

PENJUMLAHAN MOMENTUM SUDUT

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Representasi elektron lengkap memerlukan informasi ruang dan

$$\text{spin } |x, y, z, s_z = \pm\rangle = |xyz\rangle |s_z = \pm\rangle$$

$$\text{Untuk elektron bebas } |xyz\rangle = Ae^{i\vec{k}\cdot\vec{r}} : \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Untuk electron yang terikat atom keadaan ruangnya dinyatakan $|n, l, m\rangle$ sehingga eigen state : $|nlm_0, m_0\rangle = |nlm_l\rangle |m_0\rangle$ Ruang $|nlm_l\rangle = |nl\rangle |lm\rangle$

Dengan :

$$L_z |lm\rangle = m\hbar |lm\rangle \quad ; L^2 |lm\rangle = l(l+1)\hbar^2 |lm\rangle$$

$$L_+ |lm\rangle = [(l+m+1)(l-m)]^{\frac{1}{2}} \hbar |lm+1\rangle$$

$$L_- |lm\rangle = [(l-m+1)(l+m)]^{\frac{1}{2}} \hbar |lm-1\rangle$$

$$\text{Spin } |ms\rangle = |\pm\rangle \rightarrow s_z |\pm\rangle = \pm \frac{1}{2} \hbar |\pm\rangle \quad s_+ |-\rangle = \hbar |+\rangle$$

$$s_+ |+\rangle = s_- |-\rangle = 0 \quad s_- |+\rangle = \hbar |-\rangle$$

Penjumlahan spin orbit

$$\vec{J} = \vec{L} + \vec{S}$$

$$\begin{aligned}\vec{J} \cdot \vec{J} &= L^2 + S^2 + 2(L_x + L_y + L_z) \cdot (s_x + s_y + s_z) \\ &= L^2 + S^2 + 2L_z s_x + L_+ s_- + L_- s_+\end{aligned}$$

$$\begin{aligned}J^2 |jm\rangle &= J^2 \left[\alpha \left| lm - \frac{1}{2}, \frac{1}{2} \right\rangle + \beta \left| lm + \frac{1}{2}, -\frac{1}{2} \right\rangle \right] = j(j+1)\hbar^2 \left(\alpha \left| lm - \frac{1}{2}, \frac{1}{2} \right\rangle + \beta \left| lm + \frac{1}{2}, -\frac{1}{2} \right\rangle \right) \\ &= \alpha \hbar^2 \left\{ \left[1(1+1) + \frac{3}{4} + 2 \left(m - \frac{1}{2} \right) \left(\frac{1}{2} \right) \right] \left| lm - \frac{1}{2}, \frac{1}{2} \right\rangle + \left[\left(1 + m + \frac{1}{2} \right) \left(1 - m - \frac{1}{2} \right) \right]^{\frac{1}{2}} \left| lm - \frac{1}{2}, \frac{1}{2} \right\rangle \right\} \\ &= +\beta \hbar^2 \left\{ \left[1(1+1) + \frac{3}{4} + 2 \left(m + \frac{1}{2} \right) \left(-\frac{1}{2} \right) \right] \left| lm + \frac{1}{2}, -\frac{1}{2} \right\rangle + \left[\left(1 - m + \frac{1}{2} \right) \left(1 + m - \frac{1}{2} \right) \right]^{\frac{1}{2}} \left| lm + \frac{1}{2}, -\frac{1}{2} \right\rangle \right\}\end{aligned}$$

$$\text{maka : } \alpha \hbar^2 \left\{ \left[1(1+1) + \frac{3}{4} + \left(m - \frac{1}{2} \right) \right] + \beta \left[\left(1 + m + \frac{1}{2} \right) \left(1 - m - \frac{1}{2} \right) \right]^{\frac{1}{2}} \hbar^2 \right\} = \alpha j(j+1)\hbar^2$$

$$\beta \hbar^2 \left\{ 1(1+1) + \frac{3}{4} - \left(m + \frac{1}{2} \right) + \alpha \left[\left(1 + m + \frac{1}{2} \right) \left(1 + m - \frac{1}{2} \right) \right]^{\frac{1}{2}} \hbar^2 \right\} = \beta j(j+1)\hbar^2$$

diperoleh: $j = 1 + \frac{1}{2}$ dan $j = 1 - \frac{1}{2}$ buktikan!!!!

untuk $j = 1 + \frac{1}{2} \rightarrow \frac{\beta}{\alpha} = \left(\frac{1 + \frac{1}{2} - m}{1 + \frac{1}{2} - m} \right)^{\frac{1}{2}}$ buktikan!!!!

Normalisasi $\rightarrow \alpha = \sqrt{\frac{1 + \frac{1}{2} + m}{2l + 1}}$ $\beta = \sqrt{\frac{1 + \frac{1}{2} - m}{2l + 1}}$

$$\left| j = 1 + \frac{1}{2}, m \right\rangle = \sqrt{\frac{1 + \frac{1}{2} + m}{2l + 1}} \left| l, m - \frac{1}{2}, \frac{1}{2} \right\rangle + \sqrt{\frac{1 + \frac{1}{2} - m}{2l + 1}} \left| l, m + \frac{1}{2}, -\frac{1}{2} \right\rangle$$

$$\left| j = 1 - \frac{1}{2}, m \right\rangle = \sqrt{\frac{1 + \frac{1}{2} + m}{2l + 1}} \left| l, m - \frac{1}{2}, \frac{1}{2} \right\rangle + \sqrt{\frac{1 + \frac{1}{2} - m}{2l + 1}} \left| l, m + \frac{1}{2}, -\frac{1}{2} \right\rangle$$

Contoh copling 2 partikel ber spin 1/2

$$|SM\rangle = \sum_{m_s m_{s'}} \left\langle \frac{1}{2} m_s \frac{1}{2} m_{s'} \left| SM \right. \right\rangle |m_s m_{s'}\rangle \Leftrightarrow |m_s m_{s'}\rangle = \sum_{SM} \left\langle \frac{1}{2} m_s \frac{1}{2} m_{s'} \left| SM \right. \right\rangle |SM\rangle$$

untuk $S = 1, M = 1$, hanya ada satu suku pada $m_z = m_{z'} = \frac{1}{2}$

$$|1\ 1\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \left| 1\ 1 \right. \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\langle j_1 m_1 j_2 m_2 | JM \rangle = (-1)^{j_1 - m_1} \sqrt{\frac{2J_1 + 1}{2J_2 + 1}} \langle j_1 m_1 J - M | j_2 - m_2 \rangle$$

$$\left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \left| 1\ 1 \right. \right\rangle = (-1)^{\frac{1}{2} - \frac{1}{2}} \sqrt{\frac{3}{2}} \left\langle \frac{1}{2} \frac{1}{2} 1 - 1 \left| \frac{1}{2} - \frac{1}{2} \right. \right\rangle = \sqrt{\frac{3}{2}} \langle j m_s l m_{s'} | jm \rangle$$

$$\left\langle \frac{1}{2} \frac{1}{2} 1 - 1 \left| \frac{1}{2} - \frac{1}{2} \right. \right\rangle = \sqrt{\frac{l + \frac{1}{2} - m}{2l + 1}} \text{ dengan } l = 1 \text{ dan } m = -\frac{1}{2}$$

$$= \sqrt{\frac{l + \frac{1}{2} - \left(-\frac{1}{2}\right)}{2 \cdot 1 + 1}} = \sqrt{\frac{2}{3}}$$

Sehingga $|1\ 1\rangle = \left| \frac{1}{2}\ \frac{1}{2} \right\rangle$ keadaan 2 partikel dengan $S=1$ dan $M=1$

Hanya dibentuk oleh $m_s=1/2$ dan $m_s=1/2$

$$|1\ 0\rangle = \left\langle \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2} - \frac{1}{2} \middle| 1\ 0 \right\rangle + \left\langle \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2} - \frac{1}{2} \middle| 1\ 0 \right\rangle \left| \frac{1}{2}\ +\ \frac{1}{2} \right\rangle$$

$$|1\ -1\rangle = \left\langle \frac{1}{2}\ -\ \frac{1}{2}\ \frac{1}{2}\ -\ \frac{1}{2} \middle| 1\ -1 \right\rangle \left| -\ \frac{1}{2}\ -\ \frac{1}{2} \right\rangle$$

$$|0\ 0\rangle = \left\langle \frac{1}{2}\ \frac{1}{2}\ -\ \frac{1}{2}\ -\ \frac{1}{2} \middle| 0\ 0 \right\rangle + \left\langle -\ \frac{1}{2}\ -\ \frac{1}{2}\ -\ \frac{1}{2}\ \frac{1}{2} \middle| 0\ 0 \right\rangle \left| -\ \frac{1}{2}\ \frac{1}{2} \right\rangle$$

$$\begin{array}{ccc}
|SM\rangle & M & |m_s m_s\rangle \\
\left(\begin{array}{c} |11\rangle \\ |10\rangle \\ |1-1\rangle \\ |00\rangle \end{array} \right) & = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} & \left(\begin{array}{c} \left| \frac{1}{2} \frac{1}{2} \right\rangle \\ \left| \frac{1}{2} - \frac{1}{2} \right\rangle \\ \left| -\frac{1}{2} \frac{1}{2} \right\rangle \\ \left| -\frac{1}{2} - \frac{1}{2} \right\rangle \end{array} \right)
\end{array}$$

Gunakan $\langle j_1 m_1 j_2 m_2 | JM \rangle = (-1)^{j_1 - j_2 - J} \langle j_1 -m_1, j_2 -m_2 | J - M \rangle$

$$\left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \middle| 1 - 1 \right\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \middle| 1 1 \right\rangle = 1$$

Gunakan $\langle j_1 m_1 j_2 m_2 | JM \rangle = (-1)^{j_1 + j_2 - J} \langle j_2 m_2, j_1 m_1 | JM \rangle$

$$\left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} - \frac{1}{2} \middle| 10 \right\rangle = \left\langle \frac{1}{2} - \frac{1}{2} \frac{1}{2} \frac{1}{2} \middle| 10 \right\rangle$$

$$|SM\rangle = \hat{M} |m_s m_s\rangle \Rightarrow |m_s m_s\rangle = \hat{M}^{-1} |SM\rangle$$

$$\left| \frac{1}{2} - \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |00\rangle)$$