Dinamika Gelombang

MK Gelombang-Optik

Topik 3

Bagian 2

Sub Topik

- Gelombang pada zat cair
- Gelombang di udara (gelombang bunyi)
- Gelombang permukaan air

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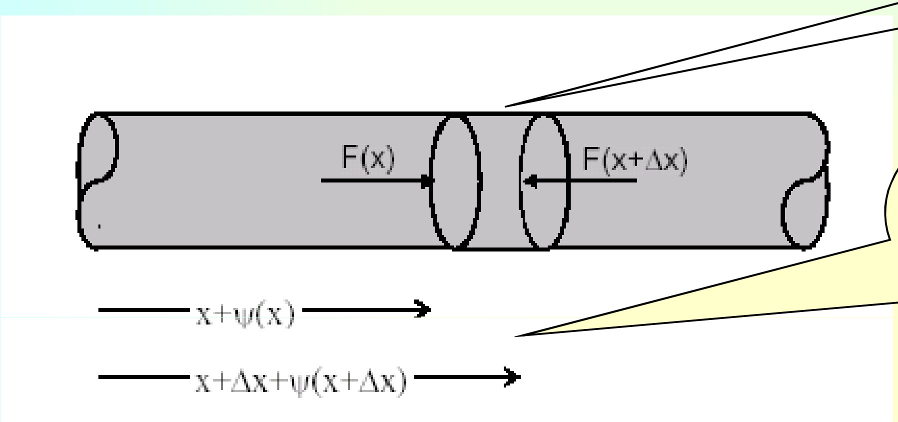
B.4 Gelombang Pada Zat Cair

Elemen Zat cair setebal Δx
dengan luas penampang A



Elemen mengalami deformasi. Perpindahan sisi kiri dan kanan elemen tsb dinyatakan dengan

$\Psi(x)\, dan\, \Psi(x+\Delta x)$

Persamaan gerak elemen Volume zat Cair

$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = F(x) - F(x+\Delta x)$$

Hubungan antara tegangan dan regangan :

$$\frac{F}{A} = -M \frac{\Delta V}{V}$$

Modulus
Bulk

Persamaan gerak elemen Volume zat Cair

$$\rho A\Delta x\frac{\partial^2\Psi}{\partial t^2}=F(x)-F(x+\Delta x)$$

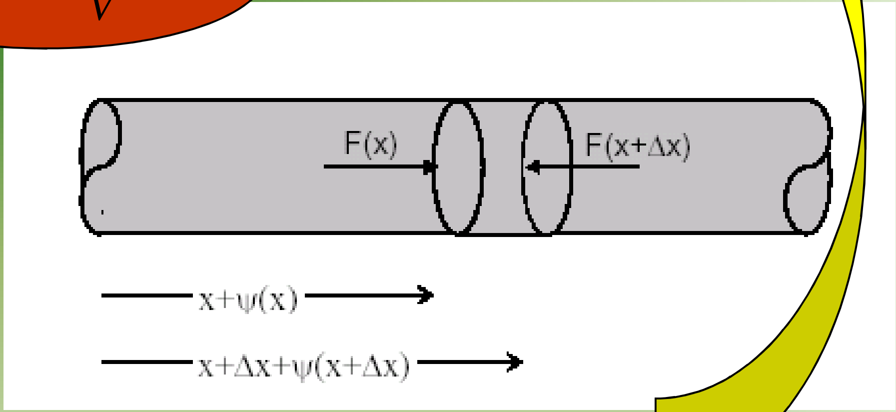
Ekspansi ke Deret Taylor

$$\rho A\Delta x\frac{\partial^2\Psi}{\partial t^2}=\left[F(x)-F(x)-\frac{\partial F}{\partial x}\Delta x\right]$$

$$\rho A\Delta x\frac{\partial^2\Psi}{\partial t^2}=-\Delta x\frac{\partial F}{\partial x}$$

Hubungan antara tegangan dan regangan :

$$\frac{F}{A}=-M\frac{\Delta V}{V}$$



$$\frac{F}{A}=-M\,\frac{A[\Delta x+\Psi(x+\Delta x)-\Psi(x)]-A\Delta x}{A\Delta x}$$

$$\frac{F}{A}=-M\,\frac{A\frac{\partial\Psi}{\partial x}\Delta x}{A\Delta x}=-M\frac{\partial\Psi}{\partial x}$$

$$F=-AM\frac{\partial\Psi}{\partial x}$$

$$\frac{\partial F}{\partial x}=-AM\frac{\partial^2\Psi}{\partial x^2}$$

$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = -\Delta x \frac{\partial F}{\partial x}$$

Substitusi

$$\frac{\partial F}{\partial x} = -AM \frac{\partial^2 \Psi}{\partial x^2}$$

$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = AM \Delta x \frac{\partial^2 \Psi}{\partial x^2}$$

$$\frac{\partial^2 \Psi}{\partial t^2} = \frac{M}{\rho} \frac{\partial^2 \Psi}{\partial x^2}$$

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{M}{\rho} \frac{\partial^2 \Psi}{\partial x^2} = 0$$

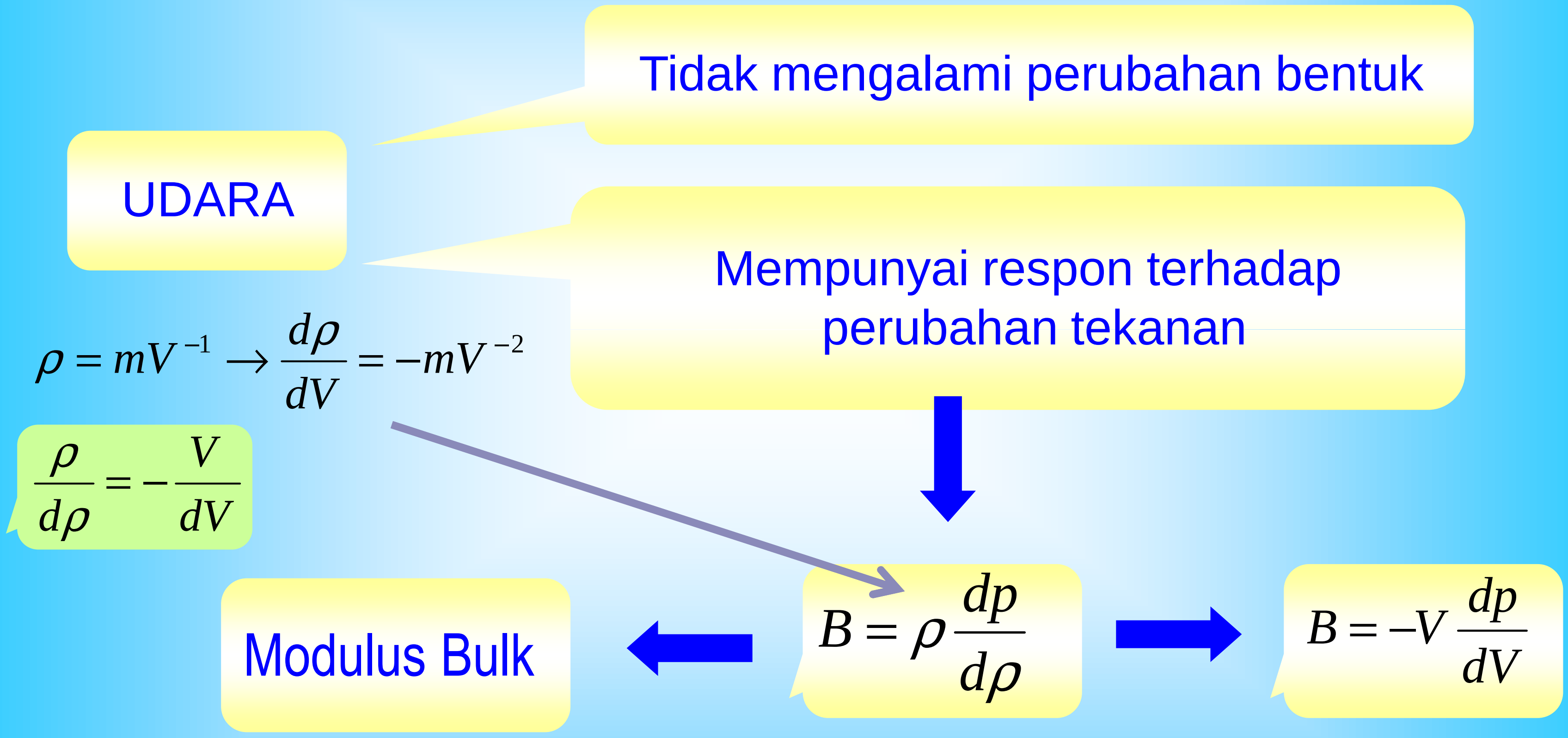
Bandingkan dengan Persamaan Umum gelombang

$$\frac{\partial^2 \Psi}{\partial t^2} - v^2 \frac{\partial^2 \Psi}{\partial x^2} = 0$$

Cepat Rambat Gelombang :

$$v = \sqrt{\frac{M}{\rho}}$$

C. Gelombang di Udara (Gelombang Bunyi)



UDARA

Tidak mengalami perubahan bentuk

Mempunyai respon terhadap perubahan tekanan

$$\rho = mV^{-1} \rightarrow \frac{d\rho}{dV} = -mV^{-2}$$

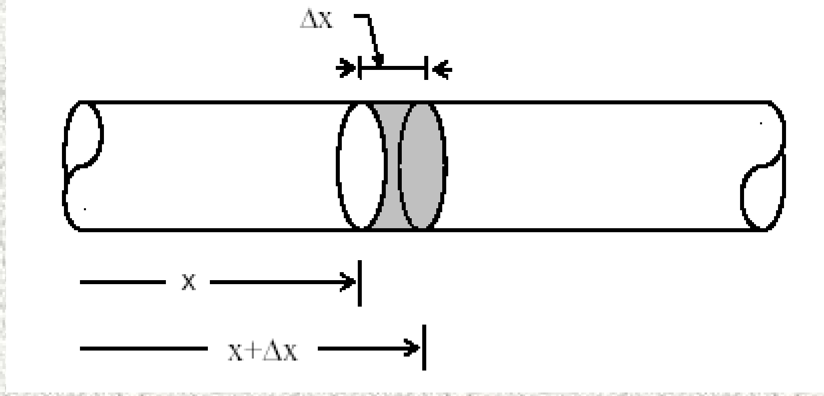
$$\frac{\rho}{d\rho} = -\frac{V}{dV}$$

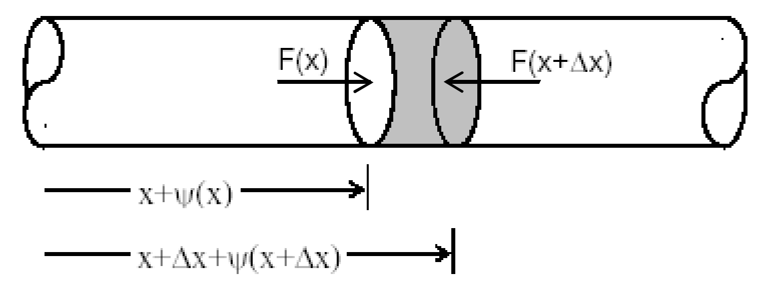
Modulus Bulk

$$B = \rho \frac{dp}{d\rho}$$

$$B = -V \frac{dp}{dV}$$

C.1 Cepat Rambat Gelombang Bunyi





Hukum II Newton

$$ma = F$$

$$\rho A\Delta x \frac{\partial^2\Psi}{\partial t^2} = A\left[p(x) - p(x+\Delta x)\right]$$

Ekspansi ke Deret Taylor

$$\rho \frac{\partial^2\Psi}{\partial t^2} = -\frac{\partial p}{\partial x}$$

Dalam perambatannya berlaku hukum kekekalan massa

$$\rho A\left[\Delta x+\Psi(x+\Delta x)-\Psi(x)\right]=\rho_0 A\Delta x=c$$

Ekspansi ke Deret Taylor

$$\rho A\Delta x\left\{1+\frac{\partial\Psi}{\partial x}\right\}=c \;\Rightarrow\; \rho\left\{1+\frac{\partial\Psi}{\partial x}\right\}=C$$

$$\frac{\partial\Psi}{\partial x}<<1$$

$$+\frac{\partial\Psi}{\partial x}\Big)+\rho\frac{\partial^2\Psi}{\partial x^2}=0 \;\Rightarrow\; \frac{\partial\rho}{\partial x}+\rho\frac{\partial^2\Psi}{\partial x^2}=0 \;\Rightarrow\; \frac{\partial\rho}{\partial x}=-\rho\frac{\partial^2\Psi}{\partial x^2}$$

Cepat rambat Gelombang bunyi di udara

Mod

$$\rho\frac{\partial^2\Psi}{\partial t^2}=-\frac{\partial p}{\partial x}=-\frac{\partial p}{\partial\rho}\frac{\partial\rho}{\partial x} \;\Rightarrow\; \rho\frac{\partial^2\Psi}{\partial t^2}=-\frac{\partial p}{\partial\rho}\frac{\partial\rho}{\partial x}$$

$$B=\rho\frac{\partial p}{\partial\rho} \;\Rightarrow\; \frac{\partial p}{\partial\rho}=\frac{B}{\rho}$$

$$\rho\frac{\partial^2\Psi}{\partial t^2}=B\frac{\partial^2\Psi}{\partial x^2}$$

$$\frac{\partial^2\Psi}{\partial t^2}-\frac{B}{\rho}\frac{\partial^2\Psi}{\partial x^2}=0$$

$$\frac{\partial^2\Psi}{\partial t^2}-v^2\frac{\partial^2\Psi}{\partial x^2}=0$$

$$\sqrt{B}$$

Gelombang dalam gas bersifat adiabatik

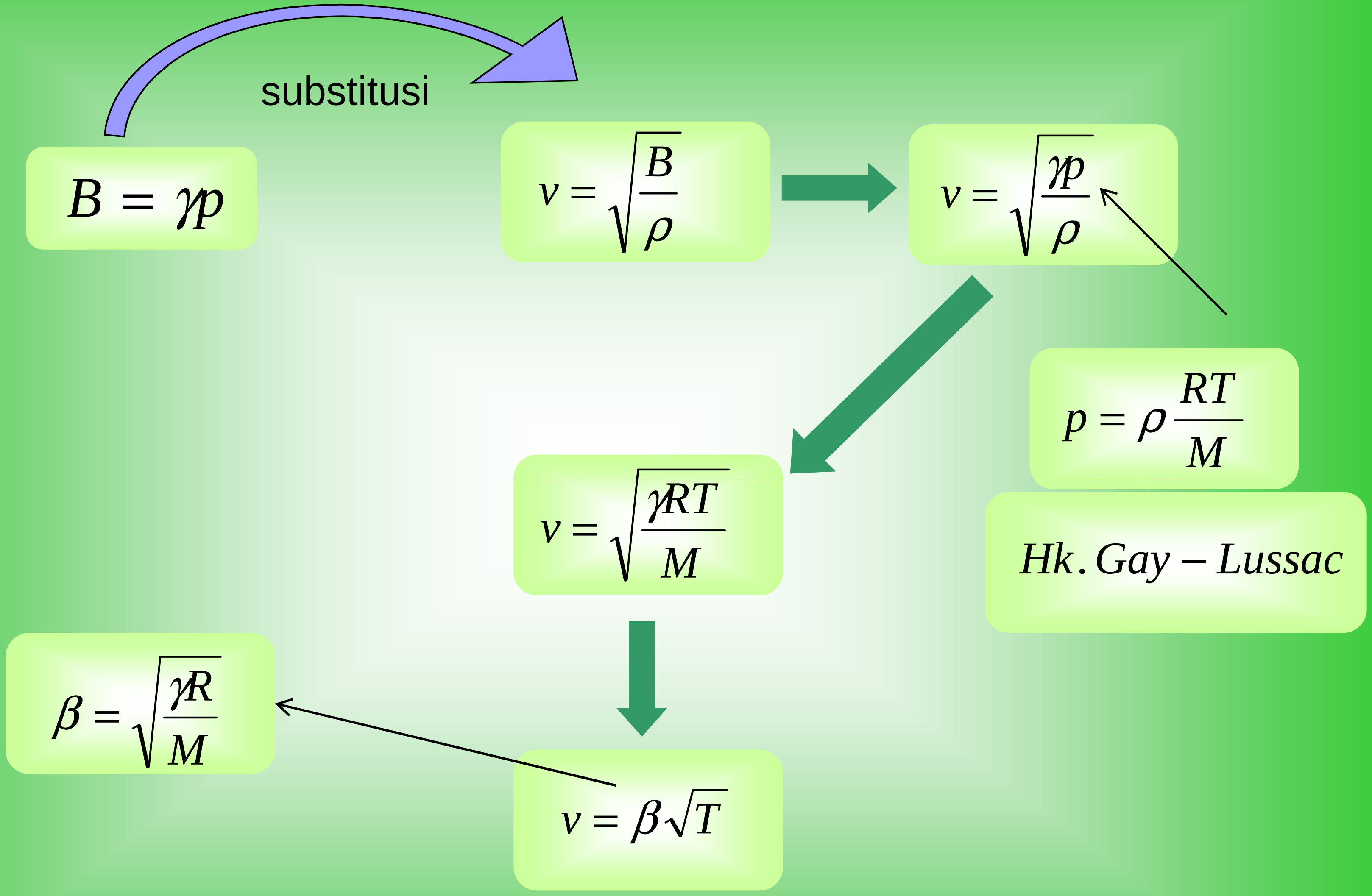
$$pV^{\gamma} = c \quad \rightarrow \quad p\rho^{-\gamma} = c$$

$$dp\rho^{-\gamma} - p\gamma\rho^{-\gamma-1}d\rho = 0$$

$$\frac{dp}{d\rho} = \frac{\gamma p}{\rho}$$

$$\uparrow\downarrow$$

$$B = \rho\frac{dp}{d\rho} \quad \rightarrow \quad B = \gamma p$$



substitusi

$$B = \gamma p$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

$$p = \rho \frac{RT}{M}$$

$$Hk.\,Gay - Lussac$$

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v = \beta \sqrt{T}$$

$$\beta = \sqrt{\frac{\gamma R}{M}}$$

C.2 Intensitas Gelombang Bunyi

Dari $B = -\frac{p}{d\psi / dx}$ Diperoleh hubungan antara gelombang tekanan dan gelombang pergeseran $p = -B\frac{d\Psi}{dx}$

Daya atau arus energi gelombang bunyi:

$$P = p.A\frac{\partial \Psi}{\partial t} \rightarrow P = -B\frac{\partial \Psi}{\partial x}A\frac{\partial \Psi}{\partial t}$$

$$P = B.A.v\left(\frac{\partial \Psi}{\partial x}\right)^2$$

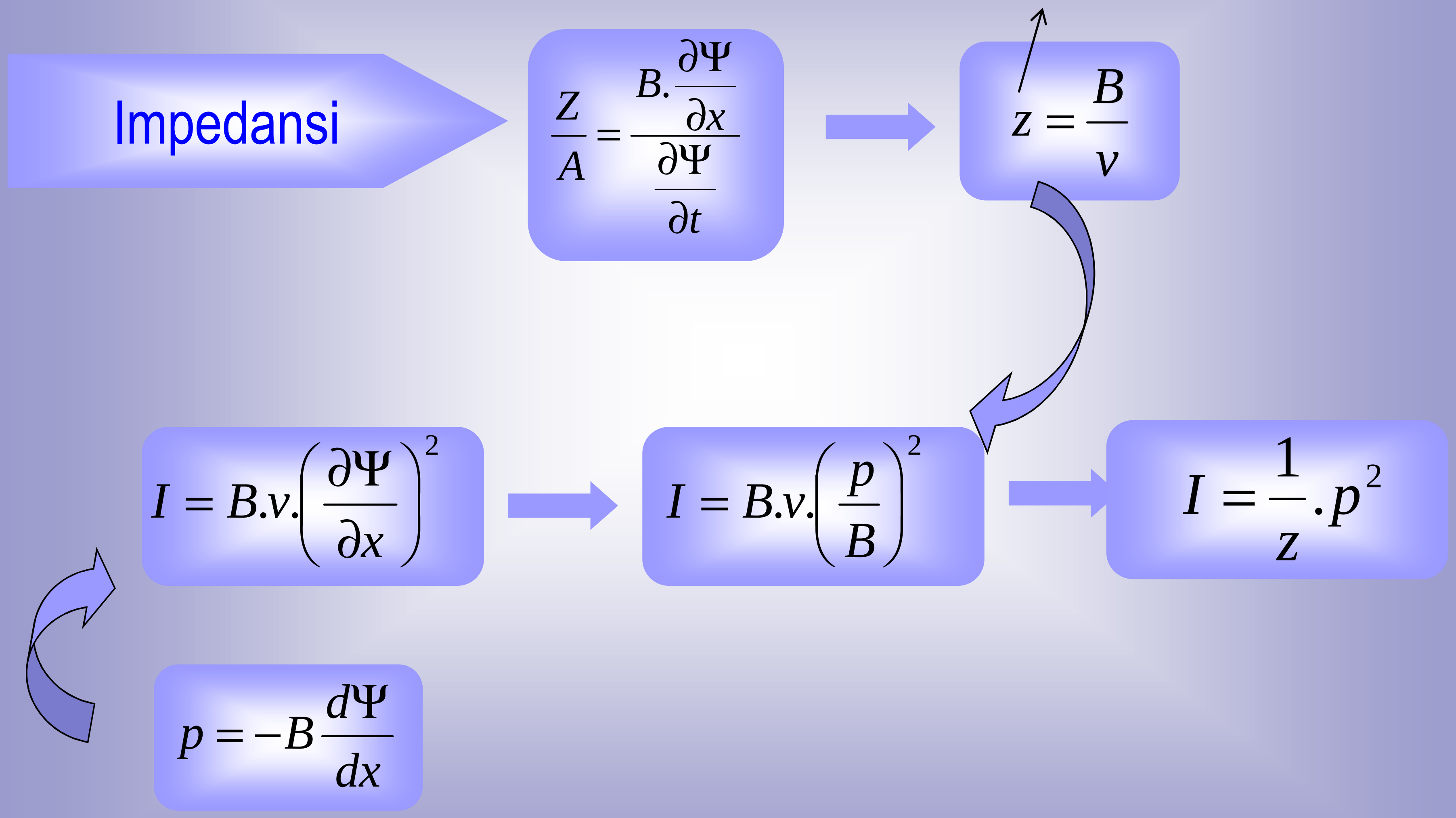
Rapat arus energi atau Intensitas gelombang bunyi *P/A*

$$I = B.v.\left(\frac{\partial \Psi}{\partial x}\right)^2$$

$$p = -B.\frac{\partial\Psi}{\partial x} = \frac{F}{A}$$

$$F = -Z.\frac{\partial\Psi}{\partial t}$$

Impedansi karakteristik
Impedansi jenis
Rapat Impedansi



Impedansi

$$\frac{Z}{A} = \frac{B.\frac{\partial\Psi}{\partial x}}{\frac{\partial\Psi}{\partial t}}$$

$$z = \frac{B}{v}$$

$$I = B.v.\left(\frac{\partial\Psi}{\partial x}\right)^2$$

$$I = B.v.\left(\frac{p}{B}\right)^2$$

$$I = \frac{1}{z}.p^2$$

$$p = -B\frac{d\Psi}{dx}$$

Intensitas gelombang bunyi sering dinyatakan sebagai taraf intensitas β dalam satuan decibel (dB), yang menyatakan tingkat relatif dan didefinisikan sebagai berikut::

$$\beta = \log \frac{I}{I_0} \; Bel \quad \rightarrow \quad \beta = 10.\log \frac{I}{I_0} \; dB$$

Dengan:

$$I_0 = 10^{-12} W / m^2 = \text{Intensitas acuan}$$

Gelombang Permukaan Air

Anggap Air Memiliki sifat – sifat sebagai berikut

a. Non viskos, Viskositas yang disebabkan oleh gesekan internal, diabaikan.

b. Amplitudo gelombang relatif lebih kecil dibanding panjang gelombangnya.

c. Gaya-gaya yang bekerja hanyalah gaya gravitasi dan tegangan permukaan.

d. Inkompresibel, Volume tidak berubah karena perubahan tekanan, jadi rapat massanya konstan.

Selain itu air dipandang sebagai air ideal, dengan sifat sifat :

a. Berlaku hukum kekekalan massa :

$$\nabla \cdot (\rho v) = -\frac{\partial \rho}{\partial t}$$

$$\frac{\partial \rho}{\partial t} = 0 \Rightarrow$$

Inkompresibel

$$\Downarrow$$

$$\nabla \cdot (\rho v) = 0$$

$$v = \frac{\partial \Psi}{\partial t} \Rightarrow$$

$$\Downarrow$$

$$\nabla \cdot \left(\rho \frac{\partial \Psi}{\partial t} \right) = 0 \Rightarrow \nabla \cdot \Psi = \text{Konstan}$$

b. Tidak ada gelembung.

$$\oint \Psi\cdot\hat{n}\,dA=0$$

Teorema Divergensi

$$\int \nabla\cdot\Psi dV=0$$

$$\nabla\cdot\Psi=0 \Longrightarrow \frac{\partial\Psi_x}{\partial x}+\frac{\partial\Psi_y}{\partial y}=0$$

c. Tidak ada pusaran.

$$\oint \vec{v}\cdot d\vec{\ell}=0$$

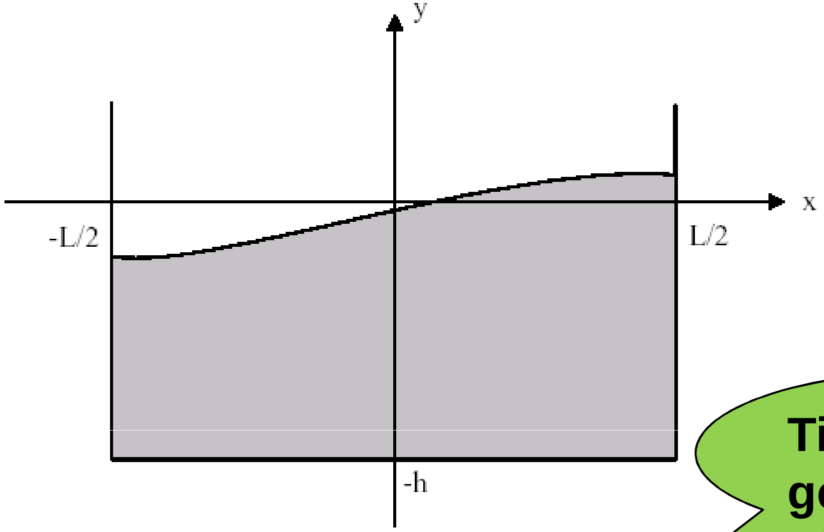
Teorema Stokes (Rotasi)

$$\int \nabla\times\vec{v}\cdot\hat{n}\,dA=0$$

$$\nabla\times\frac{\partial\Psi}{\partial t}=0 \Longrightarrow \frac{\partial}{\partial t}\left(\nabla\times\Psi\right)=0 \Longrightarrow \left(\nabla\times\Psi\right)=0$$

$$\hat{k}\left(\frac{\partial\Psi_y}{\partial x}-\frac{\partial\Psi_x}{\partial y}\right)=0$$

D.1. Penerapan Syarat Batas



Syarat batas di x = 0 :

$\psi_y = 0$, maka ψ_y mengandung faktor sin(kx).

$\psi_y = \cos(\omega t)\sin(kx)\,f(y)$ ….(1)

Syarat batas di $x = \pm\frac{L}{2}$:

$\psi_x = 0$, maka y_x mengandung faktor cos(kx).

$\psi_x = \cos(\omega t)\cos(kx)\,g(y)$ ….(2)

Tidak ada gelembung

Pers. 1

$$\frac{\partial \Psi_x}{\partial x} + \frac{\partial \Psi_y}{\partial y} = 0$$

Pers. 2

$$\left(\frac{\partial \Psi_y}{\partial x} - \frac{\partial \Psi_x}{\partial y}\right) = 0$$

Tidak ada pusaran

$$-kg(y) + \frac{df(y)}{dy} = 0 \;.......(3)$$

$$k\,f(y) - \frac{dg(y)}{dy} = 0 \;.......(4)$$

Persamaan 3

Diferensiasikan terhadap y

$$-k\frac{dg(y)}{dy}+\frac{d^2f(y)}{dy^2}=0$$

$$\frac{dg(y)}{dy}=\frac{1}{k}\frac{d^2f(y)}{dy^2} \quad ....(5)$$

Substitusi ke persamaan 4

$$k\,f(y)-\frac{1}{k}\frac{d^2f(y)}{dy^2}=0$$

$$\frac{d^2f(y)}{dy^2}-k^2\,f(y)=0$$

Solusi Persamaan

$$f(y)=Ae^{ky}+Be^{-ky}$$

Persamaan 4

Diferensiasikan terhadap y

$$k\frac{df(y)}{dy}-\frac{d^2g(y)}{dy^2}=0$$

$$\frac{df(y)}{dy}=\frac{1}{k}\frac{d^2g(y)}{dy^2} \quad ....(6)$$

Substitusi ke persamaan 3

$$-k\,g(y)+\frac{1}{k}\frac{d^2g(y)}{dy^2}=0$$

$$\frac{d^2g(y)}{dy^2}-k^2\,g(y)=0$$

Solusi Persamaan

$$g(y)=Ce^{ky}+De^{-ky}$$

Syarat Batas : y = -h: $\Psi_y = 0$ **Maka f (-h) = 0**

$$f(-h) = Ae^{-kh} + Be^{kh} = 0$$

$$B = -Ae^{-2kh}$$

$$f(y) = Ae^{ky} + (-Ae^{-2kh})e^{-ky}$$

$$f(y) = A\left(e^{ky} - e^{-k(2h+y)}\right)$$

$$\text{dari pers } (3): -kg(y) + \frac{df(y)}{dy} = 0 \rightarrow g(y) = A\left(e^{ky} + e^{-k(2h+y)}\right)$$

Persamaan Gelombang arah x dan y pada persamaan (1) dan (2)

$$\Psi_y = A\cos(\omega t)\sin(kx)\left\{e^{ky} - e^{-k(2h+y)}\right\}.......(7)$$

$$\Psi_x = A\cos(\omega t)\cos(kx)\left\{e^{ky} + e^{-k(2h+y)}\right\}.......(8)$$

Ekspansi ke deret pangkat

Kasus khusus

a. Bila h >> , maka

$$\Psi_y = Ae^{ky}\cos(\omega t)\sin(kx).......(9)$$

$$\Psi_x = Ae^{ky}\cos(\omega t)\cos(kx).......(10)$$

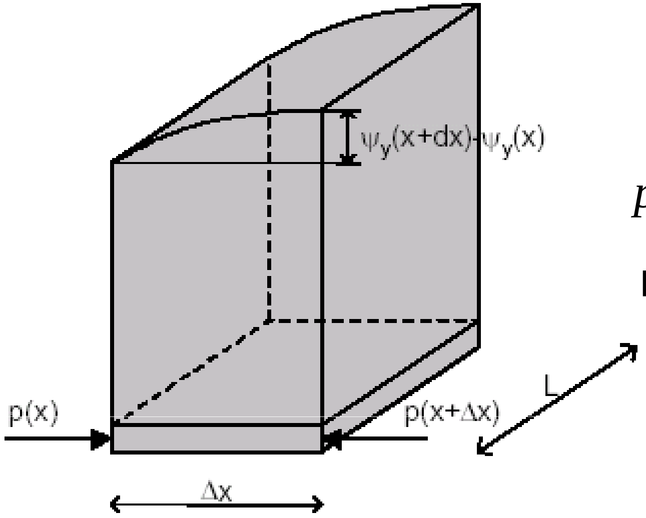
b. Bila h << , maka :

$$\Psi_y = 2Ak(y+h)\cos(\omega t)\sin(kx).......(11)$$

$$\Psi_x = 2A\cos(\omega t)\cos(kx).......(12)$$

D.2. Hubungan Dispersi Gelombang Permukaan Air

Persamaan Gerak



$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = L\Delta y[\,p(x) - p(x+\Delta x)]$$

$$p(x) = \rho g \Psi_y(x)$$

Hukum hidrostatika

Deret Taylor

$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = L\Delta y \rho g[\Psi_y(x) - \Psi_y(x+\Delta x)]$$

$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = -L\Delta y \Delta x \rho g \frac{\partial \Psi_y}{\partial x}$$

$$\Delta m = \Delta V\ \rho \quad \Rightarrow \quad \Delta m = L\Delta y \Delta x\ \rho$$

$$\frac{\partial^2 \Psi_x}{\partial t^2} = -g \frac{\partial \Psi_y}{\partial x}$$

$$\frac{\partial^2\Psi_x}{\partial t^2} = -g\frac{\partial\Psi_y}{\partial x}$$

$$\Psi_y = A\cos(\omega t)\sin(kx)\left\{e^{ky} - e^{-k(2h+y)}\right\}$$

$$\Psi_x = A\cos(\omega t)\cos(kx)\left\{e^{ky} + e^{-k(2h+y)}\right\}$$

$$\omega^2\left\{e^{ky} + e^{-k(2h+y)}\right\} = gk\left\{e^{ky} - e^{-k(2h+y)}\right\}$$

Syarat batas di y = 0

$$\omega^2\left\{1 + e^{-k2h}\right\} = gk\left\{1 - e^{-k2h}\right\}$$

Persamaan Dispersi

$$\omega^2 = gk\frac{1 - e^{-2kh}}{1 + e^{-2kh}}$$

D.3. Gelombang Gravitasi dan Gelombang Riak

$$\omega^2 = gk\,\frac{1-e^{-2kh}}{1+e^{-2kh}}$$

Persamaan Dispersi

Kasus Khusus

a. Bila h >> $\Rightarrow$ $e^{-2kh} \cong 0$

Persamaan dispersi menjadi :

$$\omega^2 = gk \Rightarrow \omega = \sqrt{gk} \Leftrightarrow k = \frac{2\pi}{\lambda}$$

$$\Downarrow$$

$$v_f = \frac{\omega}{k}$$

$$\Downarrow$$

Kecepatan fase

$$v_f = \sqrt{\frac{g\lambda}{2\pi}}$$

$$v_g = \frac{d\omega}{dk}$$

$$v_g = \frac{d\sqrt{gk}}{dk}$$

$$v_g = \frac{1}{2}\sqrt{\frac{g}{k}}$$

$$v_g = \frac{1}{2}\sqrt{\frac{g\lambda}{2\pi}}$$

Gelombang ini disebut Gelombang Gravitasi

Gelombang ini bersifat dispersif

$$v_f \neq v_g$$

Kecepatan Grup

b. Bila h <<, Maka e^{-2kh} *dalam deret pangkat*

$$e^{-2kh} = 1 + (-2kh) + \frac{(-2kh)^2}{2!} + ...$$

$$e^{-2kh} = 1 - (-2kh)$$

$$\omega^2 = gk\frac{1-e^{-2kh}}{1+e^{-2kh}}$$

$$\omega^2 = gk\frac{1-(1-2kh)}{1+(1-2kh)} \Rightarrow \omega^2 = gk^2h \Rightarrow \omega = k\sqrt{gh}$$

$$v_f = \frac{\omega}{k} \Rightarrow v_f = \frac{k}{k}\sqrt{gh} \Rightarrow v_f = \sqrt{g}$$

Gelombang Riak bersifat non Dispersif

$$v_g = \frac{d\omega}{dk} \Rightarrow v_g = \frac{dk\sqrt{gh}}{dk} \Rightarrow v_g = \sqrt{gh} \Rightarrow v_g = v_f$$

$$\gamma \Rightarrow \gamma k^2\Psi$$

Efek tegangan permukaan diperhitungkan

Tekanan pada elemen massa bertambah

$$p(x)=\left(\rho g+\gamma k^2\right)\Psi_y(x)$$

$$p(x+\Delta x)=\left(\rho g+\gamma k^2\right)\Psi_y(x+\Delta x)$$

$$\Delta m\frac{\partial^2\Psi_x}{\partial t^2}=L\Delta y[p(x)-p(x+\Delta x)]$$

$$\Delta m\frac{\partial^2\Psi_x}{\partial t^2}=L\Delta y\left(\rho g+\gamma k^2\right)[\Psi_y(x)-\Psi_y(x+\Delta x)]$$

Deret Taylor

$$\Delta m\frac{\partial^2\Psi_x}{\partial t^2}=L\Delta y\left(\rho g+\gamma k^2\right)\left(-\Delta x\frac{\partial\Psi_y(x)}{\partial x}\right)$$

$$\rho\frac{\partial^2\Psi_x}{\partial t^2}=-\left(\rho g+\gamma k^2\right)\frac{\partial\Psi_y}{\partial x}$$

$$\frac{\partial^2\Psi_x}{\partial t^2}=-\left(g+\frac{\gamma k^2}{\rho}\right)\frac{\partial\Psi_y}{\partial x}$$

$$\Psi_y=A\cos(\omega t)\sin(kx)\left\{e^{ky}-e^{-k(2h+y)}\right\}$$

$$\Psi_x=A\cos(\omega t)\cos(kx)\left\{e^{ky}+e^{-k(2h+y)}\right\}$$

$$\omega^2=\left(gk+\frac{\gamma k^3}{\rho}\right)\frac{1-e^{-2kh}}{1+e^{-2kh}}$$

Untuk kasus h >>, tegangan permukaan tidak diabaikan

$$e^{-2kh}\cong 0$$

$$\omega^2=\left(gk+\frac{\gamma k^3}{\rho}\right)\Rightarrow v=\sqrt{\frac{g\lambda}{2\pi}+\frac{2\pi\gamma}{\lambda\rho}}$$

Untuk kasus h << tegangan permukaan tidak diabaikan, Bagaimana dispersivitasnya?