Electrical Conduction Theories & Conducting Materials (Part-A)

Outline:

∗ Resistivity, TCR, and Matthiessen's rule

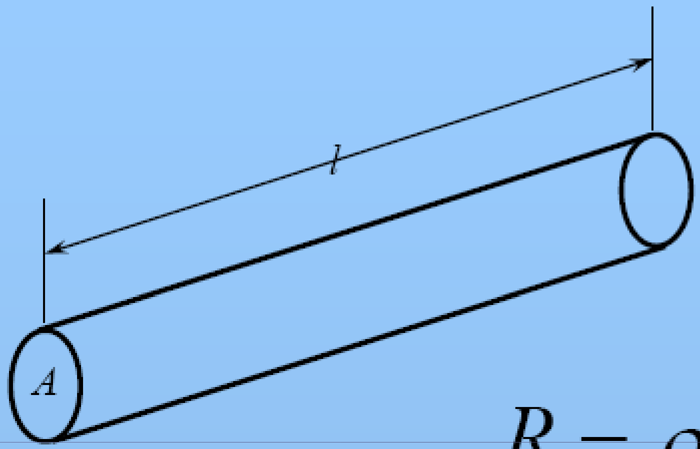
∗ Classification of metals, insulators, and semiconductors

∗ Free electron theory

∗ Hall effect

∗ Nordheim's rule

Resistivity & Temperature coefficient of resistivity (TCR)



$$R = \rho \times \frac{l}{A}$$

l → Length (m)

A → Area of the cross-section of the conductor (m^2)

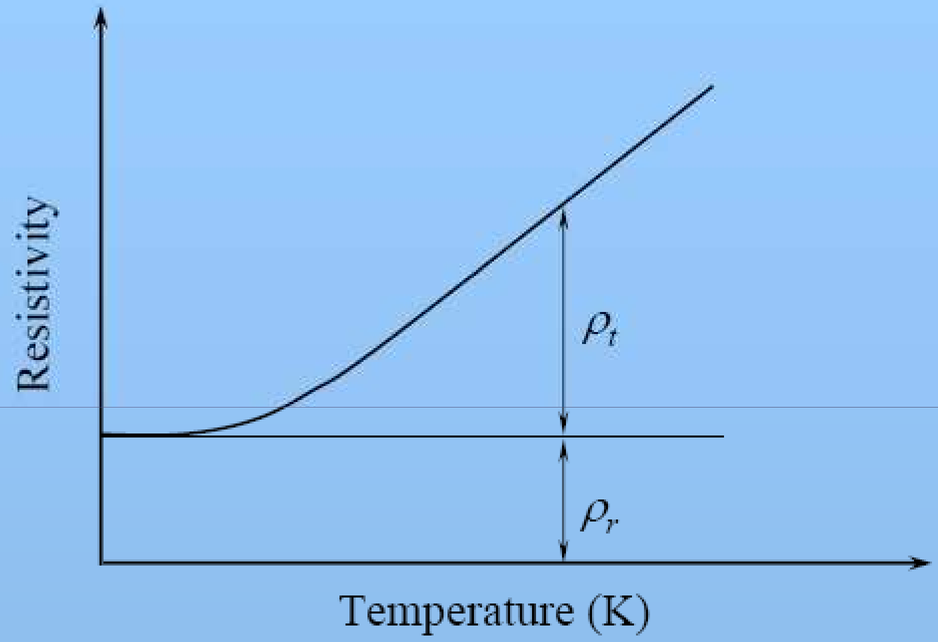
R: Electrical resistance (Ω)

ρ: Electrical resistivity (Ω·m)

$$\sigma = {}^{1}\!/_{\rho}$$

σ: Conductivity ($\Omega^{-1}m^{-1}$)

Matthiessen's rule and TCR



Matthiessen's rule: the resistivity ρ of a pure metal is the sum of a residual part ρ_r and a thermal part ρ_t.

$$\rho_{(total)} = \rho_r + \rho_t$$

Matthiessen's rule and TCR

$$\rho_{(total)} = \rho_r + \rho_t$$

$$\rho_{(total)} = \rho_r \left(1 + \rho_t / \rho_r\right)$$

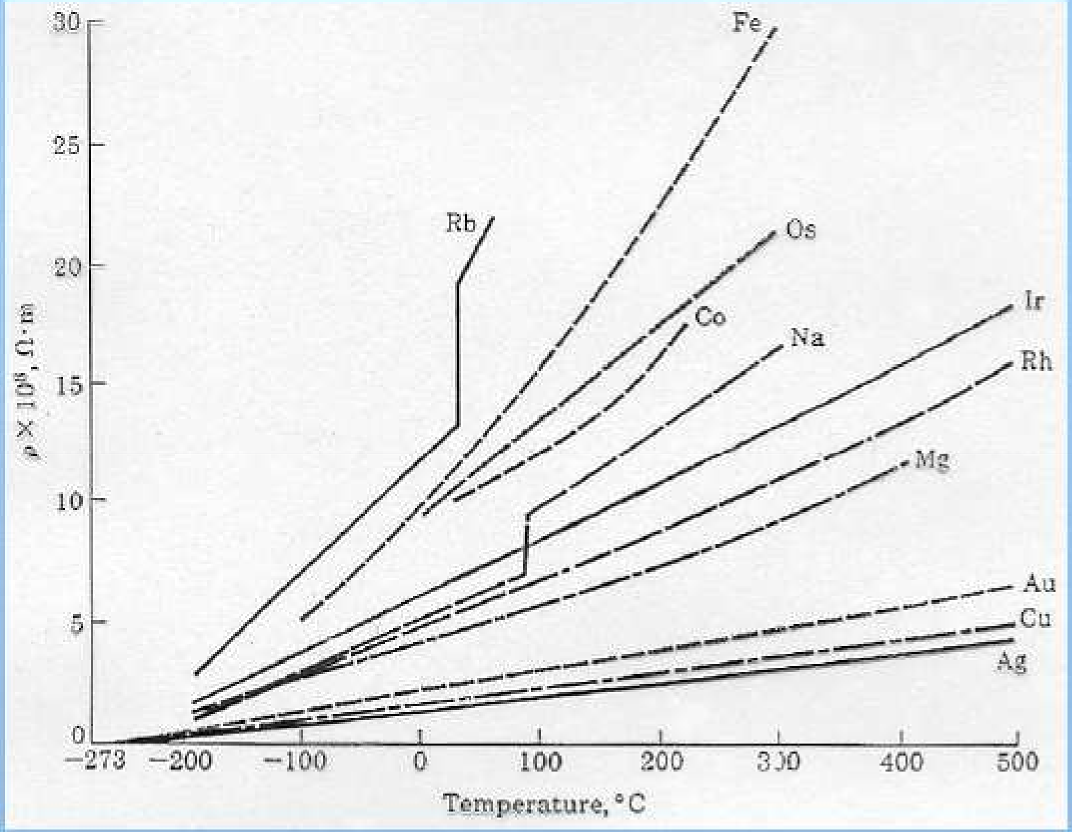
$$\rho_t / \rho_r = f(T)$$

$$\rho_{(total)} = \rho_r \left[1 + f(T)\right]$$

For most metals and alloys, ρ is approximately proportional to temperature T,

$$\rho_{(total)} = \rho_o (1 + \underset{\downarrow}{\alpha} \Delta T)$$

Temperature coefficient of resistivity (TCR)



Effect of temperature on resistivity of selected metals

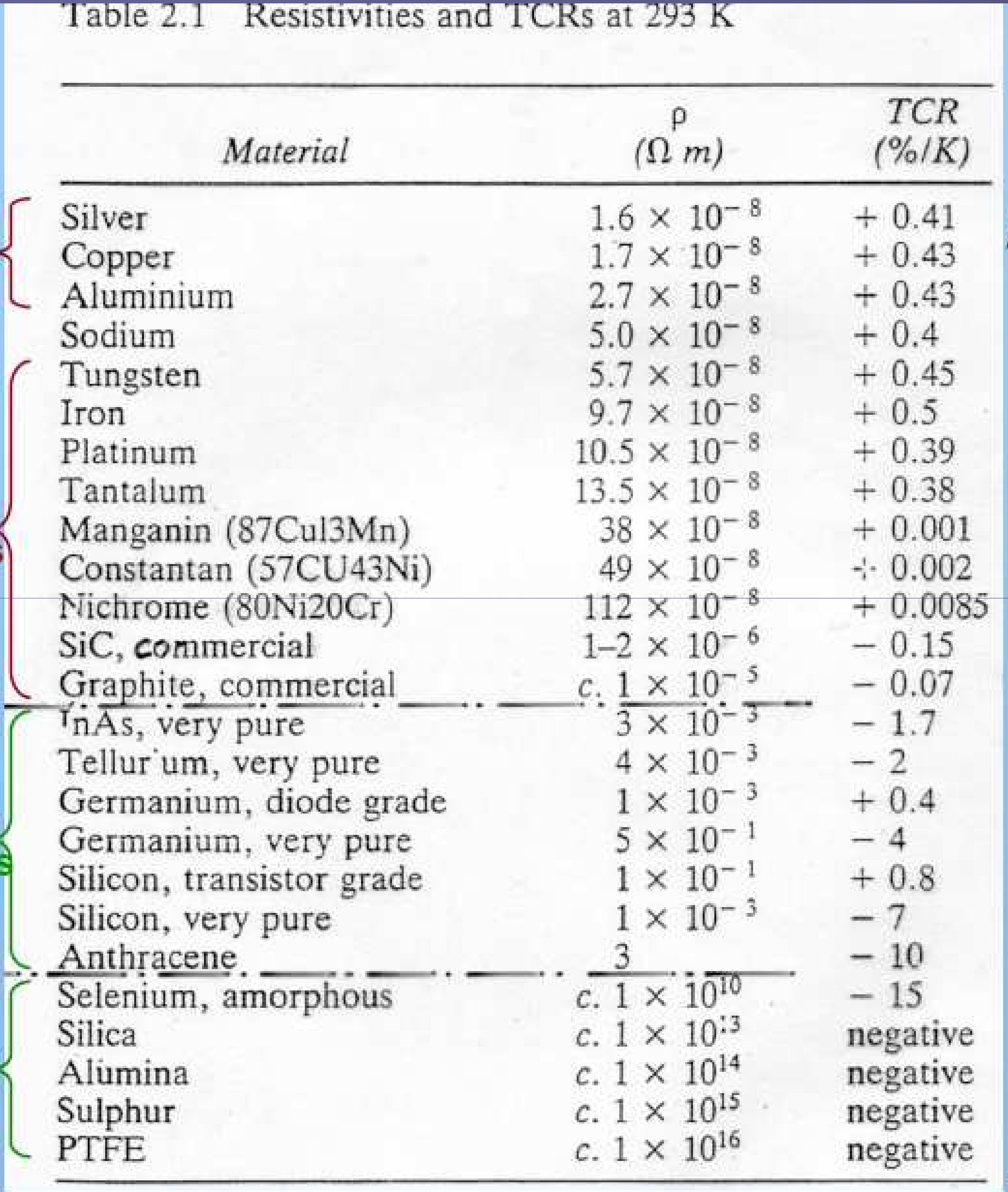


Table 2.1 Resistivities and TCRs at 293 K

	Material	ρ (Ω m)	TCR (%/K)
Conducting materials	Silver	1.6×10^{-8}	+ 0.41
	Copper	1.7×10^{-8}	+ 0.43
	Aluminium	2.7×10^{-8}	+ 0.43
	Sodium	5.0×10^{-8}	+ 0.4
Heating elements	Tungsten	5.7×10^{-8}	+ 0.45
	Iron	9.7×10^{-8}	+ 0.5
	Platinum	10.5×10^{-8}	+ 0.39
	Tantalum	13.5×10^{-8}	+ 0.38
	Manganin (87Cul3Mn)	38×10^{-8}	+ 0.001
	Constantan (57CU43Ni)	49×10^{-8}	+ 0.002
	Nichrome (80Ni20Cr)	112×10^{-8}	+ 0.0085
	SiC, commercial	$1–2 \times 10^{-6}$	− 0.15
	Graphite, commercial	c. 1×10^{-5}	− 0.07
Semi-conductors	InAs, very pure	3×10^{-3}	− 1.7
	Tellurium, very pure	4×10^{-3}	− 2
	Germanium, diode grade	1×10^{-3}	+ 0.4
	Germanium, very pure	5×10^{-1}	− 4
	Silicon, transistor grade	1×10^{-1}	+ 0.8
	Silicon, very pure	1×10^{-3}	− 7
	Anthracene	3	− 10
Insulators	Selenium, amorphous	c. 1×10^{10}	− 15
	Silica	c. 1×10^{13}	negative
	Alumina	c. 1×10^{14}	negative
	Sulphur	c. 1×10^{15}	negative
	PTFE	c. 1×10^{16}	negative

Conductivity

Drude's free electron theory (1900)

Electrons, treated as particles with certain mass and electric charge, move through the metal lattice freely and obey Newton's laws of motion and Maxwell-Boltzmann statistics

Paul Drude (1863-1906)

Drude's free electron theory

when an electric field E is applied to a metal, the force acting on an electron is:

$$F = -eE$$

According to the 2nd Newton's Law:

$$\underset{\text{Acceleration}}{\underset{\downarrow}{a}} = F/m = -eE/\underset{\text{Mass of an electron, } 9.1\times10^{-31}\text{kg}}{\underset{\downarrow}{m}} = 1.75\times10^{11}\,E$$

In a time τ, the velocity v_d of the electron will be:

$$\underset{\text{Drift velocity}}{\underset{\downarrow}{v_d}} = a\tau$$

$$v_d = -eE\tau / m$$

$$\mu = e\tau / m$$

Mobility of electron, or drift velocity in unit electrical field

Considering a unit volume of a conductor containing *n* free electron, the charge crossing unit area in unit time must be $-nev_d$, which is the **current density** *J*:

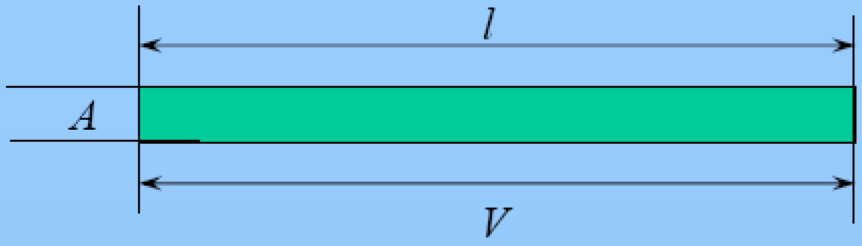
$$J = -nev_d = ne^2 E\tau / m$$

or

$$J = \sigma E$$

$$\sigma = ne^2\tau / m = ne\mu$$

Conductivity of a metal, proportional to mobility and free electron density



$$E = \frac{V}{l}$$

$$R = \rho \times \frac{l}{A}$$

Since

$$J = \sigma E = {}^{E}\!/_{\rho} = {}^{V}\!/_{\rho l}$$

We get

$$J = {}^{V}\!/_{RA}$$

$$JA = I = {}^{V}\!/_{R} \longrightarrow \text{Ohm's law}$$

Mean free path

The average distance travelled by an electron between collisions:

$$l_m = \tau v$$

Time between two collisions

Average velocity of electron, $v = v_{th} + v_d$

Since:

$$v_{th} \ \phi\phi \ v_d$$

$$\tau = \mu m / e$$

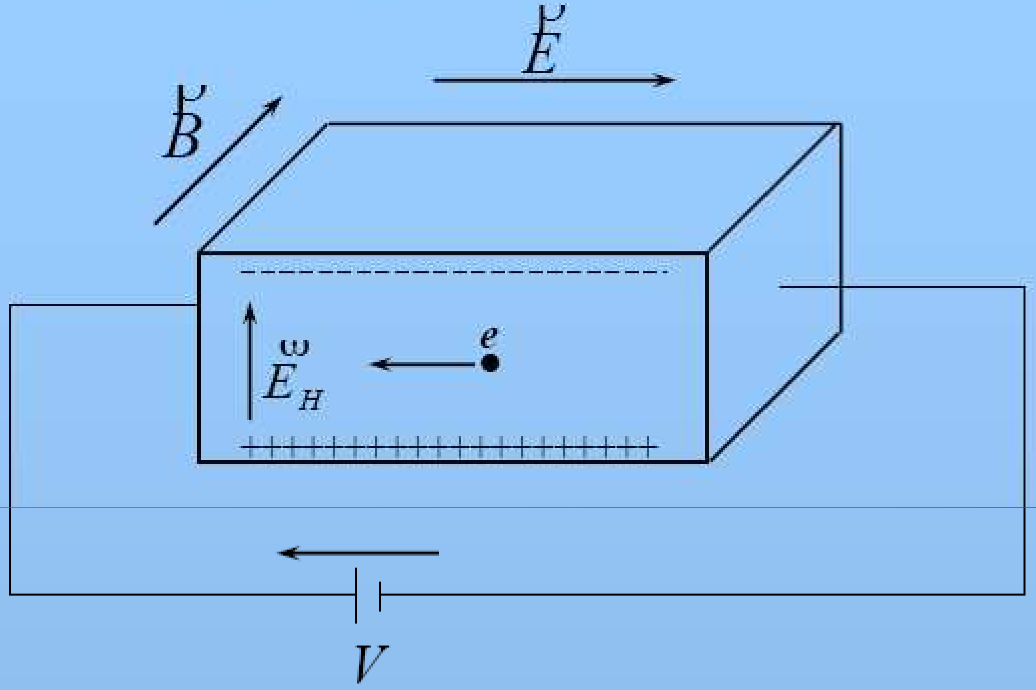
We get:

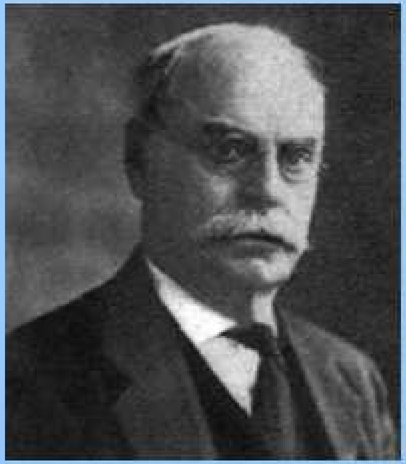
$$l_m = \mu m v_{th} / e$$

Typically, $\tau = 10^{-14} \sim 10^{-15}$Sec, electron concentration = $10^{28}/\mathrm{m}^3$, therefore:

$$\sigma = ne^2 \tau / m = 0.3 - 3 \times 10^6 / (\Omega \cdot m)$$

Hall effect (1879)





Edwin Hall (1855-1938)

Electrons moving in a magnetic field are subjected to the **Lorenz force, *F***:

$$F = -ev \times B$$

Instantaneous velocity of electron

Magnetic induction

The electrons move upwards to accumulate and produce a field, E_H, **Hall field**, which acts on the electrons as to oppose the Lorenz force;

$$eE_H = evB$$

Since:

$$v = v_{th} + v_d$$

$$v_{th} \approx 0 \longrightarrow E_H = v_d B$$

We get:

$$E_H = \mu EB$$

Also, since:

$$v_d = -J / ne$$

We then have:

$$E_H = \left(-1/_{ne}\right) JB$$

R_H, Hall constant

$$R_H = \frac{E_H}{JB}$$

$R_H ne$ values for some metals

Metal	**$R_H ne$**
Li	-1.15
Na	-1.05
K	-1.08
Rb	-1.05
Cs	-1.04
Cu	-0.68
Ag	-0.8
Au	-0.69
Pd	-0.73
Pt	-0.21
Cd	+0.5
W	+1.2
Be	+5.0

Resistivity of alloys

Impurity increases the residual part ρ_r of $\rho_{(total)}$.

$$\rho_r(x) = Ax(1-x)$$

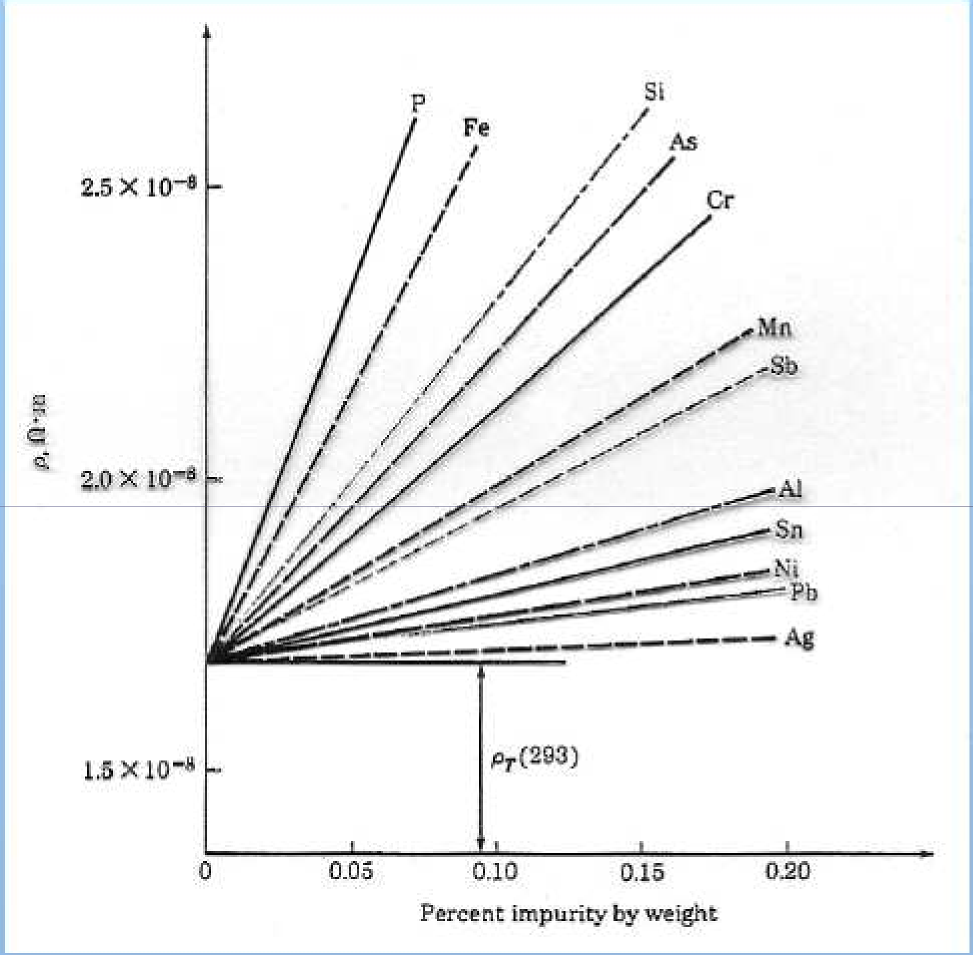
Nordheim's rule

Solution resistivity coefficient

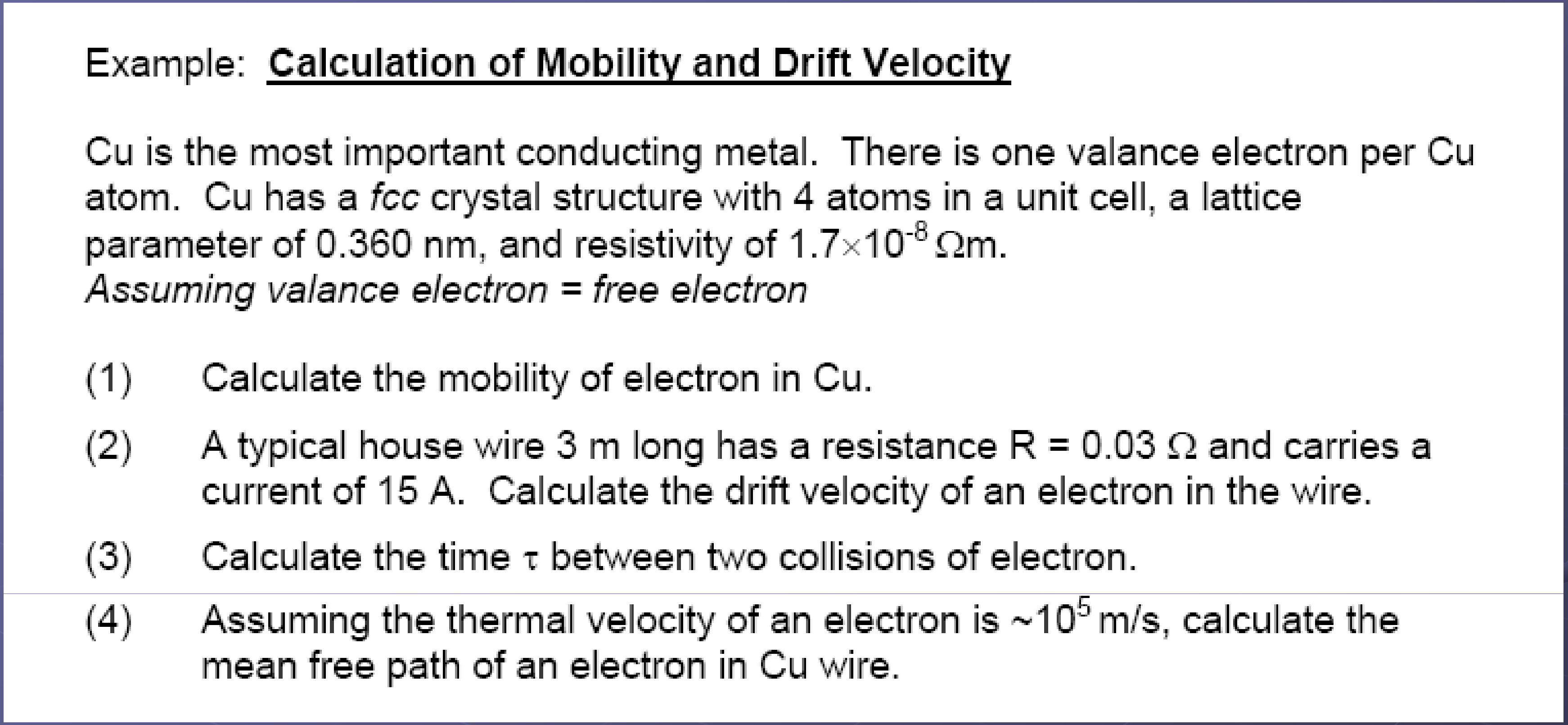
Concentration of the impurity

When $x \ll 1$, $1 - x \approx 1$, then we have:

$$\rho_r = Ax$$



Effects of small additions of various elements on the resistivity of Cu at 293K



Example: **Calculation of Mobility and Drift Velocity**

Cu is the most important conducting metal. There is one valance electron per Cu atom. Cu has a *fcc* crystal structure with 4 atoms in a unit cell, a lattice parameter of 0.360 nm, and resistivity of $1.7\times10^{-8}\ \Omega$m.

Assuming valance electron = free electron

(1) Calculate the mobility of electron in Cu.

(2) A typical house wire 3 m long has a resistance R = 0.03 Ω and carries a current of 15 A. Calculate the drift velocity of an electron in the wire.

(3) Calculate the time τ between two collisions of electron.

(4) Assuming the thermal velocity of an electron is ~10^5 m/s, calculate the mean free path of an electron in Cu wire.

1. $\mu = \sigma/(n\ e) = 1/(\rho\ n\ e)$

2. $v_d = \mu E$

3 $v_d = -e\ E\ \tau/m$

4 $l_m = \tau v_{th}$