^3 . \frac{L^3}{8\pi^3} = \frac{L^3 k^3}{6\pi^2} = \frac{V k^3}{6\pi^2}$$

$$D(k) = \frac{dN}{dk} = \frac{d}{dk}\left(\frac{V k^3}{6\pi^2}\right) = \frac{3 V k^3}{6\pi^2}$$

$$D(k) = \frac{dN}{dk} = \frac{V k^2}{2\pi^2}$$

$$D(w) = \frac{dN}{dw} = \frac{dN}{dk} . \frac{dk}{dw} = \frac{V k^2}{2\pi^2}\left(\frac{dk}{dw}\right)$$

$$D(w) = \frac{V k^2}{2\pi^2}\frac{1}{V} = \frac{V\left(\frac{w^2}{V^2}\right)}{2\pi^2}\frac{1}{V}$$

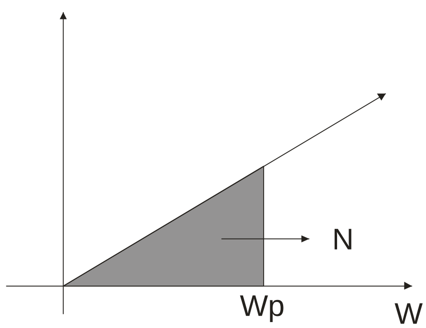
$$D(w) = \frac{V w^2}{2\pi^2 V} \quad \text{...... (2)}$$

Substitusi persamaan (2) ke (1) :

$$U = \Sigma_p \int \frac{\hbar w_{kp}}{e^{\frac{\hbar w_{kp}}{k_B T}} - 1} D(W)\, dW$$

$$U = 3 \int_0^{w_0} \frac{\hbar w}{e^{\hbar w / k_B T} - 1} \cdot \frac{V w^2}{2\pi^2 V}\, dw$$

$$U = \int_0^{w_0} \frac{\hbar w^3 V / 2\pi^2 V^3}{e^{\hbar w / k_B T} - 1}\, dw$$



$$w_D = w_{Debye}$$

$$w_D = V k_D$$

$$N(w) = N_{total}$$

$$N = \frac{\frac{4}{3}\pi k_D^3}{\left(\frac{2\pi}{L}\right)^3} = \frac{L^3 k_D^3}{6\pi^2} = \frac{V k_D^3}{6\pi^2}$$

$V = \frac{6\pi^2}{k_D^3} N$ ……. (3)

<u>Kapasitas Panas</u>

$$Cv = \frac{\partial U}{\partial T} = \frac{\partial}{\partial T}\left[3 \int_0^{w_0} \frac{\hbar w}{e^{\hbar w / k_B T} - 1} \cdot \frac{V w^2}{2\pi^2 V}\, dw\right]$$

$$Cv = \frac{3\hbar V}{2\pi^2 V^3}\int_0^{w_D} w^3 \left(\frac{d}{dT}\frac{1}{e^{\hbar w/k_B T}-1}\right) dw$$

$$= -\left(e^{\hbar w/k_B T}-1\right)^{-2} \cdot \frac{\hbar w}{k_B T^2} e^{\hbar w/k_B T}$$

$$= \frac{\hbar w}{k_B T^2}\frac{e^{\hbar w/k_B T}}{\left(e^{\hbar w/k_B T}-1\right)^2}$$

Maka :

$$Cv = \frac{3\hbar V}{2\pi^2 V^3}\int_0^{w_D} w^3 \frac{\hbar w}{k_B T^2}\frac{e^{\hbar w/k_B T}}{(e^{\hbar w/k_B T}-1)^2} dw$$

$$Cv = \frac{3\hbar^2 V}{2\pi^2 V^3}\frac{1}{k_B T^2}\int_0^{w_D} w^4 \frac{e^{\hbar w/k_B T}}{(e^{\hbar w/k_B T}-1)^2} dw$$

Misal : $x = \frac{\hbar w}{k_B T} \rightarrow w = \frac{k_B T}{\hbar} x$

$$dw = \frac{k_B T}{\hbar} dx$$

$$Cv = \frac{3\hbar^2 V}{2\pi^2 V^3}\frac{1}{k_B T^2}\int_0^{x} w^4 \frac{\left(\frac{k_B T}{\hbar}x\right)^4 e^x}{(e^x-1)^2}\frac{k_B T}{\hbar} dx$$

$$Cv = \frac{3\hbar^2 V}{2\pi^2 V^3}\frac{1}{k_B T^2}\int_0^{\hbar w/k_B T} w^4 \frac{\left(\frac{k_B T}{\hbar}\right)^4 x^4 e^x}{(e^x-1)^2}\frac{k_B T}{\hbar} dx$$

$$= \frac{3\hbar^2 V}{2\pi^2 V^3}\frac{1}{k_B T^2}\frac{k_B T}{\hbar}\left(\frac{k_B T}{\hbar}\right)^4 \int_0^{\hbar w/k_B T} \frac{x^4 e^x}{(e^x-1)^2} dx$$

$$Cv = \frac{3V {k_B}^4 T^3}{2\pi^2 V^3 \hbar^3}\int_0^{\hbar w/k_B T} \frac{x^4 e^x}{(e^x-1)^2} dx$$

Jika $\theta_D = \frac{\hbar w}{k_B}$ ingat $x = \frac{\hbar w}{k_B}$

Maka $\frac{\hbar w}{k_B} = \frac{\theta_D}{T}$

$$Cv = \frac{3Vk_B{}^4T^3}{2\pi^2V^3\hbar^3}\int_0^{\theta_D/T}\frac{x^4e^x}{(e^x-1)^2}\,dx$$

Dari persamaan (3), $V = \frac{6\pi^2}{k_D^3}N$ → ingat $w_D = \nu k_D$

$$k_D^3 = \frac{w_D{}^3}{\nu^3}$$

$$V = \frac{6\pi^2\,N\,\nu^3}{w_D{}^3}$$

$$Cv = \frac{3\left(\frac{6\pi^2\,N\,\nu^3}{w_D{}^3}\right)k_B{}^4T^3}{2\pi^2V^3\hbar^3}\int_0^{\theta_D/T}\frac{x^4e^x}{(e^x-1)^2}\,dx$$

$Cv = \frac{9Nk_B{}^4T^3}{\hbar^3w_D{}^3}\int_0^{\theta_D/T}\frac{x^4e^x}{(e^x-1)^2}\,dx$, ingat $\theta_D = \frac{\hbar w}{k_B} \rightarrow w_D = \frac{\theta_D}{\hbar}k_B$

$Cv = \frac{9Nk_B{}^4T^3}{\hbar^3\left(\frac{\theta_D}{\hbar}k_B\right)^3}\int_0^{\theta_D/T}\frac{x^4e^x}{(e^x-1)^2}\,dx$, ingat $x = \frac{\theta_D}{T}$

↘ Untuk suhu tinggi T >> → T >> θ_D → x_D << 1

Maka :

$$\frac{x^4e^x}{(e^x-1)^2} = \frac{x^4e^x}{(e^x-1)(e^x-1)} = \frac{x^4e^x}{(e^x-1)(1-e^x)}$$

Deret :

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ ....$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ ....$$

$$(e^x - 1)(e^x - 1) = (e^x - 1)(1 - e^x)$$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ ....\right)\left(1 - \left(x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ ....\right)\right)$$

$$= \left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)\left(x - \frac{x^2}{2!} + \frac{x^3}{3!}\right)$$

$$= x^2 - \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^3}{2!} - \frac{x^4}{2!2!} + \frac{x^5}{2!3!} + \frac{x^4}{3!} - \frac{x^5}{2!3!} + \frac{x^6}{3!3!}$$

$$= x^2 + \frac{x^4}{3!} - \frac{x^4}{2!2!} + \frac{x^4}{3!} + \ .....$$

$$= x^2 + \frac{2x^4}{3!} - \frac{x^4}{2!2!} = x^2 + x^4\left(\frac{2}{3.2.1} - \frac{1}{2.1.2.1}\right)$$

$$(e^x - 1)(1 - e^x) = 2\left(\frac{x^2}{2} + \frac{x^{43}}{24}\right) = 2\left(\frac{x^2}{2!} + \frac{x^4}{4!} + \ ....\right)$$

$$x_D << 1 \rightarrow \frac{x^4 e^x}{(e^x - 1)^2} = \frac{x^4 e^0}{(e^x - 1)(1 - e^x)} = \frac{x^4}{2\left\{\frac{x^2}{2!} + \frac{x^4}{4!} + \ ....\right\}} = \frac{x^4}{2 . \frac{x^2}{2!}} \approx x^2$$

$Jadi$:

$$Cv = \frac{9Nk_B T^3}{{\theta_D}^3} \int_0^{\theta_D} x^2 dx = 9Nk_B \left(\frac{T}{\theta_D}\right)^3 \frac{1}{3} x^3 \rightarrow \text{ingat } x = \frac{\theta_D}{T}$$

$$Cv = 9Nk_B \left(\frac{T}{\theta_D}\right)^3 \frac{1}{3} \frac{{\theta_D}^3}{T^3}$$

$Cv = 3Nk_B \rightarrow$ dimana $Nk_B = R$

$Cv = 3R \rightarrow$ kapasitas panas model Debye untuk T >>

↘ Untuk suhu rendah T << → T << θ_D → x_D >> 1

$$Cv = 9Nk_B\left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^4e^x}{(e^x-1)^2}dx$$

↓

①

① → integral parsial $\int u\,dv = uv - v\int du$

$\int_0^{\theta_D/T} \frac{x^4e^x}{(e^x-1)^2}dx$ → misal $u = x^4$

$$du = 4x^3dx$$

$$dv = \frac{e^x}{(e^x-1)^2}dx \rightarrow v = \int \frac{e^x}{(e^x-1)^2}dx$$

Misal: $e^x - 1 = u$

$$du = e^x\,dx$$

$$v = \int \frac{1}{u^2}du = \int u^{-2}du$$

$$v = -\frac{1}{u} = -\frac{1}{(e^x-1)}$$

$$\int_0^{\theta_D/T} \frac{x^4e^x}{(e^x-1)^2}dx = \frac{-x^4}{(e^x-1)} + 4\int \frac{x^3}{(e^x-1)}\,dx$$

$$jadi:\ Cv = 9Nk_B\left(\frac{T}{\theta_D}\right)^3\left\{\frac{-x^4}{(e^x-1)} + 4\int_0^{\sim} \frac{x^3}{(e^x-1)}dx\right\}$$

↓ ↓

② ③

② → untuk T = 0 maka $\frac{-x^4}{(e^x-1)} \approx \frac{\left(\theta_D/_T\right)^4}{e^{\theta_D/_{T-1}}} = 0$

③ → jika $4\int_0^{\sim} \frac{x^3}{(e^x-1)} dx$ → $gunakan\ \frac{1}{e^x-1} = \sum_{\alpha=1}^{\sim} e^{-\alpha x}$

$$4\int_0^{\sim} \frac{x^3}{(e^x-1)} dx = 4\int_0^{\sim} x^3 \sum_{\alpha=1}^{\alpha=n} e^{-\alpha x} dx$$

$$\rightarrow rumus \int_0^{\sim} x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$$

$$= \left[3!\ \textstyle\sum_{\alpha=1}^{\alpha=4} \frac{1}{\alpha^{3=1}} dx\right]$$

$$= 4\left[6\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots\right)\right]$$

↓

Rumus : $\frac{1}{1^{2p}} + \frac{1}{2^{2p}} + \frac{1}{3^{2p}} + \frac{1}{4^{2p}} + \cdots = \frac{2^{p-1}\pi^{2p}}{(2p)!}$

Didapat p = 2 → untuk p = 2 → $B_p = \frac{1}{30}$ (Dari tabel bernoulli)

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} = \frac{2^3\pi^4 \frac{1}{30}}{4!} = \frac{8\pi^4 \frac{1}{30}}{4.3.2.1} = \frac{\pi^4}{90}$$

$$4\int_0^{\sim} \frac{x^3}{(e^x-1)} dx = 4\left[6.\frac{\pi^4}{90}\right] = 8\left(\frac{\pi^4}{30}\right) = \frac{4\pi^4}{15}$$

Jadi :

$$Cv = 9Nk_B\left(\frac{T}{\theta_D}\right)^3 \left\{\frac{-x^4}{(e^x-1)} + 4\int_0^{\sim} \frac{x^3}{(e^x-1)} dx\right\}$$

$$Cv = 9Nk_B\left(\frac{T}{\theta_D}\right)^3 \left\{0 + \frac{4\pi^4}{15}\right\}$$

$$Cv = 9Nk_B\left(\frac{T}{\theta_D}\right)^3 \frac{4\pi^4}{15}$$

$$Cv = \frac{12}{5}\pi^4 Nk_B\left(\frac{T}{\theta_D}\right)^3$$

$Cv = 234Nk_B\left(\frac{T}{\theta_D}\right)^3$ → kapasitas panas model Debye untuk T <<

5. Turunkan bahwa kapasitas panas untuk elektron adalah :

   $Cv = \frac{\pi^2 T {k_B}^2 N}{2E_F}$

   Dari mekanika klasik diperoleh bahwa :

   Energi untuk satu derajat kebebasan $U = \frac{1}{2}k_B T$

   Sehingga untuk partikul dengan 3 derajat kebebasan adalah:

   $$U = 3\frac{1}{2}k_B T = \frac{3}{2}k_B T$$

   Dan kapasitas panas untuk 1 partikel adalah : $Cv = \frac{dU}{dT} = \frac{3}{2}k_B$

   Maka kapasitas panas untuk N buah partikel : $Cv = \frac{3}{2}Nk_B$

   Besarnya energi fermi : $E_F = \frac{\hbar^2}{2m}{k_p}^2$

   Bila $\frac{T}{T_F} \approx 0{,}01$ maka energi ferminya $E_F = k_B . T_F \rightarrow T_F = \frac{E_F}{k_B}$

   ↘ Untuk suhu rendah $k_B T \ll E_F$

   Distribusi fermi diract : $F(E) = \frac{1}{e^{(E-E_F)/k_B T}+1}$

   $T = 0$

$$E < E_F \ \rightarrow f(E) = 1$$

$$E > E_F \ \rightarrow f(E) = 0$$

Bila perubahan energinya adalah $U = U(T) - U(0)$

Dimana $U(T)$ = energi setelah elektron pindah dari keadaan dasar

$U = U(T) - U(0)$

$$U = \int_0^2 D(E) - F(E)E\, dE - \int_0^{E_F} D(E)E\, dE$$

Bila $N = \int_0^{E_F} D(E)\, F(E)\, dE = \int_0^{\sim} D(E)F(E)\, dE$

$$NE_F = \int_0^{\sim} D(E)\, F(E)E_F\, dE = \left[\int_0^{E_F} + \int_{E_F}^{\sim} \right] D(E)F(E)E_F\, dE$$

Maka :

$$U = \int_{E_F}^{\sim} D(E)F(E)(E - E_F)\, dE + \int_0^{E_F} D(E)(E_F - E)\big(1 - F(E)\big)\, dE$$

Karena integral yang bergantung pada suhu hanya F(E) maka diferensiasinya terhadap T hanya berlaku untuk suku-suku yang mengandung F(E) saja.

$$Cv = \frac{dU}{dT}$$

$$= \frac{d}{dT}\left[\int_{E_F}^{\sim} D(E)F(E)(E - E_F)\, dE\right.$$

$$\left. + \int_{0}^{E_F} D(E)(E_F - E)\, dE + \int_{0}^{E_F} D(E)(E - E_F)\, F(E)dE\right]$$

$$= \frac{d}{dT}\left[\int_{0}^{E_F} D(E)(E - E_F)\, F(E)dE + \int_{E_F}^{\sim} D(E)(E - E_F)\, F(E)dE\right]$$

$$= \frac{d}{dT} D(E)F(E)(E - E_F)\, dE$$

$$= k_B \int_{0}^{\sim} D(E)(E - E_F)\, \frac{dE(E)}{dTk_B} dE$$

$Cv = \ k_B \int_0^{\sim} D(E)(E - E_F)\ \frac{dF(E)}{dTk_B} dE$ .... (1)

Ingat : $F(E) = \frac{1}{e^{(E-E_F)/k_B T}}$

Maka $\frac{dF(E)}{dTk_B} = \frac{d}{dT}\left(e^{(E-E_F)/T} + 1\right)^{-1} \; dimana\; \tau = k_B T$

$\frac{dF(E)}{d\tau} = \ \frac{E-E_F}{\tau^2}\ \frac{e^{(E-E_F)/\tau}}{\left(e^{(E-E_F)/\tau}+1\right)^2}$ .... (2)

Persamaan (1) menjadi :

$$Cv = \ k_B \int_{0}^{\sim} D(E)(E - E_F)\ \frac{dF(E)}{d\tau} dE$$

Untuk T << maka persamaan diatas menjadi :

$$Cv = k_B D(E_F) \int_0^{\sim} (E - E_F) \frac{(E - E_F)}{\tau^2} \frac{e^{(E-E_F)/\tau}}{(e^{(E-E_F)/\tau} + 1)^2} dE$$

$$Cv = k_B D(E_F) \int_0^{\sim} (E - E_F) \frac{(E - E_F)}{\tau^2} \frac{e^{(E-E_F)/\tau}}{(e^{(E-E_F)/\tau} + 1)^2} dE$$

Misal : $x = \frac{(E-E_F)}{\tau}$

$E = 0 \rightarrow x = -E_F/\tau$

$E = \sim \rightarrow x = \sim$

$$Cv = k_B D(E_F) \int_{E_F/\tau}^{\sim} x^2 \frac{e^x}{(e^x + 1)^2} \tau dx \quad \rightarrow ingat\ \tau = k_B T$$

$$Cv = k_B D(E_F) \int_{E_F/\tau}^{\sim} x^2 \frac{e^x}{(e^x + 1)^2} k_B T dx$$

$Cv = {k_B}^2 T_D(E_F) \int_{E_F/\tau}^{\sim} x^2 \frac{e^x}{(e^x+1)^2} dx$ ..... (4)

↓

$\int_{E_F/\tau}^{\sim} x^2 \frac{e^x}{(e^x+1)^2} dx \rightarrow misal\ u = x^2$

$du = 2x\ dx$

$dv = \frac{e^x}{(e^x+1)^2} dx$

$v = -\frac{1}{(e^x+1)}$

Gunakan integral parsial

$$\int_{E_F/\tau}^{\sim} x^2 \frac{e^x}{(e^x+1)^2} dx = -\frac{x^2}{(e^x+1)} + 2\int \frac{x}{(e^x+1)} dx \quad .... (5)$$

↓ ①   ↓ ②

① $\rightarrow -\frac{x^2}{(e^x+1)} = -\frac{\left(\frac{(E-E_F)}{\tau}\right)^2}{e^{\frac{-(E-E_F)}{\tau}}+1}$ → dimana $\tau = k_B T$ untuk T = 0

Maka $\frac{(E-E_F)}{\tau} = 0$

Sehingga $-\frac{x^2}{(e^x+1)} = 0$

② $\rightarrow 2\int \frac{x}{(e^x+1)} dx \rightarrow gunakan \ \frac{1}{(e^x+1)} = \sum_{\alpha=1}^{\sim} e^{\alpha x}$

$$2\int \sum_{\alpha=1}^{\sim} e^{\alpha x}\, dx \quad rumus \quad \int_0^{\sim} x^n e^{\alpha x} dx = \frac{n!}{(-\alpha)^{n+1}}$$

$$= 2\left[1!\ \sum_{\alpha=1}^{\sim} \frac{1}{-\alpha^2}\right]$$

$$= 2\left[1\left(\frac{1}{(-1)^2} + \frac{1}{(-2)^2} + \cdots\right)\right]$$

↓

Rumus : $\frac{1}{1^{2p}} + \frac{1}{2^{2p}} + \frac{1}{3^{2p}} + \frac{1}{4^{2p}} + \cdots = \frac{2^{p-1}\pi^{2p}}{(2p)!}$

Didapat p = 1 → untuk p = 1 → $B_p = \frac{1}{6}$ (Dari tabel bernoulli)

$$\frac{1}{1^2}+\frac{1}{2^2}+\cdots= \frac{2^{2-1}\pi^2\frac{1}{6}}{2!} = \frac{\pi^2}{6}$$

Jadi :

$$2\left[1!\sum_{\alpha=1}^{\sim}\frac{1}{-\alpha^2}\right] = 2\left[\frac{1}{1^2}+\frac{1}{2^2}+\ \ldots.\right] = 2\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{3}$$

$$2\int\frac{x}{(e^x+1)}dx = \frac{\pi^2}{3}$$

Substitusi ① dan ② ke persamaan (5) maka diperoleh

$$\int_{E_F/\tau}^{\sim}x^2\frac{e^x}{(e^x+1)^2}dx = 0+\frac{\pi^2}{3} = \frac{\pi^2}{3}\ \ldots\ldots\ldots\ (6)$$

Substitusi persamaan (6) ke (4)

$$Cv = \ {k_B}^2TD(E_F)\int_{E_F/\tau}^{\sim}x^2\frac{e^x}{(e^x+1)^2}dx$$

$$Cv = \ {k_B}^2TD(E_F)\frac{\pi^2}{3} \rightarrow rapat\ keadaan\ D(E) = \frac{3N}{2E_F}$$

Maka :

$$Cv = \ {k_B}^2T\frac{3N}{2E_F}\frac{\pi^2}{3}$$

$$Cv = \ {k_B}^2T\frac{N}{2E_F}\pi^2$$

$$Cv = \frac{\pi^2Tk_B{}^2N}{2E_F} \leftarrow \text{Terbukti}$$

6. untuk elektron bebas

$$E_k = \frac{\hbar^2}{2m}(kx^2 + ky^2 + kz^2)$$

Dimana : kx, ky, kz = 0; $\pm\frac{2\pi}{L}$; $\pm\frac{4\pi}{L}$; … ….

Sehingga :

$$\psi_{(+)} = exp(i\,\pi x/a) + exp(i\,\pi x/a) = 2\cos\frac{\pi x}{a}$$

$$\psi_{(-)} = exp(i\,\pi x/a) - exp(i\,\pi x/a) = 2i\sin\frac{\pi x}{a}$$

- Misalkan gelombang berdiri pada persamaan di atas

$$\psi^*\psi = |\psi|^2$$

$$\rho = exp(-ikx)\,exp(-ikx) = 0$$

$$\rho_{(+)} = \left|\psi_{(+)}\right|^2 cos^2\frac{\pi x}{a}$$

$$\rho_{(-)} = \left|\psi_{(-)}\right|^2 sin^2\frac{\pi x}{a}$$

- Besarnya energi Gap

$$V(x) = V\cos\frac{2\pi x}{a}$$

$$E_g = \int_0^1 dx\;\; V(x)\left[\left|\psi_{(+)}\right|^2 + \left|\psi_{(-)}\right|^2\right]$$

$$= 2\int dx\,V\cos(2x\pi/a)\,(cos^2\,\pi x/a - sin^2\,\pi x/a) = 0$$

$$\psi_{(x+a)} = c\psi_{(x)}$$

$$\psi_{(x+Na)} = \psi_{(x)} = c^N\psi_{(x)}$$

$$c = exp(i\,2\pi s/N);\;\; s = 0, 1, 2, \dots \dots .., N = 1$$

Sehingga $\psi_{(x)} = U_k(x)\ exp(i\ 2\pi sx/Na)$

↘ untuk pemodelan kromy penny di atas

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + u(x)\psi = E\psi$$

Pada daerah 0 < x < a saat v = 0 maka:

$\psi = Ae^{ikx} + Be^{-ikx}$ dimana $E = \frac{\hbar^2 k^2}{2m}$

Pada daerah –b < x < 0 maka

$\psi = Ce^{Qx} + Be^{-Qx}$

$$dimana\ \ U_0 = E = \frac{\hbar^2 Q^2}{2m}$$

Dihubungkan solusi a < x < a+b harus dihubungkan pada daerah –b < x < 0

$$\psi(a < x < a + b) = \psi(-b < x < 0)e^{ik(a+b)}$$

Menggunakan syarat batas kontinuitas

A + B = C + D

$ik(A - B) = Q(C - D)$

$Ae^{ik\alpha} + Be^{-ik\alpha} = \ (Ce^{-Qb} + Be^{Qb})e^{ik(a+b)}$

$ik\left(Ae^{ika} - Be^{-ika}\right) = Q(Ce^{-Qb} - Be^{Qb})e^{ika}$

$jika\ P = \frac{Q^2 ba}{2}$ maka

$$\left(\frac{p}{ka}\right)\sin ka + \cos ka = \cos ka$$