ENERGETIKA GELOMBANG
bagian 1

Mata Kuliah GELOMBANG-OPTIK
Topik 4

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SUB POKOK BAHASAN

A. ENERGI KINETIK DAN ENERGI POTENSIAL
B. PENJABARAN PERSAMAAN GELOMBANG MELALUI KEKEKALAN ENERGI
C. RAPAT ENERGI DAN INTENSITAS GELOMBANG
D. RAPAT MOMENTUM DAN RAPAT ARUS MOMENTUM

A. ENERGI KINETIK DAN ENERGI POTENSIAL

Energi kinetik terjadi karena gerak massa: $E_k = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$

Energi potensial karena elastisitas: $E_p = \frac{1}{2}k\left[\psi(x+\Delta x)-\psi(x)\right]^2$

$$E_p = \frac{1}{2}k\left[\psi(x)+\Delta x\frac{\partial \psi}{\partial x}-\psi(x)\right]^2 \Leftarrow E_p = \frac{1}{2}k(\Delta x)^2\left[\frac{\partial \psi}{\partial x}\right]^2$$

Ekspansi ke deret Taylor

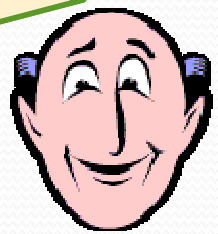
$$v=\sqrt{\frac{K}{\rho}}=\sqrt{\frac{k\Delta x}{\Delta m/\Delta x}}=\sqrt{\frac{k(\Delta x)^2}{\Delta m}} \Rightarrow k(\Delta x)^2 = \Delta m v^2$$

$$E_p = \frac{1}{2}\Delta m v^2\left[\frac{\partial \psi}{\partial x}\right]^2$$

$$\frac{\partial \psi}{\partial x} = -\frac{1}{v}\frac{d\psi}{dt}$$

$$E_p = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

Apa kesimpulannya?



B. PENJABARAN PERSAMAAN GELOMBANG MEALUI KEKEKALAN ENERGI

$$E_k = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2 \Rightarrow dE_k = \frac{1}{2}dm\left(\frac{\partial \psi}{\partial t}\right)^2 = \frac{1}{2}\rho\, dx\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E_p = \frac{1}{2}k(\Delta x)^2\left[\frac{\partial \psi}{\partial x}\right]^2 \quad E_p = \frac{1}{2}K\Delta x\left[\frac{\partial \psi}{\partial x}\right]^2 \Rightarrow dE_p = \frac{1}{2}K dx\left[\frac{\partial \psi}{\partial x}\right]^2$$

maka total energinya: $dE = dE_k + dE_p \Rightarrow E = \int_0^{n\lambda}\frac{1}{2}\left[\rho\left(\frac{\partial \psi}{\partial t}\right)^2 + K\left(\frac{\partial \psi}{\partial x}\right)^2\right]dx$

Berdasarkan kekekalan energi (energi total = tetap), maka $\frac{dE}{dt} = 0$

$$\frac{1}{2}\int\left\{\rho\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial t}\right)^2 + K\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial x}\right)^2\right\}dx = 0 \Rightarrow \frac{1}{2}\int\left\{2\rho\frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial t^2} + 2K\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial t \partial x}\right\}dx = 0$$

$$\Rightarrow \int \rho\frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial t^2}dx + \int K\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial t \partial x}dx = 0$$

Kita tahu

$$\int \rho \frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial t^2}dx + \int K \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial t \partial x}dx = 0$$

$$\frac{d}{dx}\left(\frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial t}\right) = \frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial x \partial t}$$

maka

$$\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial t \partial x} = \frac{d}{dx}\left(\frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial t}\right) - \frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial x^2}$$

$$\int \rho \frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial t^2}dx + \underbrace{K \frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial t}\Big|_0^{n\lambda}}_{=0} - \int K \frac{\partial^2 \psi}{\partial x^2}\frac{\partial \psi}{\partial t}dx = 0$$

$$\psi(x) = A\cos(kx - \omega t)$$

$$\int \left(\rho \frac{\partial^2 \psi}{\partial t^2} - K\frac{\partial^2 \psi}{\partial x^2}\right)\frac{\partial \psi}{\partial t}dx = 0$$

$$\rho \frac{\partial^2 \psi}{\partial t^2} = -kA\sin(kx - \omega t)$$

$$\frac{\partial \psi}{\partial x} = -kA\sin(kx - \omega t)$$

$$\frac{\partial \psi}{\partial t} = \omega A\sin(kx - \omega t)$$

$$\frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial t} = -k\omega A^2 \sin^2(kx - \omega t)$$

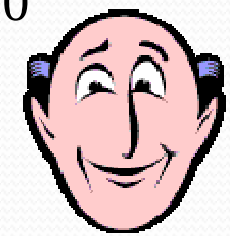
$$\rho \frac{\partial^2 \psi}{\partial t^2} - K\frac{\partial^2 \psi}{\partial x^2} = 0$$

$$\frac{\partial^2 \psi}{\partial t^2} - \frac{K}{\rho}\frac{\partial^2 \psi}{\partial x^2} = 0$$

Merupakan persamaan differensial gelombang.

$$x = 0 \to n\lambda, \Rightarrow \frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial t} = -k\omega A^2\left(\sin^2(n2\pi - \omega t) - \sin^2(-\omega t)\right) = 0$$

Sama dengan hasil yang diperoleh pada pembahasan dinamika gelombang.



C. RAPAT ENERGI DAN INTENSITAS GELOMBANG

$$E = E_k + E_p$$

$$E_k = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E_p = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E = \Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E = \Delta m v^2\left(\frac{\partial \psi}{\partial x}\right)^2$$

Sehingga Rapat energi dapat dirumuskan: $\varepsilon = \rho v^2\left(\frac{\partial \psi}{\partial x}\right)^2$

Untuk gelombang dengan persamaan: $\psi(x,t) = \psi_0 \cos(kx - \omega t)$

Maka :

$$\varepsilon = \rho\omega^2\psi_0{}^2 \sin^2(kx - \omega t)$$

$$\varepsilon = \frac{1}{2}\rho\omega^2\psi_0{}^2\{1 - \cos 2(kx - \omega t)\}$$

Karena nilai rata-rata dari $\left\langle \sin^2(kx - \omega t)\right\rangle = \frac{1}{2}$

$$\bar{\varepsilon} = \frac{1}{2}\rho\omega^2\psi_0{}^2$$

energi merambat dengan kecepatan v = ω/k juga, dan berubah-rubah secara periodik dengan frekuensi sudut **2ω**

$$\varepsilon = \rho v^2 \left(\frac{\partial \psi}{\partial x} \right)^2$$

Dari rapat energi ini, dapat diturunkan rapat daya persatuan luas penampang atau rapat arus energi (Intensitas)

Untuk gelombang dengan persamaan: $\psi(x,t) = \psi_0 \cos(kx - \omega t)$

Maka :

$$\varepsilon = \rho \omega^2 {\psi_0}^2 \sin^2(kx - \omega t)$$

$$I = \varepsilon v \quad \Rightarrow \quad I = v \rho \omega^2 {\psi_0}^2 \sin^2(kx - \omega t)$$

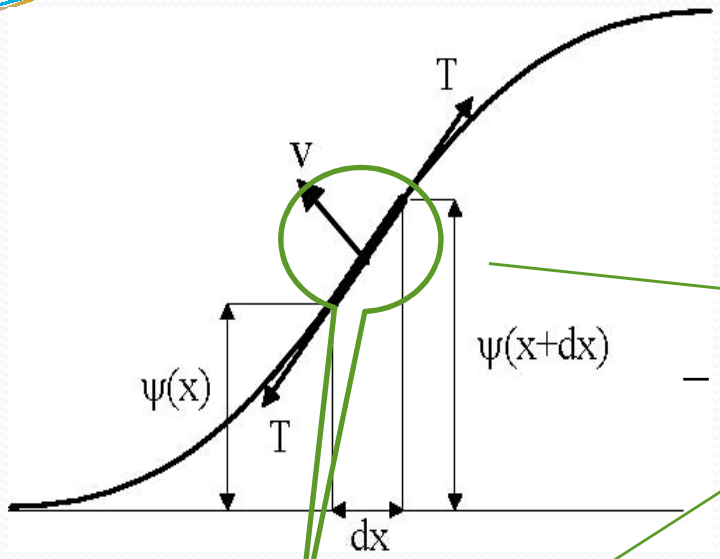
Dari rapat energi rata-rata

$$\bar{\varepsilon} = \frac{1}{2} \rho \omega^2 {\psi_0}^2$$

Dapat diturunkan rapat daya rata-rata persatuan luas penampang atau rapat arus energi rata-rata (Intensitas rata-rata)

$$\bar{I} = \bar{\varepsilon} v \quad \Rightarrow \quad \bar{I} = \frac{1}{2} \rho \omega^2 {\psi_0}^2 v$$

D. RAPAT MOMENTUM

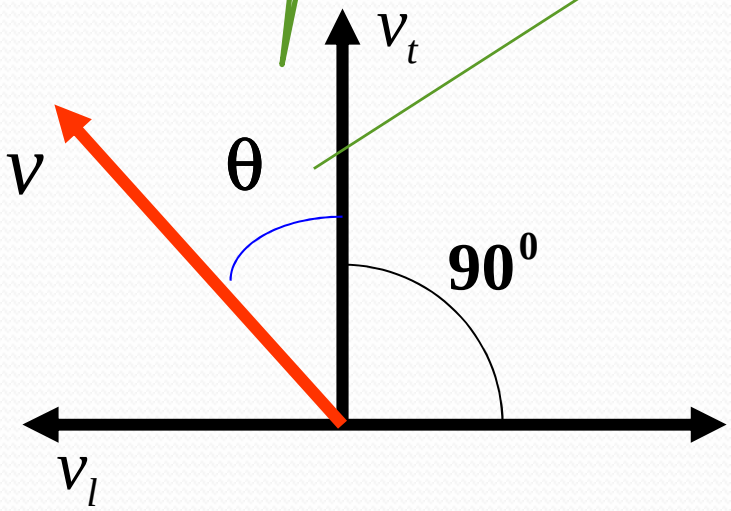


Rapat momentum = momentum persatuan volume

$$p = \frac{dm}{dV} v_l \quad \text{atau} \quad p = \rho v_l$$

$$-\frac{v_l}{v_t} = \tan\theta = \frac{\partial \psi}{\partial x} \text{ atau } v_l = -v_t \tan\theta = -v_t \frac{\partial \psi}{\partial x}$$

$$\frac{\partial \psi}{\partial x} = -\frac{1}{v}\frac{\partial \psi}{\partial t} \qquad v_l = -\frac{\partial \psi}{\partial t}\frac{\partial \psi}{\partial x}$$



$$v_l = \frac{1}{v}\left(\frac{\partial \psi}{\partial t}\right)^2 \Leftarrow v_l = -\frac{\partial \psi}{\partial t}\left(-\frac{1}{v}\frac{\partial \psi}{\partial t}\right)$$

$$p = \rho v_l \Rightarrow \boxed{p = \frac{\rho}{v}\left(\frac{\partial \psi}{\partial t}\right)^2}$$

rapat momentum

$$p = \frac{\rho}{v}\left(\frac{\partial \psi}{\partial t}\right)^2$$

Dari rapat momentum ini, dapat diturunkan rapat aliran momentum persatuan waktu atau rapat arus momentum g

$$g = pv$$

Jadi gelombang mengangkut daya persatuan luas penampang (aliran rapat energi = intensitas), dan momentum (dinyatakan oleh aliran rapat momentum persatuan waktu).

substitusi

$$g = \frac{\rho}{v}\left(\frac{\partial \psi}{\partial t}\right)^2 v$$

$$\varepsilon = pv$$

Rapat Energi

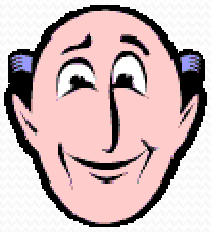
$$\varepsilon = \rho v^2 \left(\frac{\partial \psi}{\partial x}\right)^2$$

$$g = \rho\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$g = \varepsilon$$

$$\varepsilon = \rho\left(\frac{\partial \psi}{\partial t}\right)^2$$

Rapat Arus Momentum



Untuk gelombang dengan persamaan: $\psi(x,t) = \psi_0 \cos(kx - \omega t)$

$$\bar{p} = \frac{1}{2}\frac{\rho}{v}\omega^2 {\psi_0}^2$$

$$\bar{g} = \frac{1}{2}\rho\omega^2 {\psi_0}^2$$

Intensitas: $I = \varepsilon\, v$ atau $I = g\, v$

Intensitas rata-rata: $\bar{I} = \bar{\varepsilon}\, v$ atau $\bar{I} = \bar{g}\, v$