KINEMATIKA GELOMBANG

TOPIK 2

Mata Kuliah GELOMBANG-OPTIK

SUB TOPIK

PERSAMAAN DIFFERENSIAL GELOMBANG

SOLUSI PERSAMAAN GELOMBANG

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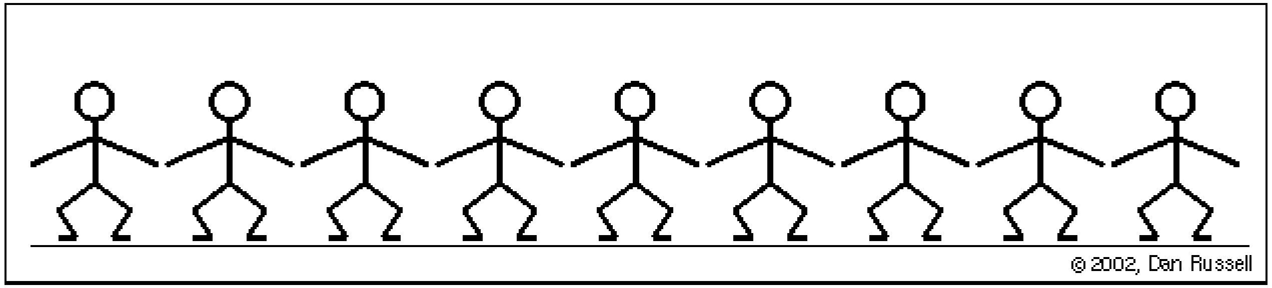
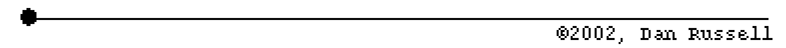
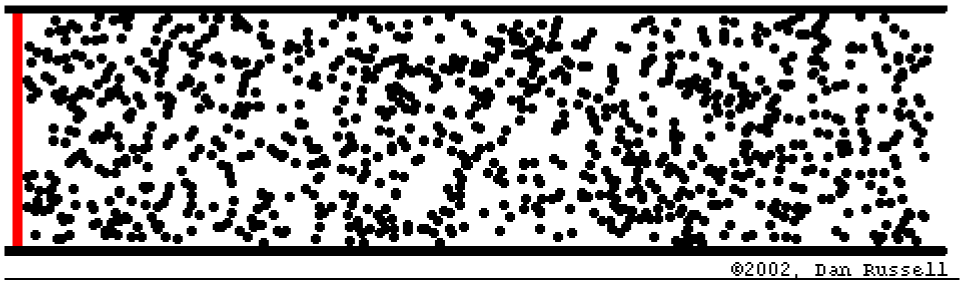
ANDHY SETIAWAN

PENGANTAR

ILUSTRASI PERAMBATAN PULSA

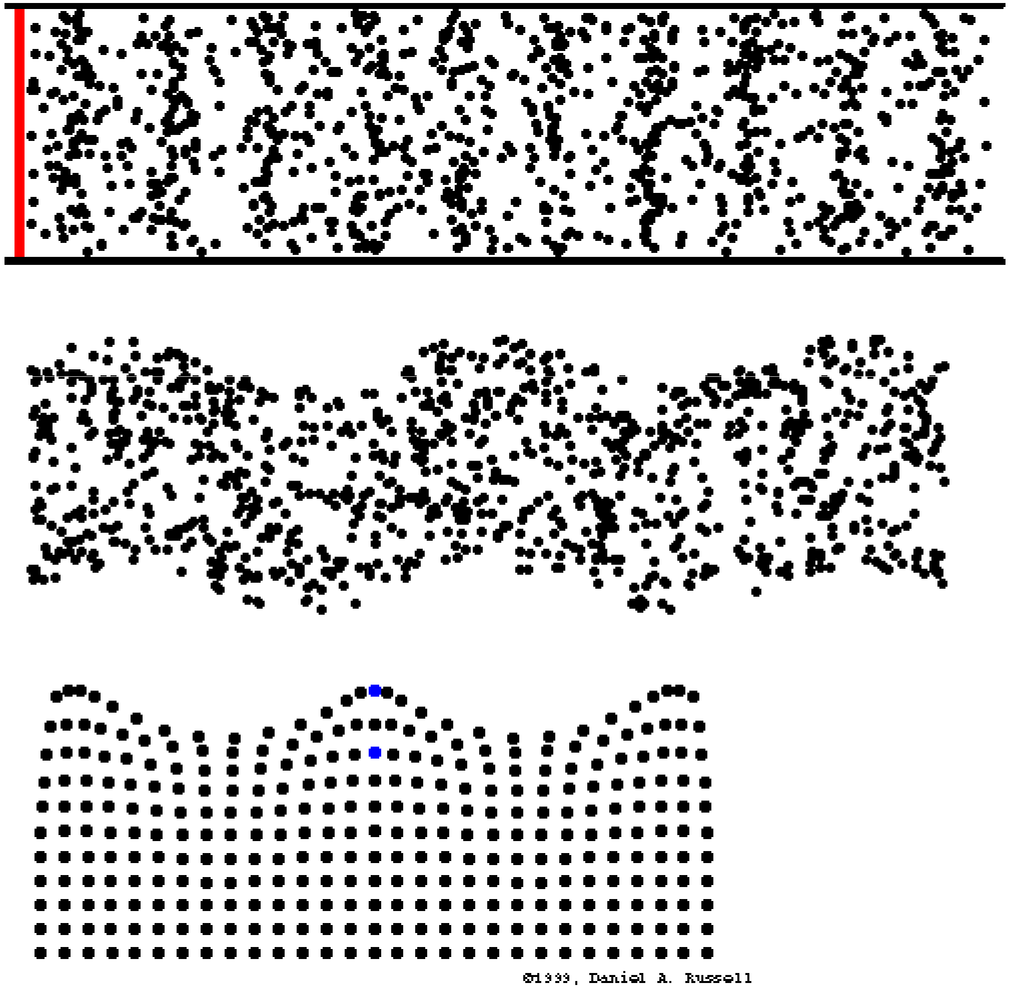






PENGANTAR

ILUSTRASI PERAMBATAN GELOMBANG



PERSAMAAN DIFFERENSIAL GELOMBANG
arah rambat dan sudut fase

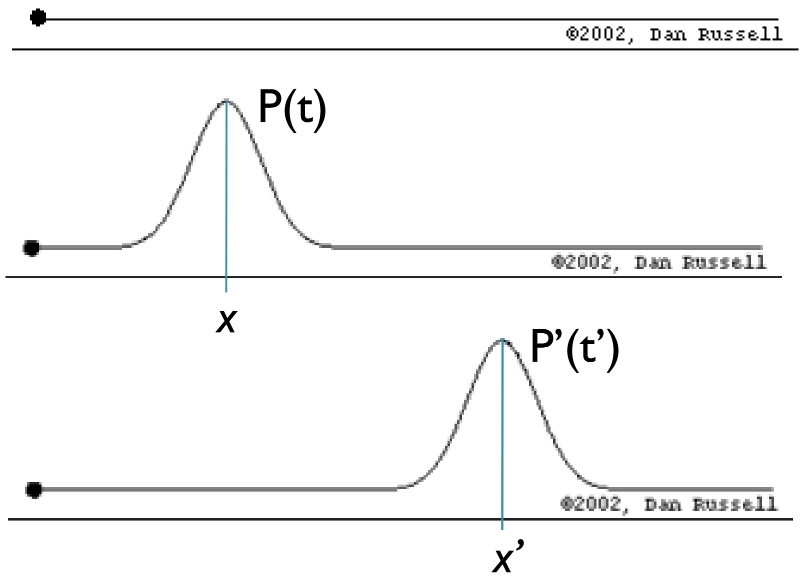
- Sistem osilasi $\psi(t)$
- fungsi gelombang $\psi(x,t)$ atau $\psi(r,t)$
- Tinjau: merambat arah *x*, kecepatan konstan *v*. → $\psi(x,t) = f(x \pm vt)$

$$\psi(x,t) = f(\phi), \text{ dengan } \phi = x \pm vt$$

$$\phi = \text{sudut fase}$$

PERSAMAAN DIFFERENSIAL GELOMBANG

arah rambat dan sudut fase



Sudut fase titik P : $\phi = x - vt$

Setelah t' : $\phi' = x' - vt'$

$$\phi = \phi'$$

$$x - vt = x' - vt'$$

$$x - vt = x + \Delta x - v(t + \Delta t)$$

$$0 = \Delta x - v\,\Delta t$$

$$\Delta x = v\,\Delta t$$

Maka $\Delta x > 0$, sehingga :

sudut fase $\phi = x - vt$ → arah rambat ke kanan

sudut fase $\phi = x + vt$ → arah rambat ke kiri (coba buktikan)

PERSAMAAN DIFFERENSIAL GELOMBANG

penurunan persamaan

- $\phi = x \pm vt$ konstan → kedudukan setiap titik yang sama

Kecepatan fase

$$\frac{d\phi}{dt} = 0 \rightarrow \frac{d(x \pm vt)}{dt} = 0 \rightarrow \frac{dx}{dt} \pm v = 0 \rightarrow v = \mp \frac{dx}{dt}$$

- Perubahan fungsi terhadap *x* dan *t*

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial \phi}\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial \phi} \rightarrow \frac{\partial \psi}{\partial \phi} = \frac{\partial \psi}{\partial x}$$

$$\frac{\partial \psi}{\partial x} \mp \frac{1}{v}\frac{\partial \psi}{\partial t} = 0$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial \phi}\frac{\partial \phi}{\partial t} = \pm v\frac{\partial \psi}{\partial \phi} \rightarrow \frac{\partial \psi}{\partial \phi} = \pm \frac{1}{v}\frac{\partial \psi}{\partial t}$$

PERSAMAAN DIFFERENSIAL GELOMBANG

penurunan persamaan

- Turunan kedua terhadap *x* dan *t*

$$\frac{\partial^2\psi}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial\phi}\right)=\frac{\partial}{\partial\phi}\left(\frac{\partial\psi}{\partial x}\right)=\frac{\partial}{\partial\phi}\left(\frac{\partial\psi}{\partial\phi}\right)=\frac{\partial^2\psi}{\partial\phi^2}$$

$$\frac{\partial^2\psi}{\partial t^2}=\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial t}\right)=\frac{\partial}{\partial t}\left(\pm v\frac{\partial\psi}{\partial\phi}\right)=(\pm v)\frac{\partial}{\partial\phi}\left(\frac{\partial\psi}{\partial t}\right)=(\pm v)\frac{\partial}{\partial\phi}\left(\pm v\frac{\partial\psi}{\partial\phi}\right)=v^2\frac{\partial^2\psi}{\partial\phi^2}$$

$$\frac{\partial^2\psi}{\partial\phi^2}=\frac{\partial^2\psi}{\partial x^2}$$

$$\frac{\partial^2\psi}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}=0$$

$$\frac{\partial^2\psi}{\partial\phi^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

$$\nabla^2\psi-\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}=0$$

Merupakan ungkapan gelombang datar
(Front wave berupa bidang datar)

Untuk koordinat bola

$$\nabla^2\psi=\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial\psi}{\partial r}$$

$$\frac{\partial^2\psi}{\partial r^2}+\frac{2}{r}\frac{\partial\psi}{\partial r}-\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}=0$$

(Buktikan)

PERSAMAAN DIFFERENSIAL GELOMBANG

prinsip superpoisi

- Jika ψ_1 dan ψ_2 solusi dari pers. Gelombang, maka berlaku:

$$\frac{\partial^2\psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

$$\frac{\partial^2\psi_1}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\psi_1}{\partial t^2} = 0$$

$$\frac{\partial^2\psi_2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\psi_2}{\partial t^2} = 0$$

dijumlahkan

$$\frac{\partial^2(\psi_1+\psi_2)}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2(\psi_1+\psi_2)}{\partial t^2} = 0$$

Jadi (ψ_1 + ψ_2) merupakan solusi dari pers. Gelombang juga

Prinsip superposisi

SOLUSI PERSAMAAN GELOMBANG

Solusi paling sederhana dari persamaan : $\frac{\partial^2\Psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2} = 0$ adalah

$\psi(x,t) = \psi_o \cos k\,(x-vt)$ → $\psi_o = \psi_{maks}$

k = bilangan gelombang/vektor gelombang (menunjukan arah rambat gelombang)

$\psi(x,t) = \psi_o \cos k\,(x-vt)$ → $\psi(x,t) = \psi_o \cos (kx - kvt)$ → $\psi(x,t) = \psi_o \cos (kx - \omega t)$

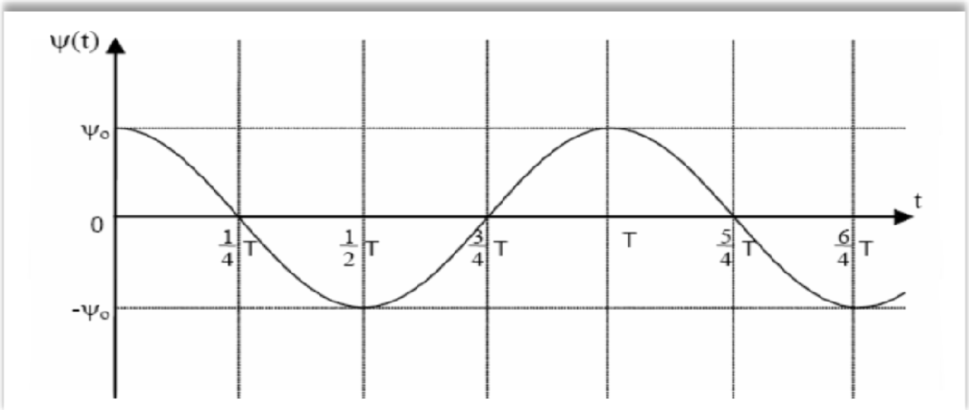
k = frekuensi spatial

ω = frekuensi temporal

→ $k = \frac{2\pi}{\lambda}$, $\omega = \frac{2\pi}{T}$ →

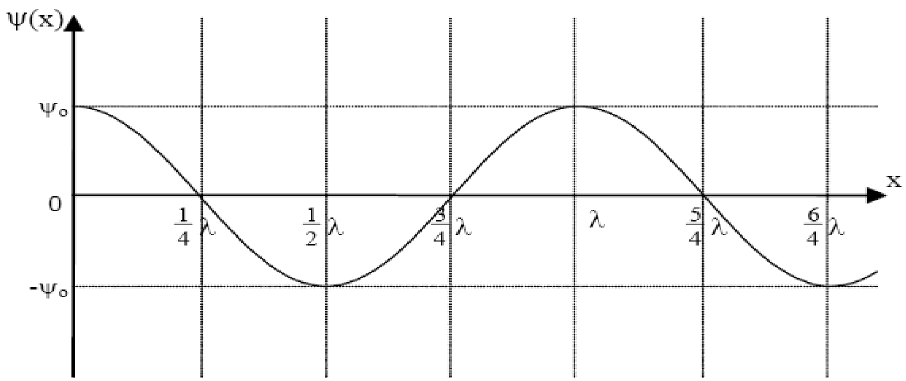
T = perioda temporal

λ = perioda spatial



→ Mengungkapkan pola eksitasi gelombang

Gelombang dalam sisi temporal



→ Mengungkapkan perambatan gelombang

Gelombang dalam sisi spatial

Sehingga solusi persamaan gelombang dapat pula diungkapkan dengan:

$$\Psi(x,t) = A\cos[2\pi(\frac{x}{\lambda} - \frac{t}{T})]$$

SUPERPOSISI DUA GELOMBANG

Misalkan dua buah gelombang dengan arah getar pada bidang yang sama, masing-masing frekuensinya ω_1 dan ω_2 serta bilangan gelombangnya k_1 dan k_2

$\psi_1(x,t) = A\cos(k_1 x - \omega_1 t)$ dan $\psi_2(x,t) = A\cos(k_2 x - \omega_2 t)$

Hasil superposisinya adalah:

$$\Psi(x,t) = A[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)]$$

$$\Psi(x,t) = 2A\left[\cos\left\{\frac{(k_1 - k_2)x - (\omega_1 - \omega_2)t}{2}\right\}\cos\left\{\frac{(k_1 + k_2)x - (\omega_1 + \omega_2)t}{2}\right\}\right]$$

$\Delta k = k_1 - k_2$ $\Delta\omega = \omega_1 - \omega_2$ Maka:

$$\Psi(x,t) = 2A\left[\cos\left\{\frac{\Delta kx - \Delta\omega t}{2}\right\}\cos\left\{\frac{(2k_1 - \Delta k)x - (2\omega_1 - \Delta\omega)t}{2}\right\}\right]$$

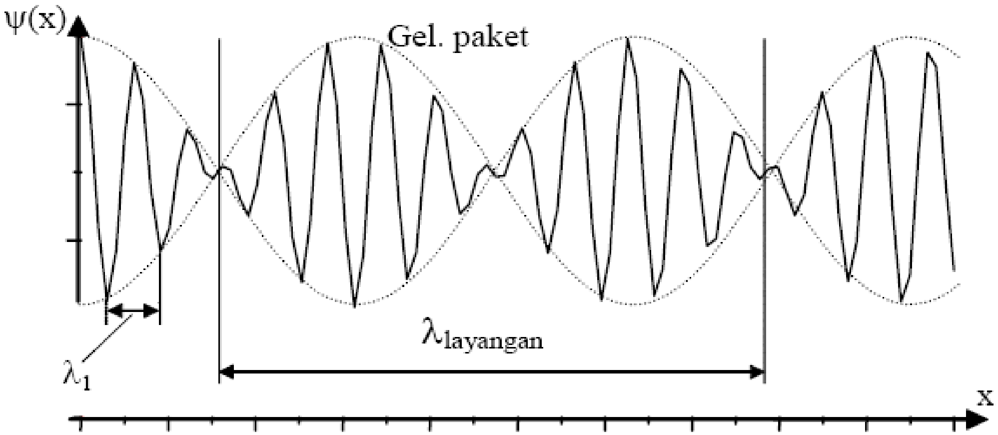
Untuk t=0

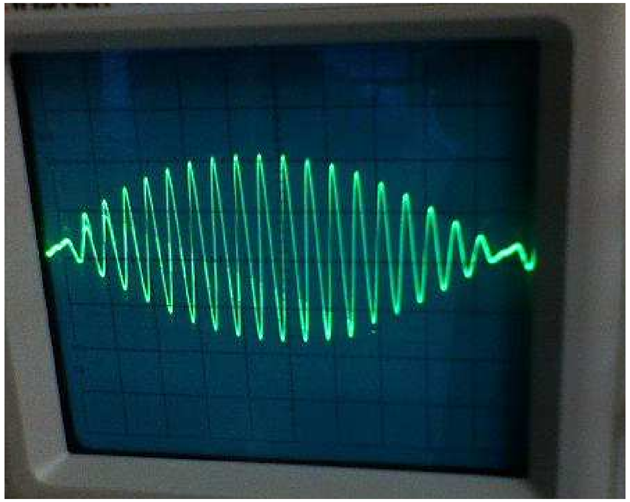
$$\Psi(x,t) = 2A\left[\cos\frac{\Delta kx}{2}\cos\frac{(2k_1 - \Delta k)x}{2}\right]$$

Δk sangat kecil, sehingga $2k_1 - \Delta k \approx 2k_1$

$$\Psi(x,t) = 2A\left[\cos\frac{\Delta kx}{2}\cos k_1 x\right]$$

Bila kita gambarkan hasil superposisinya, maka :



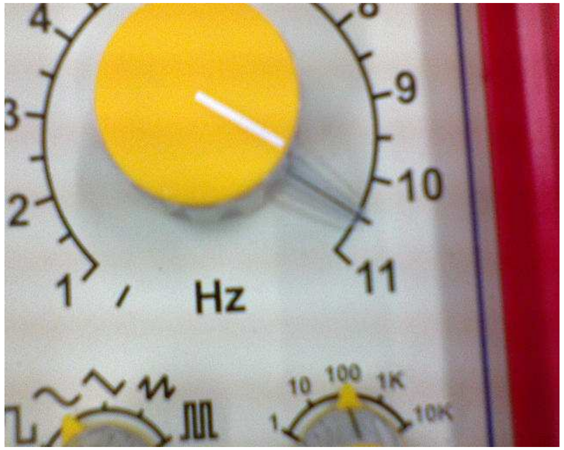


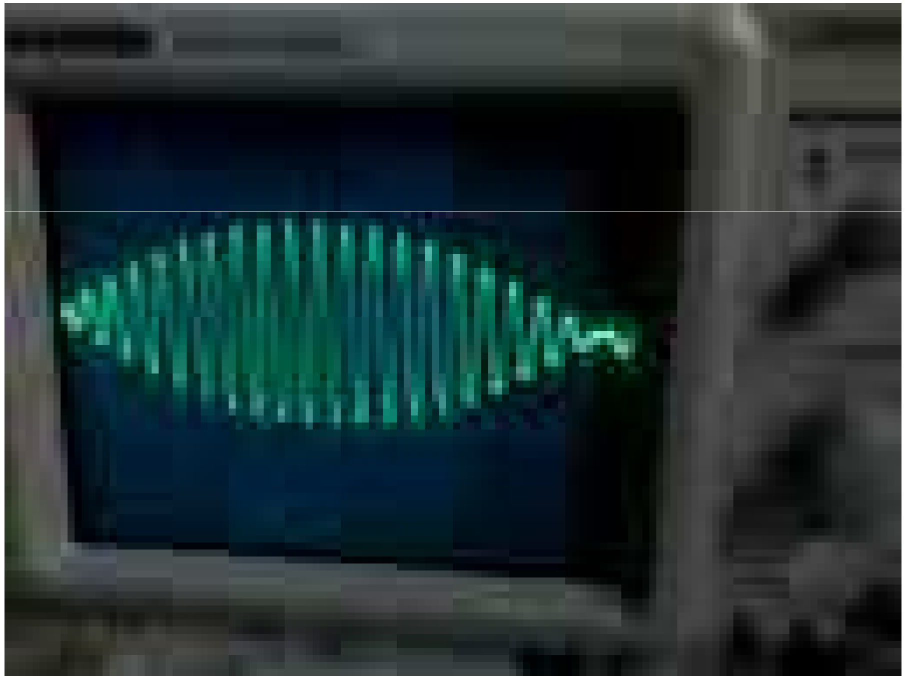
Hasil superposisi kedua gelombang dengan perbedaan frekuensi yang kecil ini disebut *layangan*, hasilnya berupa gelombang paket yang terselubung (envelope), dan kecepatan gelombang paket ini disebut dengan kecepatan group.

Kecepatan fase: $v = \dfrac{\omega_1}{k_1}$

Kecepatan group: $v_g = \dfrac{\Delta\omega}{\Delta k} = \dfrac{d\omega}{dk} = \dfrac{d(kv)}{dk} = v + k\dfrac{dv}{dk}$

Layangan





SUPERPOISI DUA GELOMBANG arah getar saling tegak lurus

Tinjauan dua gelombang dengan frekuensi yang sama dan arah getar yang tegak lurus:

Misal arah getarnya Y dan Z:

$\psi_y (t) = A_1 \, sin \, (\omega t+\phi_1)$

$\psi_z (t) = A_2 \, sin \, (\omega t+\phi_2)$

Superposisi keduanya menghasilkan:

$$-\frac{\Psi_y}{A_1}\sin\varphi_2 + \frac{\Psi_z}{A_2}\sin\varphi_1 = \sin(\omega t)\{\cos(\varphi_2)\sin(\varphi_1) - \cos(\varphi_1)\sin(\varphi_2)\}$$

$$\frac{\Psi_y}{A_1}\cos\varphi_2 - \frac{\Psi_z}{A_2}\cos\varphi_1 = \cos(\omega t)\{\cos(\varphi_2)\sin(\varphi_1) - \cos(\varphi_1)\sin(\varphi_2)\}$$

Kuadratkan kedua persamaan, kemudian dijumlahkan, menghasilkan:

$$sin^2(\delta) = \left(\frac{\Psi_y}{A_1}\right)^2 + \left(\frac{\Psi_z}{A_2}\right)^2 - \frac{2\Psi_y\Psi_z}{A_1A_2}cos(\delta)$$

Dengan beda sudut fase: $\delta = \phi_1 - \phi_2$

Persamaan ini merupakan persamaan umum elips, karena itu superposisinya disebut terpolarisasi elips.

Untuk beberapa kasus khusus, yaitu: $\delta = \pi/2, 3\pi/2, 5\pi/2$......, persamaanya jadi:

$$\left(\frac{\Psi_y}{A_1}\right)^2 + \left(\frac{\Psi_z}{A_2}\right)^2 = 1$$

Terjadi polarisasi elips putar kanan, dan bila amplitudo kedua gelombang sama ($A_1=A_2$), maka superposisinya terpolarisasi lingkaran putar kanan.

Bila: $\delta = 0, 2\pi, 4\pi$,.... Persamaan menjadi: $\Psi_y = \frac{A_1}{A_2}\Psi_z$

Terjadi polarisasi linier