PENJUMLAHAN MOMENTUM SUDUT

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Representasi elektron lengkap memerlukan informasi ruang dan spin $\left|x,\ y,z,s_z=\pm\right\rangle=\left|xyz\right\rangle\left|s_z=\pm\right\rangle$

Untuk elektron bebas $\left|xyz\right\rangle=Ae^{i\vec{k}.\vec{r}}$: $\chi_+=\begin{pmatrix}1\\0\end{pmatrix},\ \ \chi_-=\begin{pmatrix}0\\1\end{pmatrix}$

Untuk electron yang terikat atom keadaan ruangnya dinyatakan $\left|n,l,m\right\rangle$ sehingga eigen state : $\left|n\,l\,m_0,m_0\right\rangle=\left|n\,l\,m_l\right\rangle\left|m_0\right\rangle$ Ruang $\left|n\ l\ m_l\right\rangle=\left|n\,l\right\rangle\left|l\,m\right\rangle$

Dengan :

$$L_Z\left|l\,m\right\rangle m\hbar\left|l\,m\right\rangle \qquad ;L^2\left|l\,m\right\rangle=l(l+1)\hbar^2\left|l\,m\right\rangle$$

$$L_+\left|l\,m\right\rangle=\left[(l+m+1)(l-m)\right]^{\frac{1}{2}}\hbar\left|l\,m+1\right\rangle$$

$$L_-\left|l\,m\right\rangle=\left[(l-m+1)(l+m)\right]^{\frac{1}{2}}\hbar\left|l\,m-1\right\rangle$$

$$Spin\left|ms\right\rangle=\left|\pm\right\rangle\rightarrow s_z\left|\pm\right\rangle=\pm\frac{1}{2}\hbar\left|\pm\right\rangle \qquad s_+\left|-\right\rangle=\hbar\left|+\right\rangle$$

$$s_+\left|+\right\rangle=s_-\left|-\right\rangle=0 \qquad s_-\left|+\right\rangle=\hbar\left|-\right\rangle$$

Penjumlahan spin orbit

$$\vec{J} = \vec{L} + \vec{S}$$

$$\vec{J}.\vec{J} = L^2 + S^2 + 2(L_x + L_y + L_z).(s_x + s_y + s_z)$$

$$= L^2 + S^2 + 2L_z s_x + L_+ s_- + L_- s_+$$

$$\mathrm{J}^2 \left| \mathrm{jm} \right\rangle = \mathrm{J}^2 \left[\alpha \left| \mathrm{lm} - \frac{1}{2}, \frac{1}{2} \right\rangle + \beta \left| \mathrm{lm} + \frac{1}{2}, -\frac{1}{2} \right\rangle \right] = \mathrm{j}(\mathrm{j}+1)\hbar^2 \left(\alpha \left| \mathrm{lm} - \frac{1}{2}, \frac{1}{2} \right\rangle + \beta \left| \mathrm{lm} \frac{1}{2}, -\frac{1}{2} \right\rangle \right)$$

$$= \alpha \hbar^2 \left\{ \left[\mathrm{l}(\mathrm{l}+1) + \frac{3}{4} + 2\left(\mathrm{m} - \frac{1}{2} \right)\left(\frac{1}{2} \right) \right] \left| \mathrm{lm} - \frac{1}{2}, \frac{1}{2} \right\rangle + \left[\left(\mathrm{l} + \mathrm{m} + \frac{1}{2} \right)\left(\mathrm{l} - \mathrm{m} - \frac{1}{2} \right) \right]^{\frac{1}{2}} \left| \mathrm{lm} - \frac{1}{2}, \frac{1}{2} \right\rangle \right\}$$

$$= +\beta \hbar^2 \left\{ \left[\mathrm{l}(\mathrm{l}+1) + \frac{3}{4} + 2\left(\mathrm{m} + \frac{1}{2} \right)\left(-\frac{1}{2} \right) \right] \left| \mathrm{lm} + \frac{1}{2}, -\frac{1}{2} \right\rangle + \left[\left(\mathrm{l} - \mathrm{m} + \frac{1}{2} \right)\left(\mathrm{l} + \mathrm{m} - \frac{1}{2} \right) \right]^{\frac{1}{2}} \left| \mathrm{lm} - \frac{1}{2}, \frac{1}{2} \right\rangle \right\}$$

$$\text{maka} : \alpha \hbar^2 \left\{ \left[\mathrm{l}(\mathrm{l}+1) + \frac{3}{4} + \left(\mathrm{m} - \frac{1}{2} \right) \right] + \beta \left[\left(\mathrm{l} + \mathrm{m} + \frac{1}{2} \right)\left(\mathrm{l} - \mathrm{m} - \frac{1}{2} \right) \right]^{\frac{1}{2}} \hbar^2 \right\} = \alpha \mathrm{j}(\mathrm{j}+1)\hbar^2$$

$$\beta \hbar^2 \left\{ \mathrm{l}(\mathrm{l}+1) + \frac{3}{4} - \left(\mathrm{m} + \frac{1}{2} \right) + \alpha \left[\left(\mathrm{l} + \mathrm{m} + \frac{1}{2} \right)\left(\mathrm{l} + \mathrm{m} - \frac{1}{2} \right) \right]^{\frac{1}{2}} \hbar^2 \right\} = \beta \mathrm{j}(\mathrm{j}+1)\hbar^2$$

diperoleh: $j = l + \frac{1}{2}$ dan $j = l - \frac{1}{2}$ buktikan!!!!!

untuk $j = l + \frac{1}{2} \rightarrow \frac{\beta}{\alpha} = \left(\frac{l + \frac{1}{2} - m}{l + \frac{1}{2} - m} \right)^{\frac{1}{2}}$ buktikan!!!!!

Normalisasi $\rightarrow \alpha = \sqrt{\frac{l + \frac{1}{2} + m}{2l + 1}}$ $\beta = \sqrt{\frac{l + \frac{1}{2} - m}{2l + 1}}$

$$\left| j = l + \frac{1}{2}, m \right\rangle = \sqrt{\frac{l + \frac{1}{2} + m}{2l+1}} \left| lm - \frac{1}{2}, \frac{1}{2} \right\rangle + \sqrt{\frac{l + \frac{1}{2} - m}{2l+1}} \left| lm + \frac{1}{2}, -\frac{1}{2} \right\rangle$$

$$\left| j = l - \frac{1}{2}, m \right\rangle = \sqrt{\frac{l + \frac{1}{2} + m}{2l+1}} \left| lm - \frac{1}{2}, \frac{1}{2} \right\rangle + \sqrt{\frac{l + \frac{1}{2} - m}{2l+1}} \left| lm + \frac{1}{2}, -\frac{1}{2} \right\rangle$$

Contoh copling 2 partikel ber spin $1/2$

$$\left|SM\right\rangle = \sum_{m_s m} \left\langle \frac{1}{2}\mathrm{m}_s \ \frac{1}{2}\mathrm{m}_{s'} \ \middle| SM \right\rangle \left|\mathrm{m}_s \mathrm{m}_{s'}\right\rangle \Leftrightarrow \left|\mathrm{m}_s \mathrm{m}_{s'}\right\rangle = \sum_{SM} \left\langle \frac{1}{2}\mathrm{m}_s \ \frac{1}{2}\mathrm{m}_{s'} \ \middle| SM \right\rangle \left|SM\right\rangle$$

$$untuk \ S=1, M=1, hanya\, ada \ satu \ suku \ pada \ \mathrm{m}_z = \mathrm{m}_{z'} = \tfrac{1}{2}$$

$$\left|1\,1\right\rangle = \left\langle \frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2} \middle| 1\,1 \right\rangle \left| \frac{1}{2}\,\frac{1}{2} \right\rangle$$

$$\left\langle j_1 m_1 j_2 m_2 \mid JM \right\rangle = (-1)^{j_1 - m_1} \sqrt{\frac{2J_1+1}{2J_2+1}} \left\langle j_1 m_1 J - M \mid j_2 - m_2 \right\rangle$$

$$\left\langle \frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2} \middle| 1\,1 \right\rangle = (-1)^{\frac{1}{2}-\frac{1}{2}} \sqrt{\frac{3}{2}} \left\langle \frac{1}{2}\,\frac{1}{2} 1 -1 \middle| \frac{1}{2} - \frac{1}{2} \right\rangle = \sqrt{\frac{3}{2}} \left\langle j m_s l m_{s'} \mid jm \right\rangle$$

$$\left\langle \frac{1}{2}\,\frac{1}{2} 1 -1 \middle| \frac{1}{2} - \frac{1}{2} \right\rangle = \sqrt{\frac{l+\frac{1}{2}-m}{2l+1}} \, dengan\, \mathrm{l} = 1 \ \ dan \ \mathrm{m} = -\frac{1}{2}$$

$$= \sqrt{\frac{l+\frac{1}{2}-\left(-\frac{1}{2}\right)}{2.1+1}} = \sqrt{\frac{2}{3}}$$

Sehingga $\left|1\,1\right\rangle = \left|\frac{1}{2}\,\frac{1}{2}\right\rangle$ keadaan 2 partikel dengan S=1 dan M=1

Hanya dibentuk oleh m_s =1/2dan $m_{s'}$=1/2

$$\left|10\right\rangle = \left\langle \frac{1}{2}\frac{1}{2}\frac{1}{2}-\frac{1}{2}\middle|10\right\rangle + \left\langle \frac{1}{2}\frac{1}{2}-\frac{1}{2}\frac{1}{2}\middle|10\right\rangle\left|\frac{1}{2}+\frac{1}{2}\right\rangle$$

$$\left|1-1\right\rangle = \left\langle \frac{1}{2}-\frac{1}{2}\frac{1}{2}-\frac{1}{2}\middle|1-1\right\rangle\left|-\frac{1}{2}-\frac{1}{2}\right\rangle$$

$$\left|0\,0\right\rangle = \left\langle \frac{1}{2}\frac{1}{2}-\frac{1}{2}-\frac{1}{2}\middle|00\right\rangle + \left\langle -\frac{1}{2}-\frac{1}{2}-\frac{1}{2}\frac{1}{2}\middle|00\right\rangle\left|-\frac{1}{2}\frac{1}{2}\right\rangle$$

$$|SM\rangle \qquad M \qquad |m_s m_{s'}\rangle$$

$$\begin{pmatrix} |1\,1\rangle \\ |1\,0\rangle \\ |1-1\rangle \\ |0\,0\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} \left|\frac{1}{2}\,\frac{1}{2}\right\rangle \\ \left|\frac{1}{2}-\frac{1}{2}\right\rangle \\ \left|-\frac{1}{2}\,\frac{1}{2}\right\rangle \\ \left|-\frac{1}{2}-\frac{1}{2}\right\rangle \end{pmatrix}$$

$$Gunakan \langle j_1 m_1 j_2 m_2 | JM\rangle = (-1)^{j_1 - j_2 - J} \langle j_1 - m_1, j_2 - m_2 | J - M\rangle$$

$$\left\langle \frac{1}{2} - \frac{1}{2} \frac{1}{2} - \frac{1}{2} \right| 1 - 1 \Big\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right| 1 1 \Big\rangle = 1$$

$$Gunakan: \left\langle j_1 m_1 j_2 m_2 \middle| JM \right\rangle = (-1)^{j_1+j_2-J} \left\langle j_2 m_2, j_1 m_1 \middle| JM \right\rangle$$

$$\left\langle \frac{1}{2}\frac{1}{2}\frac{1}{2}-\frac{1}{2} \middle| 10 \right\rangle = \left\langle \frac{1}{2}-\frac{1}{2}\frac{1}{2}\frac{1}{2} \middle| 1\,0 \right\rangle$$

$$\left| SM \right\rangle = \hat{M} \left| m_s m_{s'} \right\rangle \Rightarrow \left| m_s m_{s'} \right\rangle = \hat{M}^{-1} \left| SM \right\rangle$$

$$\left| \frac{1}{2}-\frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} \left(\left|10\right\rangle + \left|00\right\rangle \right)$$