PROJECTILE MOTION
Senior High School Physics

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2006

Part 1.

Part 2.

Introduction

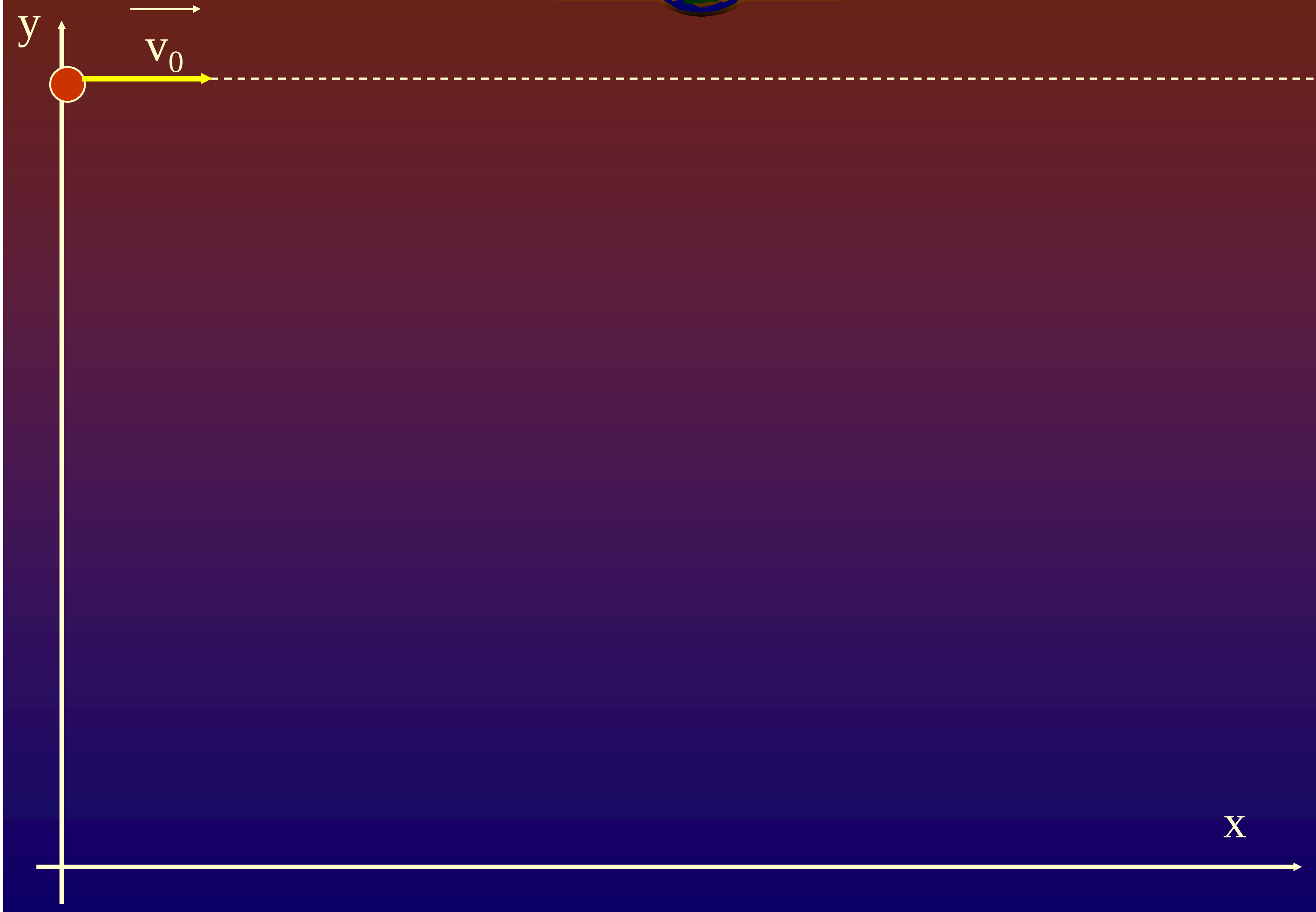
❖ Projectile Motion:

Motion through the air without a propulsion

❖ Examples:

Part 1.

Motion of Objects Projected Horizontally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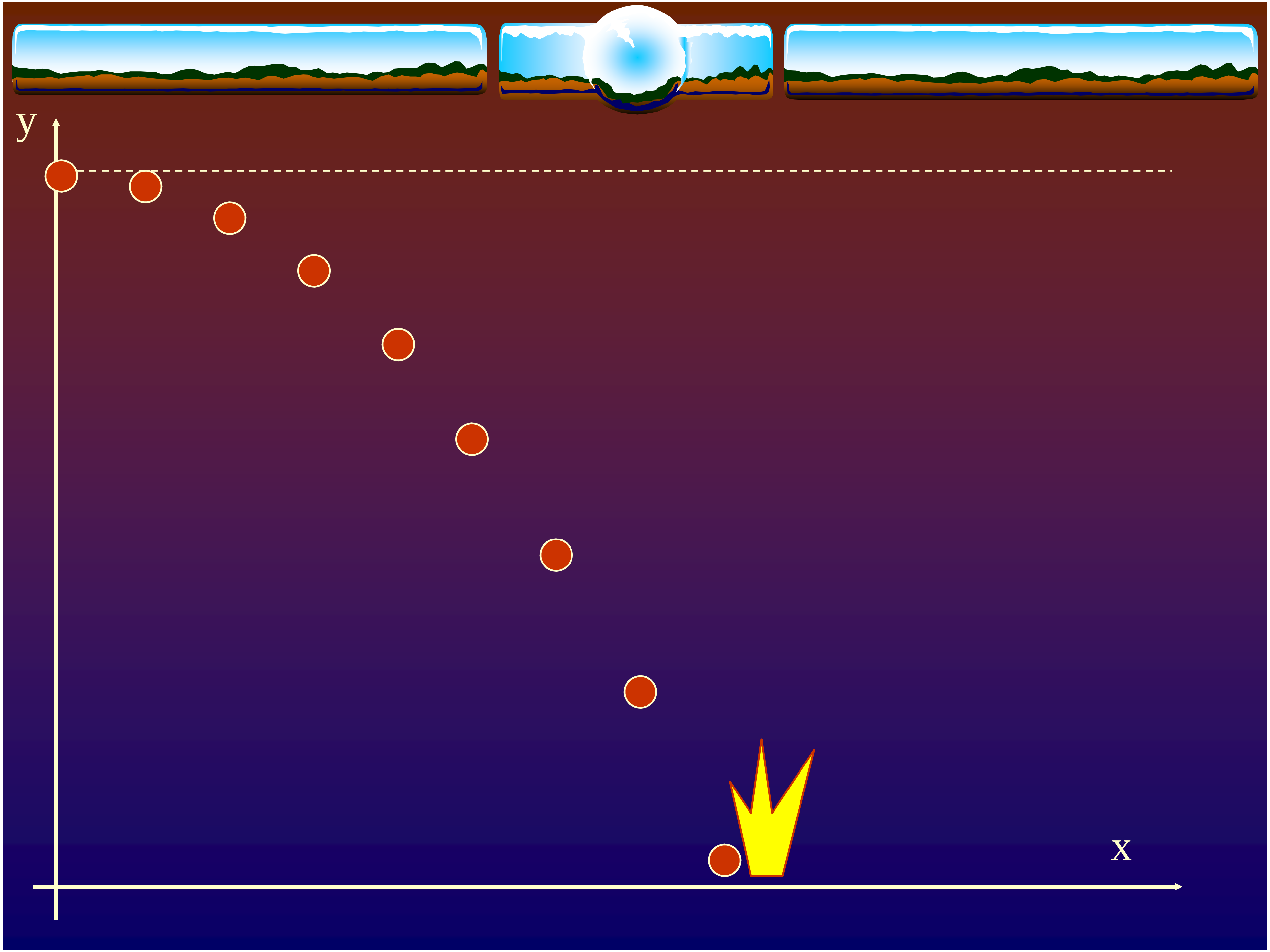
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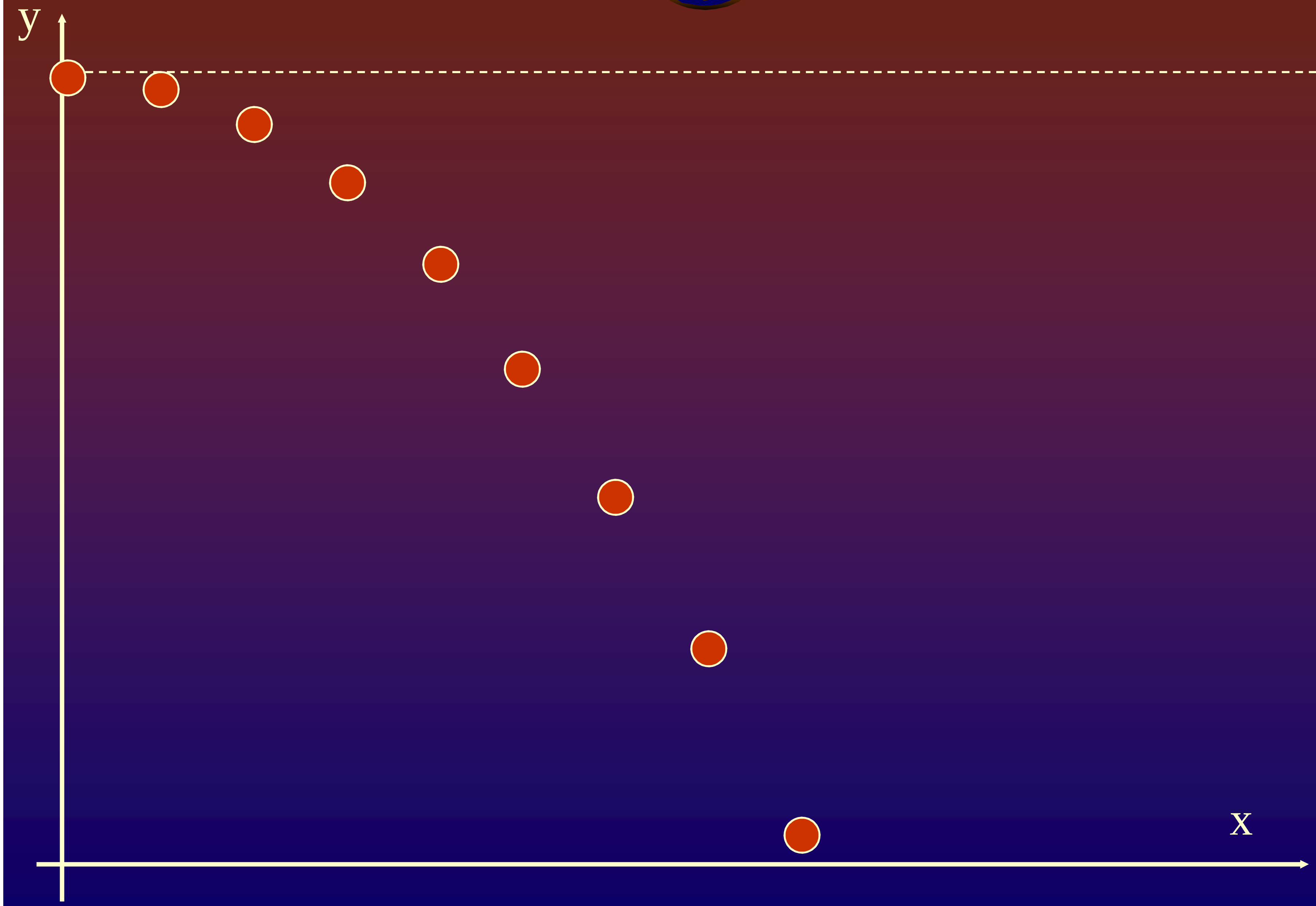
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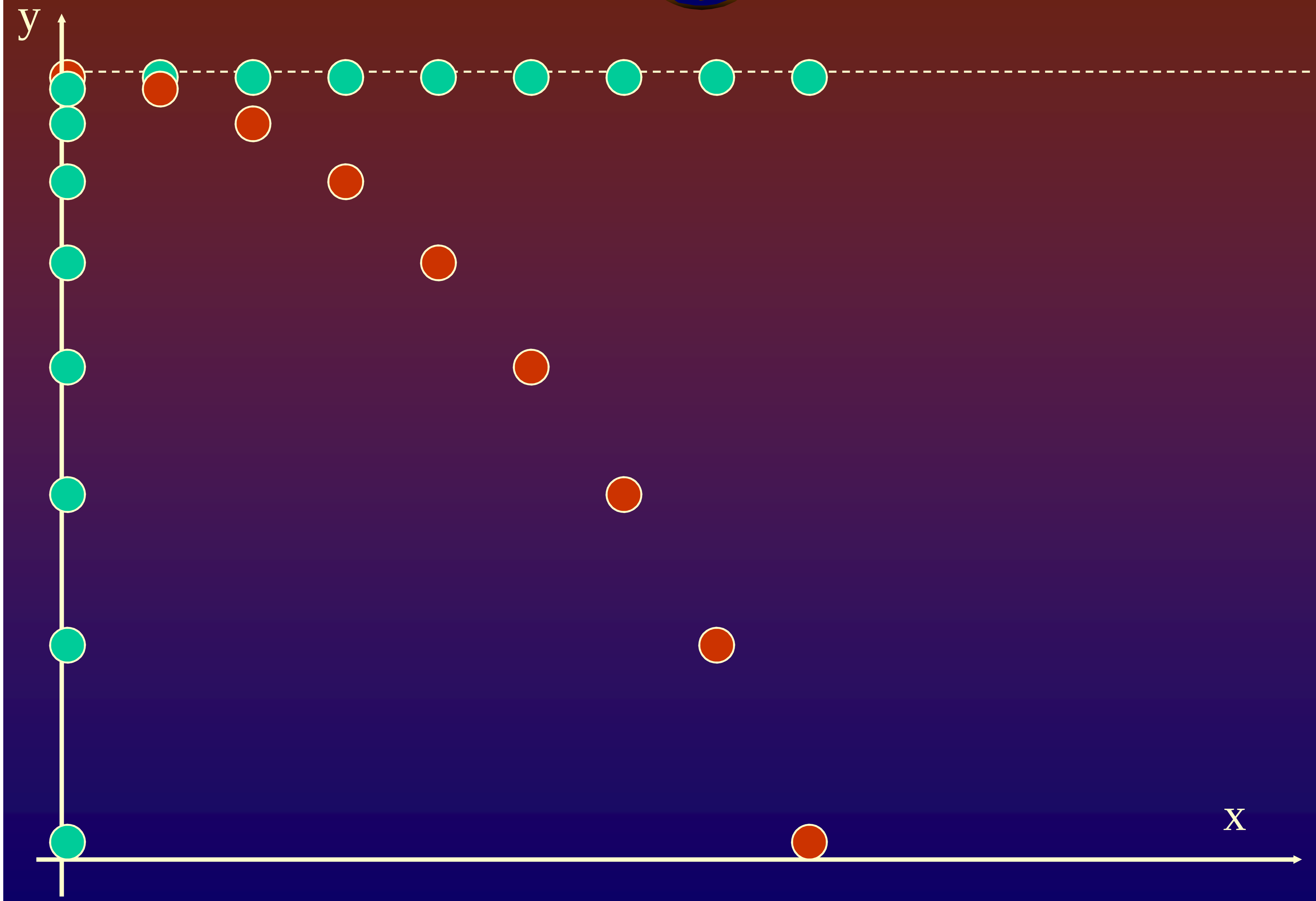
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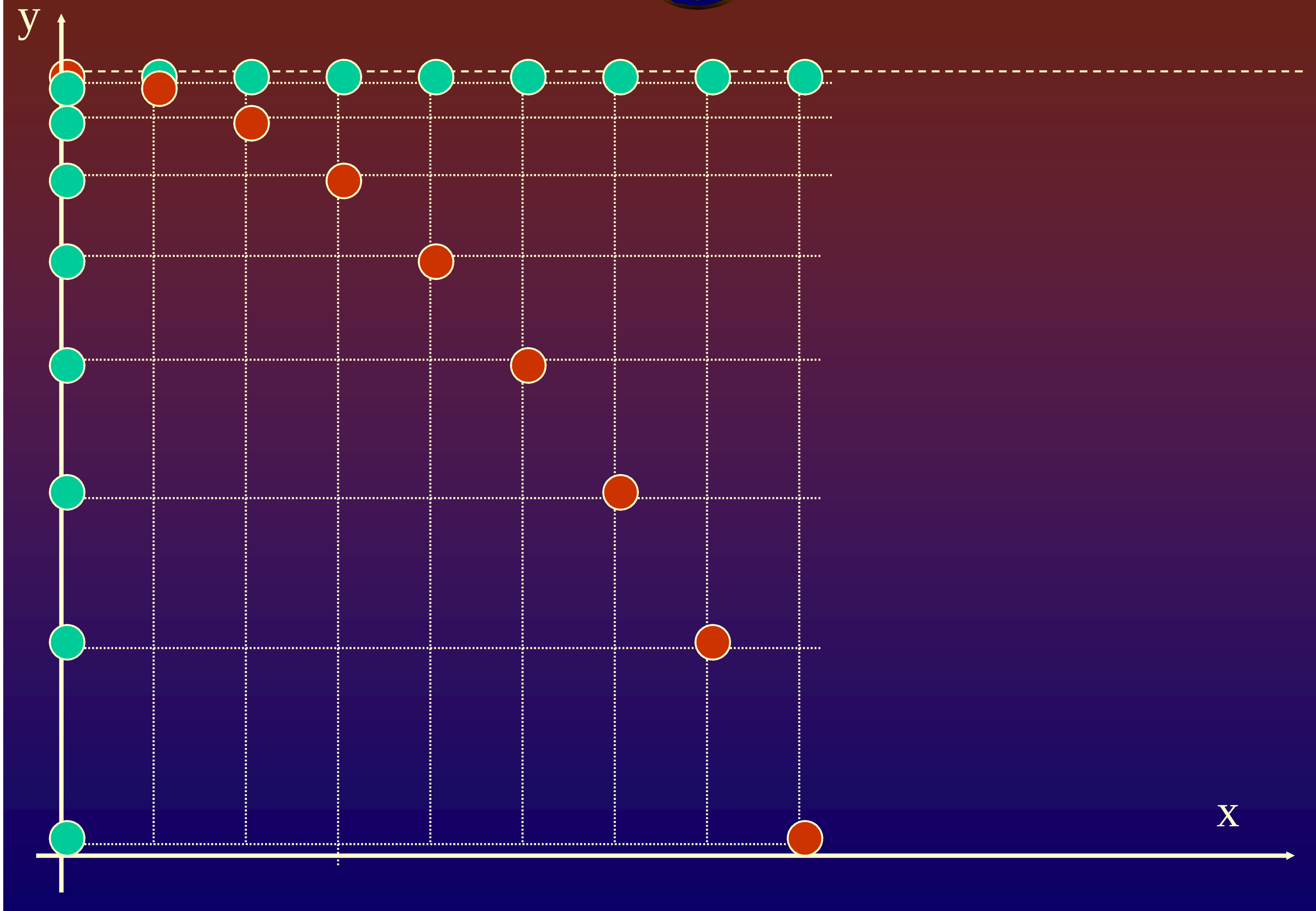
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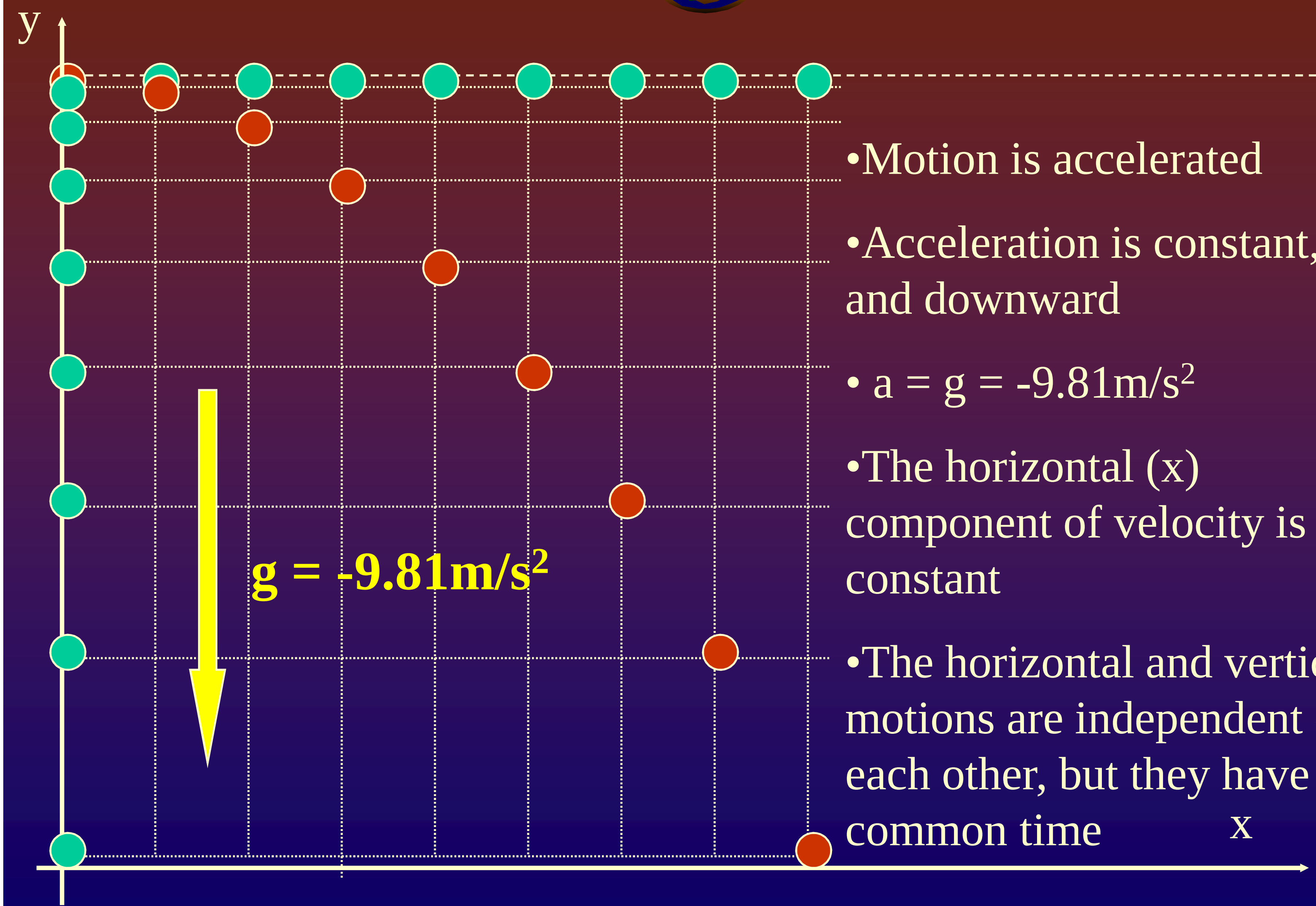


- Motion is accelerated
- Acceleration is constant, and downward
- $a = g = -9.81 m/s^2$
- The horizontal (x) component of velocity is constant
- The horizontal and vertical motions are independent of each other, but they have a common time

ANALYSIS OF MOTION

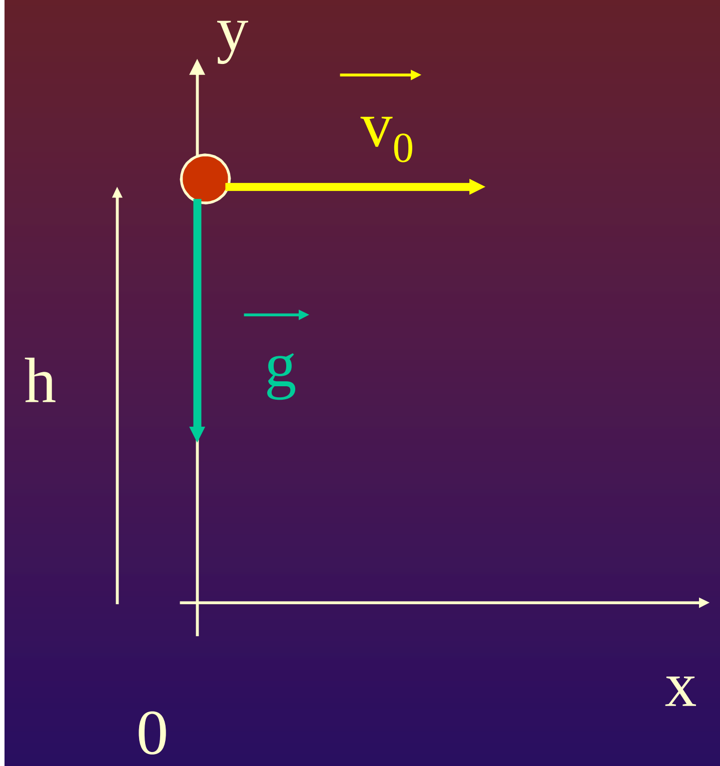
ASSUMPTIONS:

- x-direction (horizontal): uniform motion
- y-direction (vertical): accelerated motion
- no air resistance

QUESTIONS:

- What is the trajectory?
- What is the total time of the motion?
- What is the horizontal range?
- What is the final velocity?

Frame of reference:



Equations of motion:

	X Uniform m.	Y Accel. m.
ACCL.	$a_x = 0$	$a_y = g = -9.81\ m/s^2$
VELC.	$v_x = v_0$	$v_y = g\ t$
DSPL.	$x = v_0\ t$	$y = h + ½\ g\ t^2$

Trajectory

$x = v_0\, t$

$y = h + \frac{1}{2}\, g\, t^2$

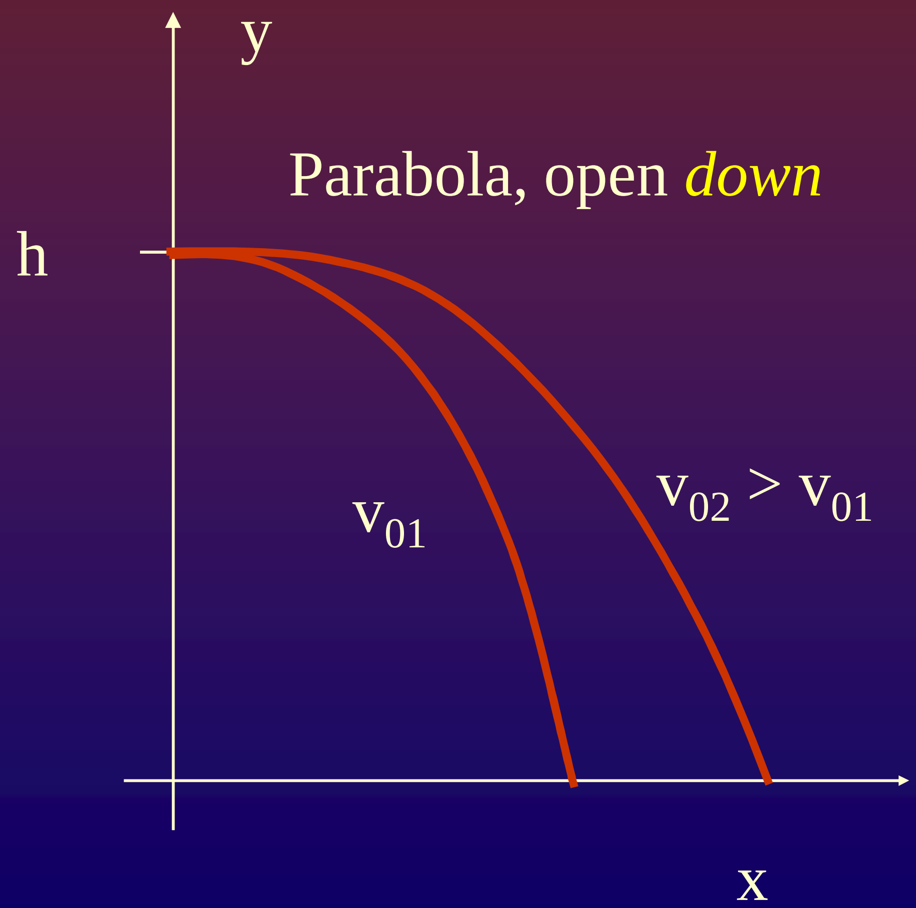
Eliminate time, t

$t = x/v_0$

$y = h + \frac{1}{2}\, g\, (x/v_0)^2$

$y = h + \frac{1}{2}\, (g/v_0^2)\, x^2$

$$y = \frac{1}{2}\, (g/v_0^2)\, x^2 + h$$



Total Time, Δt

$\Delta t = t_f - t_i$

$y = h + ½\, g\, t^2$

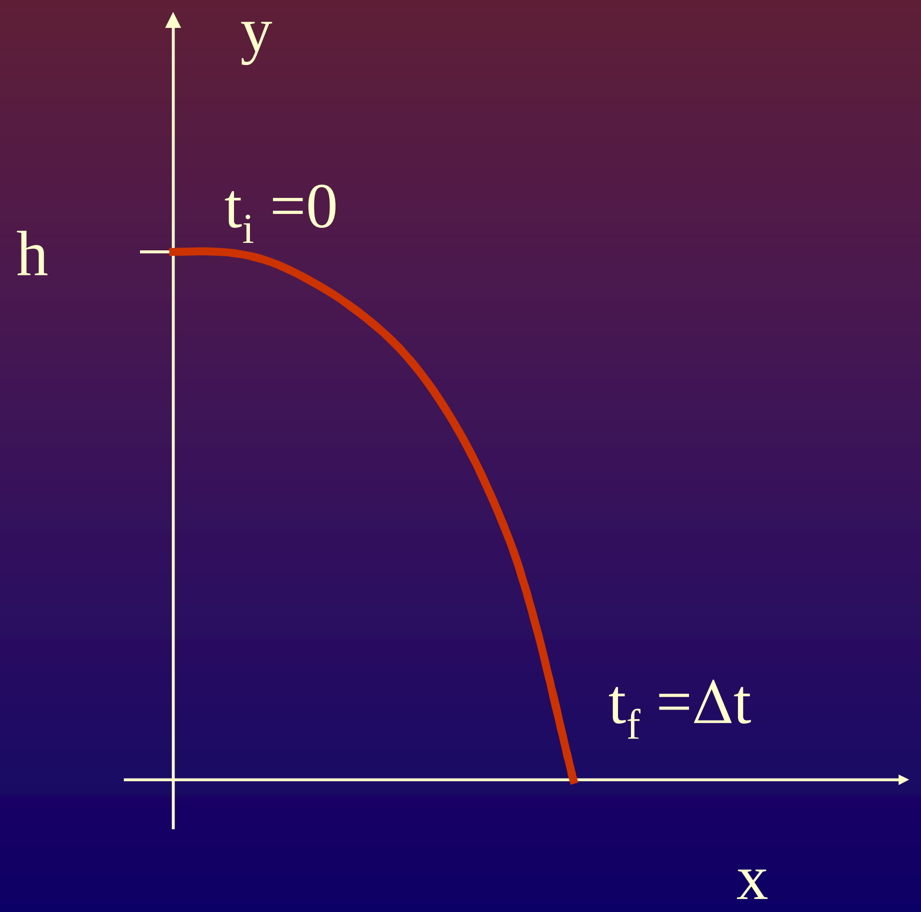
final $y = 0$

$0 = h + ½\, g\, (\Delta t)^2$

Solve for Δt:

$$\Delta t = \sqrt{2h/(-g)}$$

$$\Delta t = \sqrt{2h/(9.81\text{ms}^{-2})}$$



Total time of motion depends only on the initial height, h

Horizontal Range, Δx

$$x = v_0 t$$

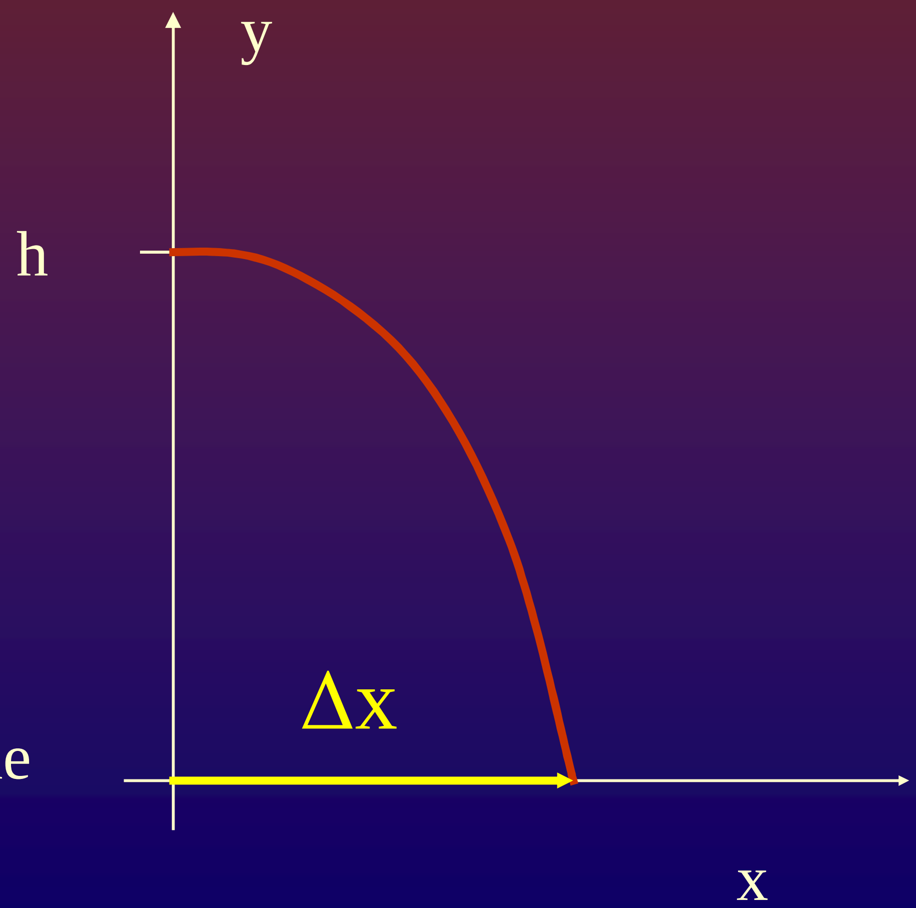
final y = 0, time is the total time Δt

$$\Delta x = v_0 \Delta t$$

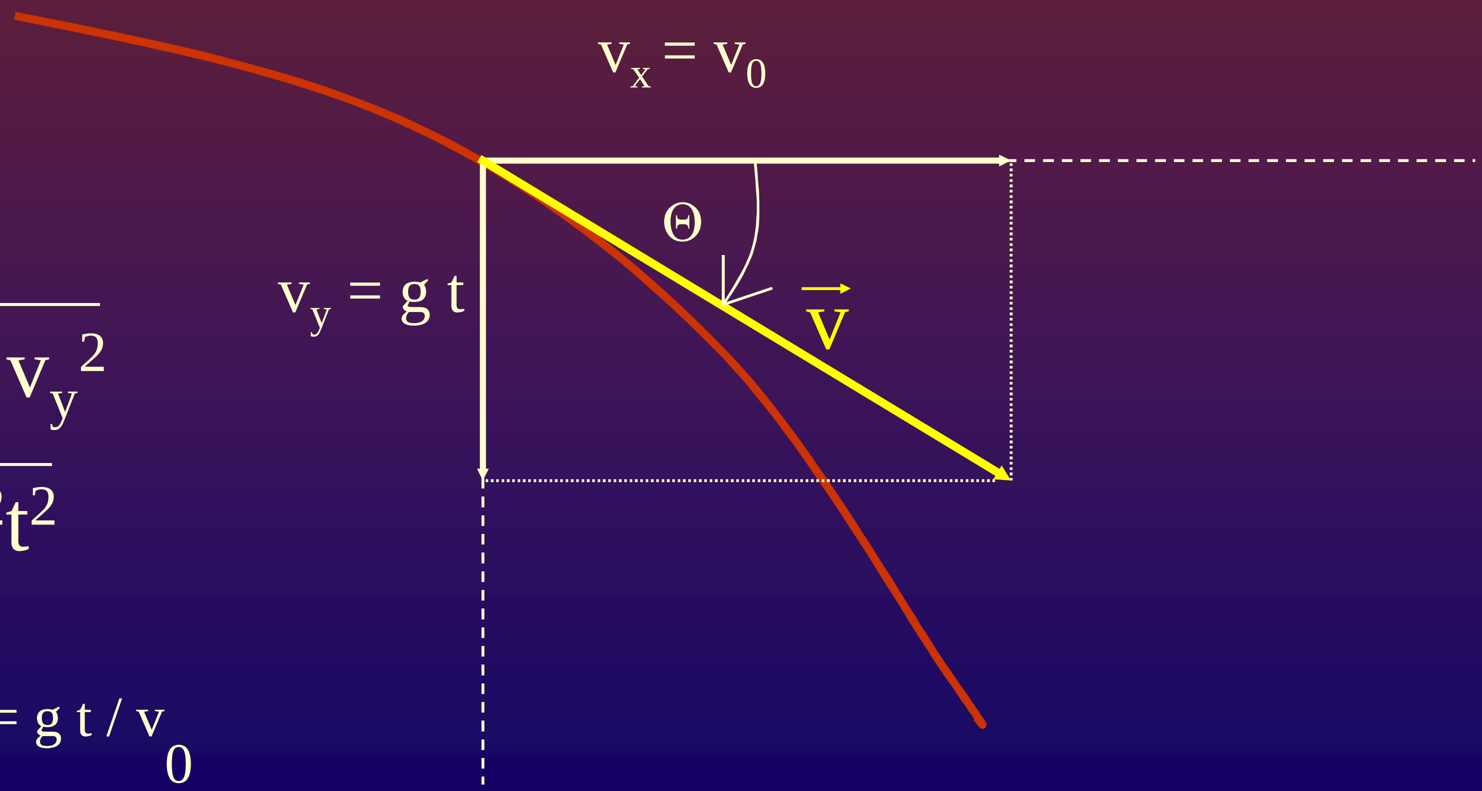
$$\Delta t = \sqrt{2h/(-g)}$$

$$\Delta x = v_0 \sqrt{2h/(-g)}$$

Horizontal range depends on the initial height, h, and the initial velocity, v_0



VELOCITY

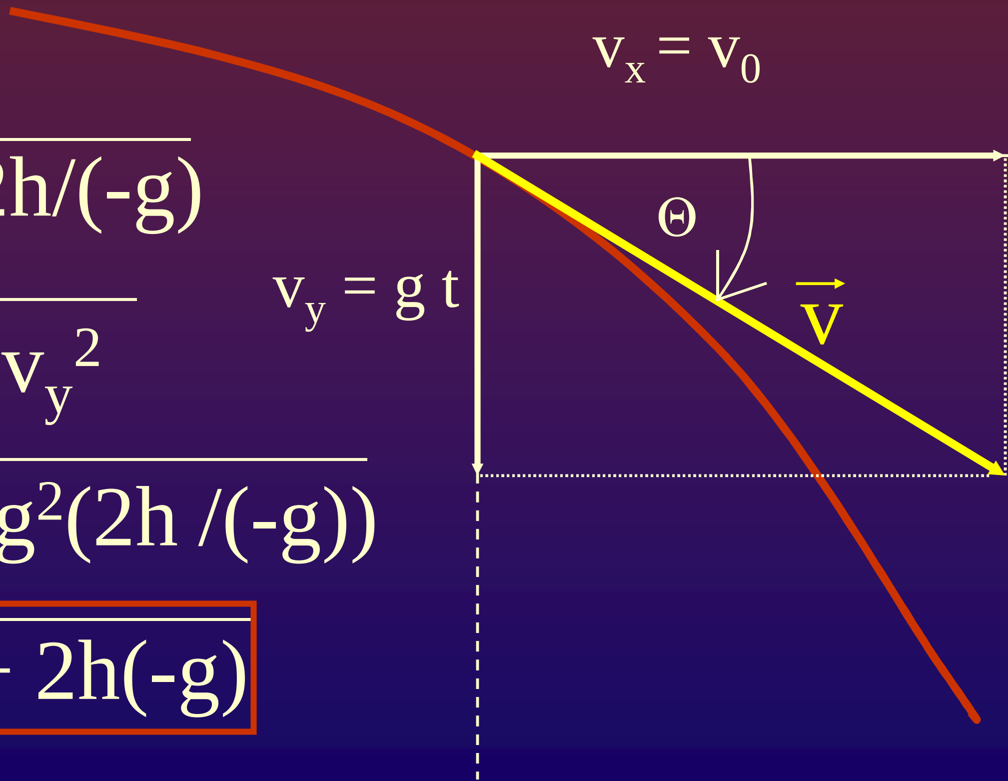


$$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

$$= \sqrt{v_0^2 + g^2 t^2}$$

$$\mathrm{tg}\,\Theta = v_y / v_x = g\,t / v_0$$

FINAL VELOCITY



$$\Delta t = \sqrt{2h/(-g)}$$

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

$$v = \sqrt{v_0^2 + g^2(2h/(-g))}$$

$$v = \sqrt{v_0^2 + 2h(-g)}$$

$$\operatorname{tg}\Theta = g\,\Delta t / v_0$$

$$= -(-g)\sqrt{2h/(-g)} / v_0$$

$$= -\sqrt{2h(-g)} / v_0$$

Θ is *negative* (below the horizontal line)

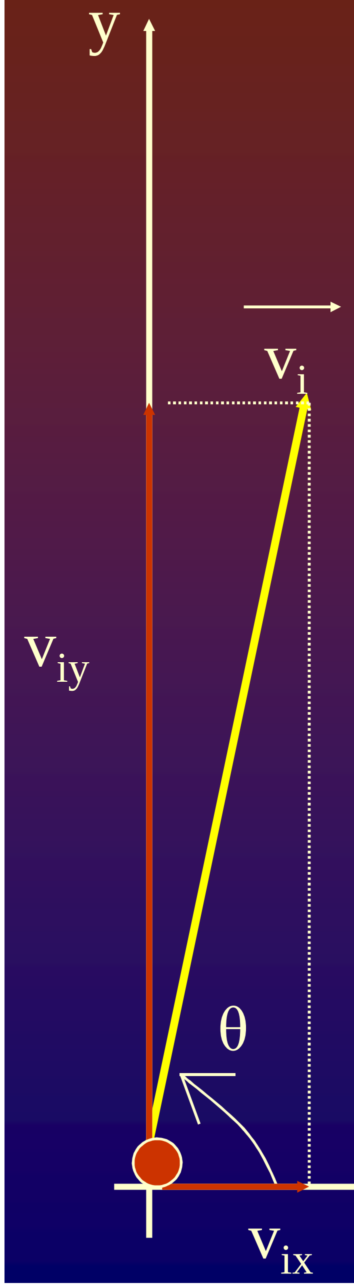
HORIZONTAL THROW - Summary

h – initial height, v_0 – initial horizontal velocity, $g = -9.81 m/s^2$

Trajectory	Half -parabola, open down
Total time	$\Delta t = \sqrt{2h/(-g)}$
Horizontal Range	$\Delta x = v_0 \sqrt{2h/(-g)}$
Final Velocity	$v = \sqrt{v_0^2 + 2h(-g)}$ $tg\,\Theta = -\sqrt{2h(-g)} / v_0$

Part 2.

Motion of objects projected at an angle



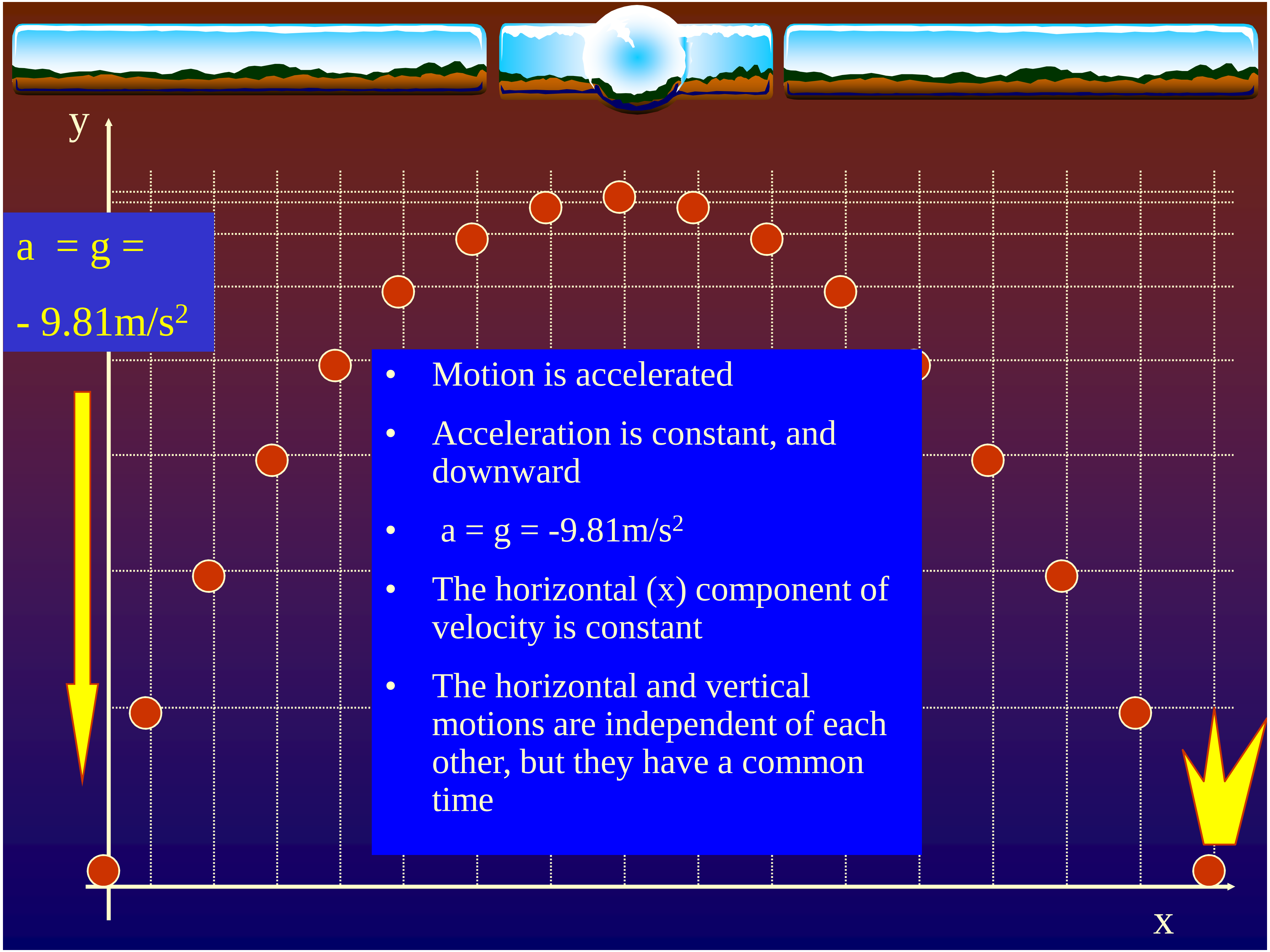
Initial position: $x = 0, y = 0$

Initial velocity: $\vec{vi} = v_i\ [\Theta]$

Velocity components:

x- direction : $v_{ix} = v_i \cos \Theta$

y- direction : $v_{iy} = v_i \sin \Theta$



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- Acceleration is constant, and downward
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ANALYSIS OF MOTION:

ASSUMPTIONS

- x-direction (horizontal): uniform motion
- y-direction (vertical): accelerated motion
- no air resistance

QUESTIONS

- What is the trajectory?
- What is the total time of the motion?
- What is the horizontal range?
- What is the maximum height?
- What is the final velocity?

Equations of motion:

	X Uniform motion	Y Accelerated motion
ACCELERATION	$a_x = 0$	$a_y = g = -9.81\ m/s^2$
VELOCITY	$v_x = v_{ix} = v_i \cos \Theta$ $v_x = v_i \cos \Theta$	$v_y = v_{iy} + g\,t$ $v_y = v_i \sin \Theta + g\,t$
DISPLACEMENT	$x = v_{ix}\,t = v_i\,t \cos \Theta$ $x = v_i\,t \cos \Theta$	$y = h + v_{iy}\,t + \frac{1}{2}\,g\,t^2$ $y = v_i\,t \sin \Theta + \frac{1}{2}\,g\,t^2$

Equations of motion:

	X Uniform motion	Y Accelerated motion
ACCELERATION	$a_x = 0$	$a_y = g = -9.81\ m/s^2$
VELOCITY	$v_x = v_i \cos \Theta$	$v_y = v_i \sin \Theta + g\,t$
DISPLACEMENT	$x = v_i\, t \cos \Theta$	$y = v_i\, t \sin \Theta + ½\, g\, t^2$

Trajectory

$x = v_i\, t \cos \Theta$

$y = v_i\, t \sin \Theta + \frac{1}{2} g\, t^2$

Eliminate time, t

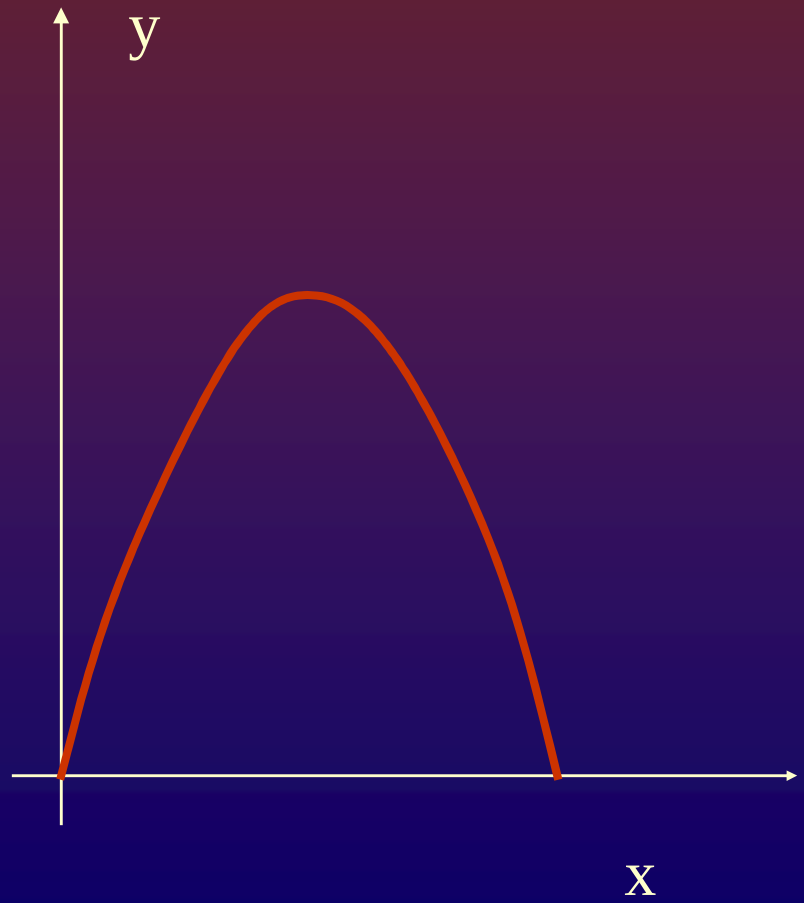
$t = x/(v_i \cos \Theta)$

$$y = \frac{v_i x \sin \Theta}{v_i \cos \Theta} + \frac{g x^2}{2 v_i^2 \cos^2 \Theta}$$

$$y = x \tan \Theta + \frac{g}{2 v_i^2 \cos^2 \Theta} x^2$$

$y = bx + ax^2$

Parabola, open *down*



Total Time, Δt

$y = v_i\, t \sin \Theta + \frac{1}{2}\, g\, t^2$

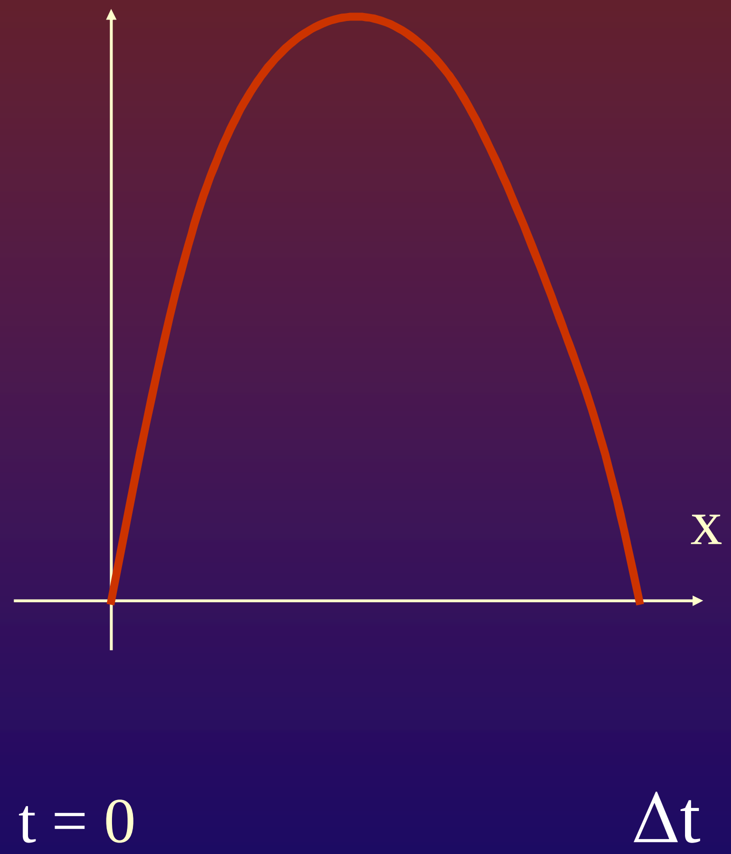
final height $y = 0$, after time interval Δt

$0 = v_i\, \Delta t \sin \Theta + \frac{1}{2}\, g\, (\Delta t)^2$

Solve for Δt:

$0 = v_i \sin \Theta + \frac{1}{2}\, g\, \Delta t$

$$\Delta t = \frac{2\, v_i \sin \Theta}{(-g)}$$



Horizontal Range, Δx

$$x = v_i \, t \cos \Theta$$

final y = 0, time is the total time Δt

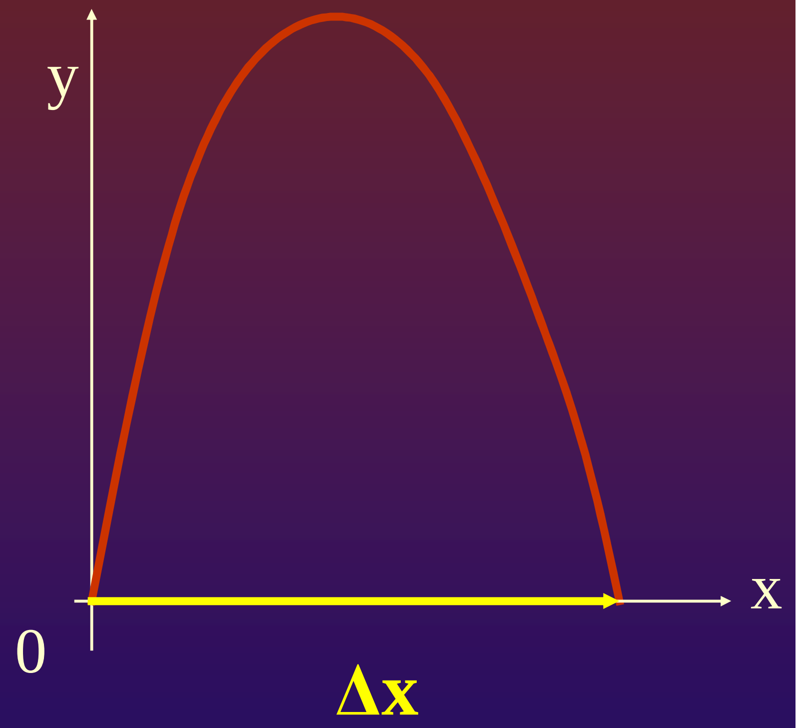
$$\Delta x = v_i \, \Delta t \cos \Theta$$

$$\Delta t = \frac{2 \, v_i \sin \Theta}{(-g)}$$

$$\sin(2\Theta) = 2 \sin \Theta \cos \Theta$$

$$\Delta x = \frac{2 v_i^{\,2} \sin \Theta \cos \Theta}{(-g)}$$

$$\Delta x = \frac{v_i^{\,2} \sin(2\Theta)}{(-g)}$$



Horizontal Range, Δx

$$\Delta x = \frac{v_i^{\,2} \sin(2\Theta)}{(-g)}$$

Θ (deg)	sin (2 Θ)
0	0.00
15	0.50
30	0.87
45	1.00
60	0.87
75	0.50
90	0

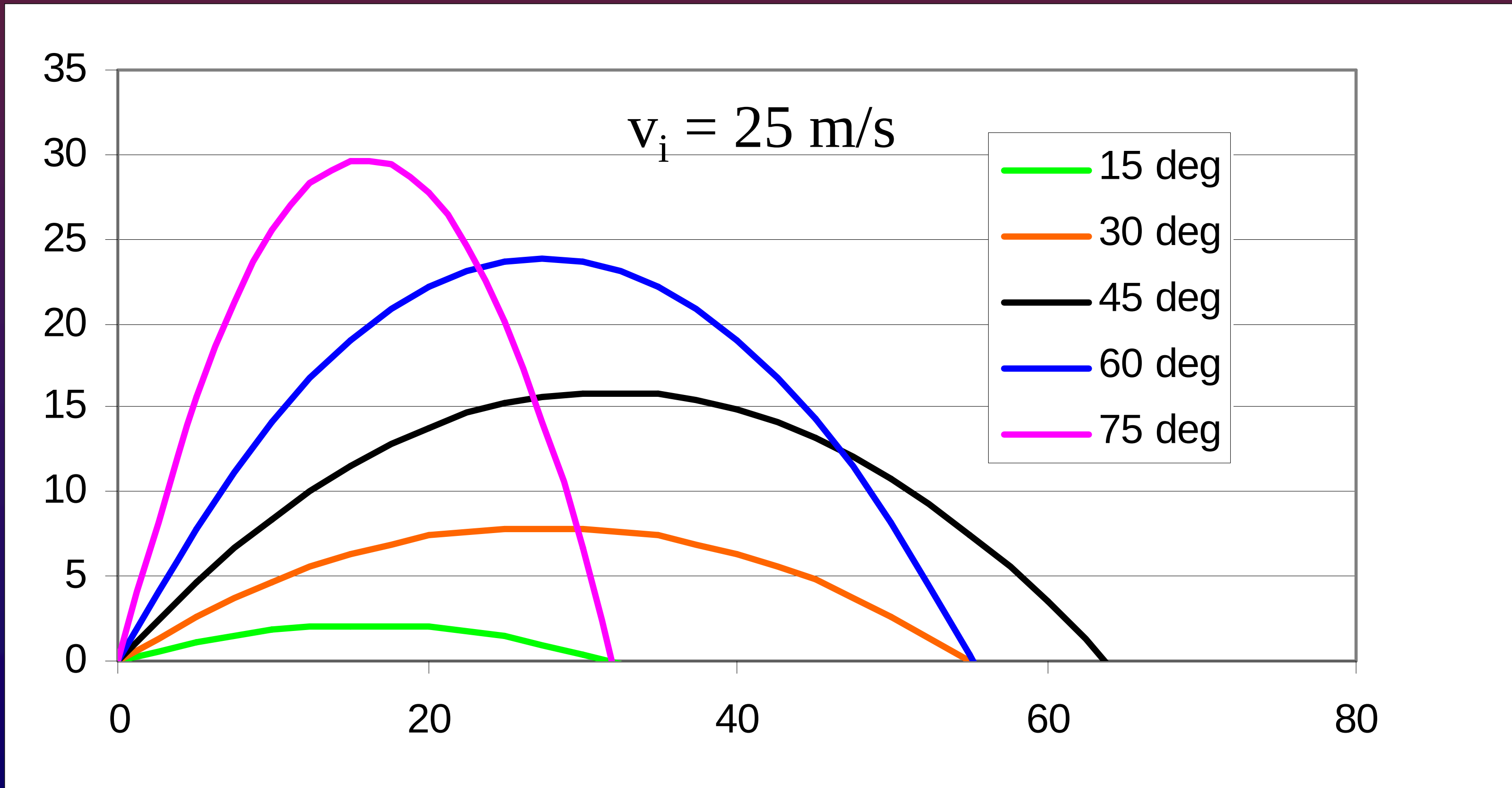
•CONCLUSIONS:

•Horizontal range is greatest for the throw angle of 45^0

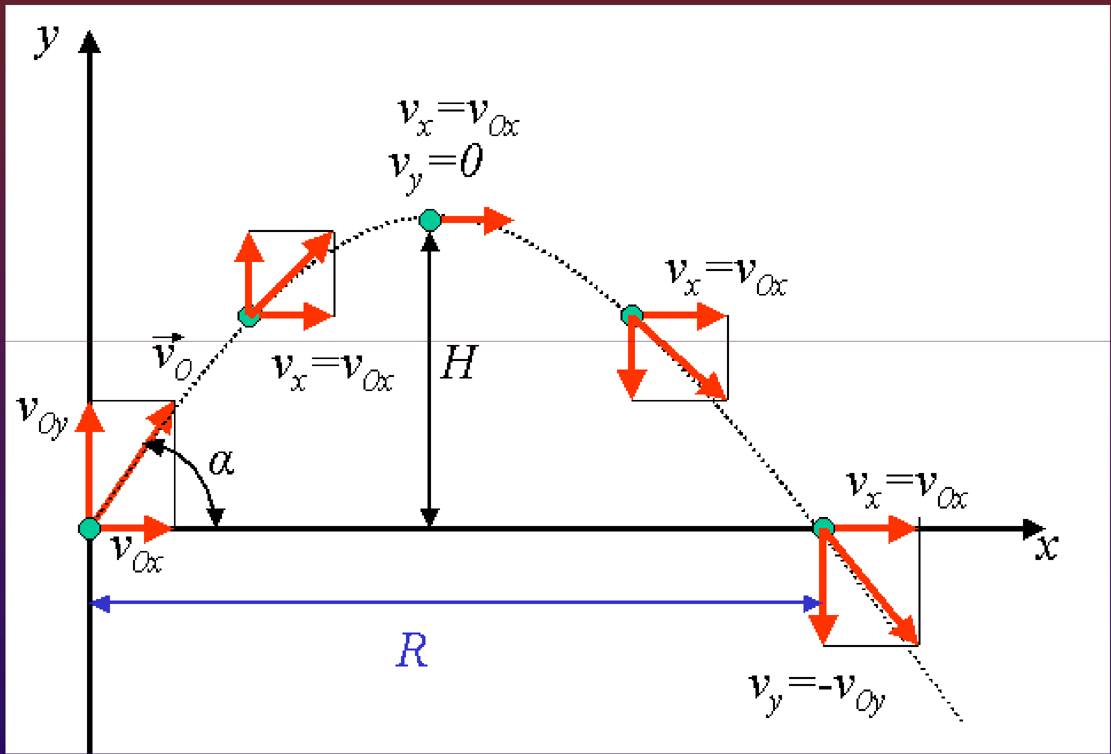
• Horizontal ranges are the same for angles Θ and $(90^0 - \Theta)$

Trajectory and horizontal range

$$y = x\tan\Theta + \frac{g}{2v_i^2\cos^2\Theta}x^2$$



Velocity



- Final speed = initial speed *(conservation of energy)*
- Impact angle = - launch angle *(symmetry of parabola)*

Maximum Height

$v_y = v_i \sin \Theta + g\,t$

$y = v_i\, t \sin \Theta + \frac{1}{2} g\, t^2$

At maximum height $v_y = 0$

$0 = v_i \sin \Theta + g\, t_{up}$

$$t_{up} = \frac{v_i \sin \Theta}{(-g)}$$

$t_{up} = \Delta t/2$

$h_{max} = v_i\, t_{up} \sin \Theta + \frac{1}{2} g\, t_{up}^{\ 2}$

$h_{max} = v_i^2 \sin^2 \Theta/(-g) + \frac{1}{2} g(v_i^2 \sin^2 \Theta)/g^2$

$$h_{max} = \frac{v_i^2 \sin^2 \Theta}{2(-g)}$$

Projectile Motion – Final Equations

(0,0) – initial position, $\vec{v}_i = v_i\ [\Theta]$– initial velocity, $g = -9.81 m/s^2$

Trajectory	Parabola, open down
Total time	$\Delta t = \dfrac{2\ v_i \sin \Theta}{(-g)}$
Horizontal range	$\Delta x = \dfrac{v_i^{\ 2} \sin (2\,\Theta)}{(-g)}$
Max height	$h_{max} = \dfrac{v_i^2 \sin^2 \Theta}{2(-g)}$

PROJECTILE MOTION - SUMMARY

- Projectile motion is motion with a constant horizontal velocity combined with a constant vertical acceleration
- The projectile moves along a parabola