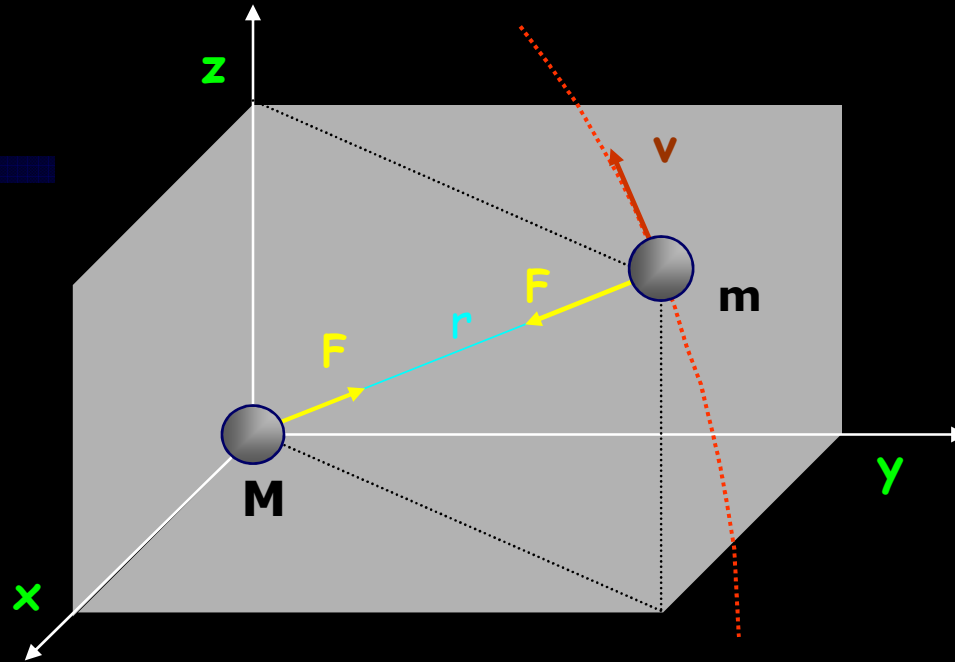


# Sistim Dua Benda Langit



$$m_1 \frac{d^2 \mathbf{r}}{dt^2} = -G \frac{m_1 m_2}{r^2}$$

$$m_1 \frac{d^2 x_1}{dt^2} = -G m_1 m_2 \frac{x_1 - x_2}{r^3}$$

$$m_1 \frac{d^2 y_1}{dt^2} = -G m_1 m_2 \frac{y_1 - y_2}{r^3}$$

$$m_1 \frac{d^2 z_1}{dt^2} = -G m_1 m_2 \frac{z_1 - z_2}{r^3}$$

**Didefinisikan:**

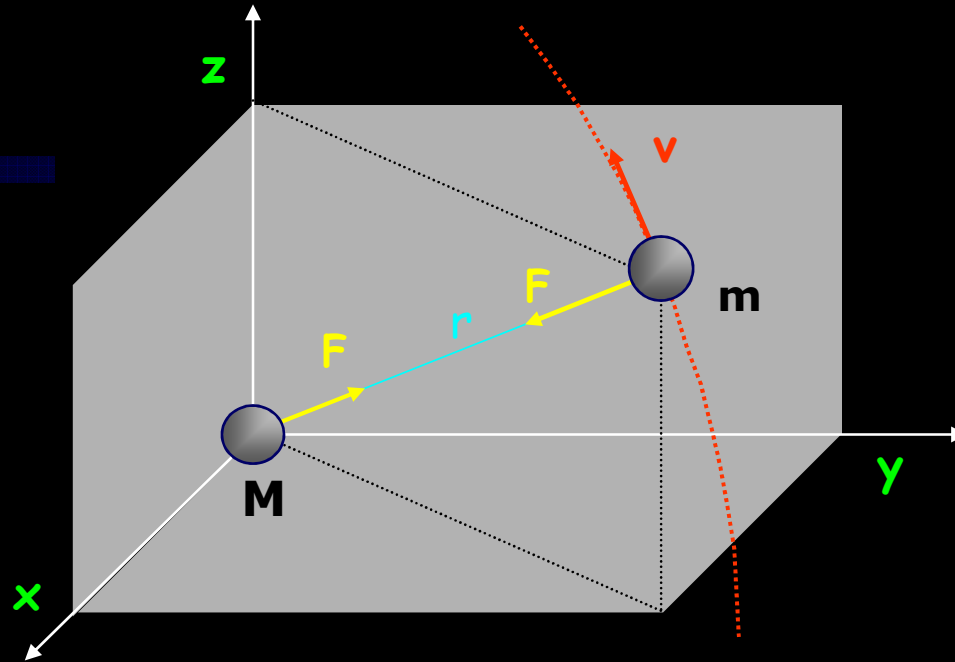
$$\mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1$$

$$\mathbf{y} = \mathbf{y}_2 - \mathbf{y}_1$$

$$\mathbf{z} = \mathbf{z}_2 - \mathbf{z}_1$$

$$\mathbf{M} = m_1 + m_2$$

# Sistim Dua Benda Langit



$$\frac{d^2x}{dt^2} = -G \frac{Mx}{r^3}$$

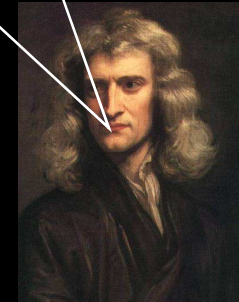
$$\frac{d^2y}{dt^2} = -G \frac{My}{r^3}$$

$$\frac{d^2z}{dt^2} = -G \frac{Mz}{r^3}$$

$$a_1 x + a_2 y + a_3 z = 0$$

Persamaan bidang datar

m bergerak pada bidang datar yang melalui M



# Sistim Dua Benda Langit

$$a_1 x + a_2 y + a_3 z = 0$$

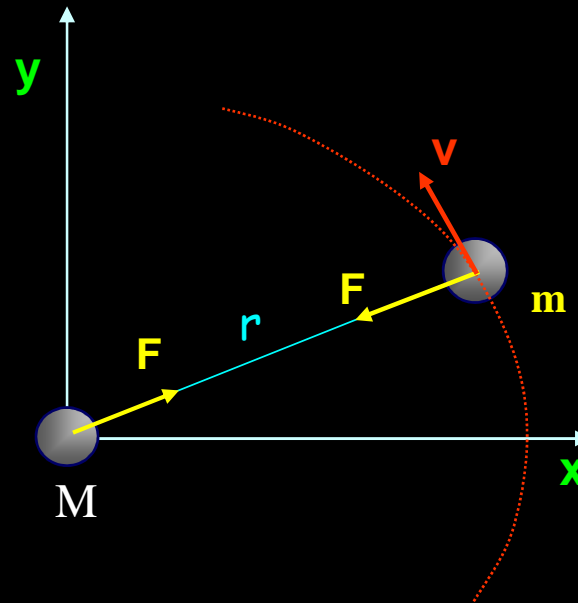
Persamaan bidang datar

m bergerak pada  
bidang datar yang  
melalui M



# Sistim Dua Benda Langit

Untuk penyederhanakan,  
ambil bidang orbit  
dalam bidang (x, y).



Gerak benda hanya ditentukan oleh dua persamaan yang mengandung variabel x dan y, yaitu:

$$\frac{d^2 \mathbf{x}}{dt^2} = -\mathbf{G} \frac{M \mathbf{x}}{r^3}$$

dikalikan dengan

$$2 \frac{dx}{dt}$$

$$\frac{d^2 \mathbf{y}}{dt^2} = -\mathbf{G} \frac{M \mathbf{y}}{r^3}$$

dikalikan dengan

$$2 \frac{dy}{dt}$$

$$\frac{d}{dt} \left\{ \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right\} = -\frac{2GM}{r^3} \left( x \frac{dx}{dt} + y \frac{dy}{dt} \right)$$

# Sistim Dua Benda Langit

$$\frac{d}{dt} \left\{ \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right\} = - \frac{2GM}{r^3} \left( x \frac{dx}{dt} + y \frac{dy}{dt} \right)$$

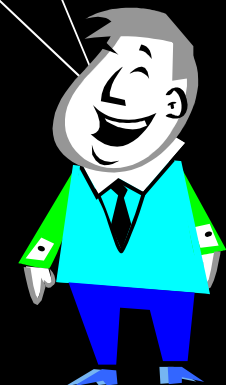
Dalam koordinat polar:

$$\left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\theta}{dt} \right)^2 - 2 \frac{GM}{r} = h$$

Solusinya:

$$r = \frac{h^2}{G(m_1 + m_2)(1 + e \cos \theta)}$$

Persamaan konik



# Energi Sistim Dua Benda Langit

$$\frac{d^2x}{dt^2} = -G \frac{Mx}{r^3}$$

$$\frac{d^2y}{dt^2} = -G \frac{My}{r^3}$$

$$\frac{d^2z}{dt^2} = -G \frac{Mz}{r^3}$$

$$\frac{d}{dt} \left[ \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 + \left( \frac{dz}{dt} \right)^2 \right] = -\frac{2GM}{r^3} \left( x \frac{dx}{dt} + y \frac{dy}{dt} + z \frac{dz}{dt} \right)$$

$$\frac{dv^2}{dt} = -2 \frac{GM}{r^2} \left( \frac{dr}{dt} \right) \Rightarrow v^2 = -2 \frac{GM}{r} + C$$

$$EK = \frac{1}{2} m_1 v^2 = \frac{GMm_1}{r} + \frac{1}{2} m_1 C$$

$$EP = -\frac{GMm_1}{r}$$

$$EK + EP = \frac{1}{2} m_1 C = \text{Konstan}$$

Energi total sistim tetap