

SOLUSI Latihan I FUNGSI KOMPLEKS

1. Jika $F(z) = u(x,y) + iv(x,y)$ maka tentukan fungsi $u(x,y)$ dan $v(x,y)$ dari fungsi kompleks berikut :

- a. $\frac{1}{z}$ b. $\text{Sinh } z$ c. $z^2 - \bar{z}^2$ d. $\ln z$

jawab:

- a. $f(z) = 1/z = 1/(x+iy) = x/(x^2+y^2) - iy/(x^2+y^2)$
b. $f(z) = \sinh z = \sinh(x+iy) = 1/2 (\cos y(e^x - e^{-x}) + i(e^x + e^{-x})\sin y)$
c. $f(z) = z^2 - \bar{z}^2 = (x+iy)^2 - (x-iy)^2 = i4xy$
d. $f(x) = \ln z = \ln(x+iy) = \ln\sqrt{x^2+y^2} + i \arctg y/x$

2. A. Dengan menggunakan persamaan Cauchy-Riemann, periksalah fungsi-fungsi pada soal 1 apakah analitik atau tidak :

Jawab :

- a. $f(z) = 1/z = 1/(x+iy) = x/(x^2+y^2) - iy/(x^2+y^2)$
 $\partial U/\partial y = -\partial V/\partial x = -2xy/(x^2+y^2)^2$ analitik
b. $f(z) = \sinh z = \sinh(x+iy) = 1/2 (\cos y(e^x - e^{-x}) + i(e^x + e^{-x})\sin y)$
 $\partial U/\partial y = -\partial V/\partial x = -1/2 (e^x - e^{-x})\sin y$ analitik
c. $f(z) = z^2 - \bar{z}^2 = (x+iy)^2 - (x-iy)^2 = i4xy$
 $\partial U/\partial y$ tidak sama dengan $-\partial V/\partial x$ tidak analitik
d. $f(x) = \ln z = \ln(x+iy) = \ln\sqrt{x^2+y^2} + i \arctg y/x$
 $\partial U/\partial y = -\partial V/\partial x = y/(x^2+y^2)$ analitik

B. Jika $u(x,y) = \ln(x^2+y^2)$ tentukanlah fungsi $f(z) = u(x,y) + iv(x,y)$

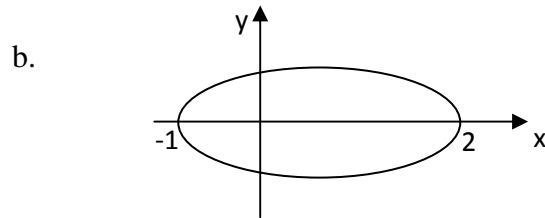
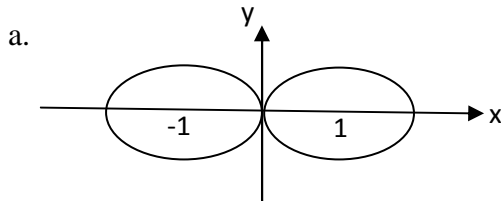
JAWAB:

$$\partial U/\partial x = \partial V/\partial y = x/(x^2+y^2)$$

$$V(x,y) = \int \frac{x}{x^2+y^2} dy = \text{arc tg } y/x$$

3. Gunakan teorema Cauchy untuk menyelesaikan integral berikut

$\oint_C \frac{dz}{z^2-1}$ dimana C adalah lintasan tertutup sbb:



JAWAB:

a. $\oint_C \frac{dz}{(z-1)(z+1)} = 2\pi i \left(\frac{1}{2} + \frac{1}{2} \right) = 2\pi i$
 b. $\oint_C \frac{dz}{(z-1)(z+1)} = 2\pi i \left(\frac{1}{2} - \frac{1}{2} \right) = 0$

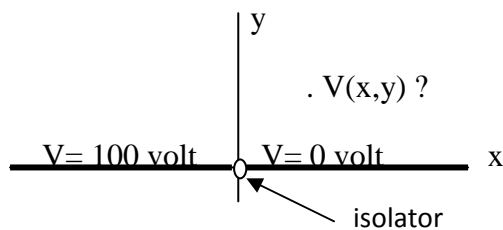
4. Dengan menggunakan teorema Residu tentukanlah solusi dari integral berikut:

a. $\int_0^{2\pi} \frac{\cos \theta d\theta}{13-12\cos 2\theta}$ b. $\int_{-\infty}^{\infty} \frac{x dx}{(x^2-2x+2)^2}$ c. $\int_0^{\infty} \frac{\cos(\ln x)}{x^2+1} dx$

JAWAB:

a. $\int_0^{2\pi} \frac{\cos \theta d\theta}{13-12\cos 2\theta} = \frac{1}{2i} \oint \frac{(z^2+1) dz}{13z^2-6z^4-6} = 0$ kutub berada diluar $|z| = 1$
 b. $\int_{-\infty}^{\infty} \frac{x dx}{(x^2-2x+2)^2} = \oint \frac{z dz}{(z-1)^2(z-2)^2} = 2\pi i \left(-\frac{i}{4} \right) = \pi/2$
 c. $\int_0^{\infty} \frac{\cos(\ln x)}{x^2+1} dx = 1/2 \oint \frac{e^{i \ln z}}{z^2+1} dz = 2i\pi \left(\frac{e^{-\pi/2}}{2i} \right) = \pi \frac{e^{-\pi/2}}{2}$

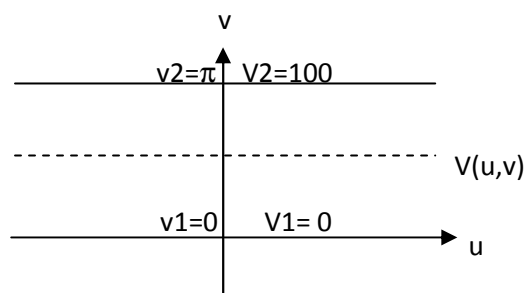
5.



- Tentukan Rumusan Tegangan $V(x,y)$ seperti pada gambar di atas
- Tentukan tegangan pada titik $x=1, y=1$!

JAWAB:

a. $F(Z) = \ln Z = \ln r + i\theta$
 $v = \theta$
 $v_1 = 0, V_1 = 0$ volt
 $v_2 = \pi, V_2 = 100$ volt



$$(V_2 - V_1)/\pi = (V(u,v) - V_1)/v$$

$$(100 - 0)/\pi = V(u,v)/v$$

$$V(u,v) = (100/\pi)v$$

$$V(x,y) = (100/\pi)\arctan y/x$$

b. Jika $x=1$, $y=1$ maka

$$V(x,y) = (100/\pi)\arctan y/x$$

$$V(1,1) = (100/\pi)\arctan 1$$

$$= (100/\pi)(\pi/4) = 25 \text{ volt}$$