

# MODULASI GELOMBANG

Mata Kuliah GELOMBANG-OPTIK

TOPIK 5

Bagian 1

## SUB TOPIK

Pendahuluan

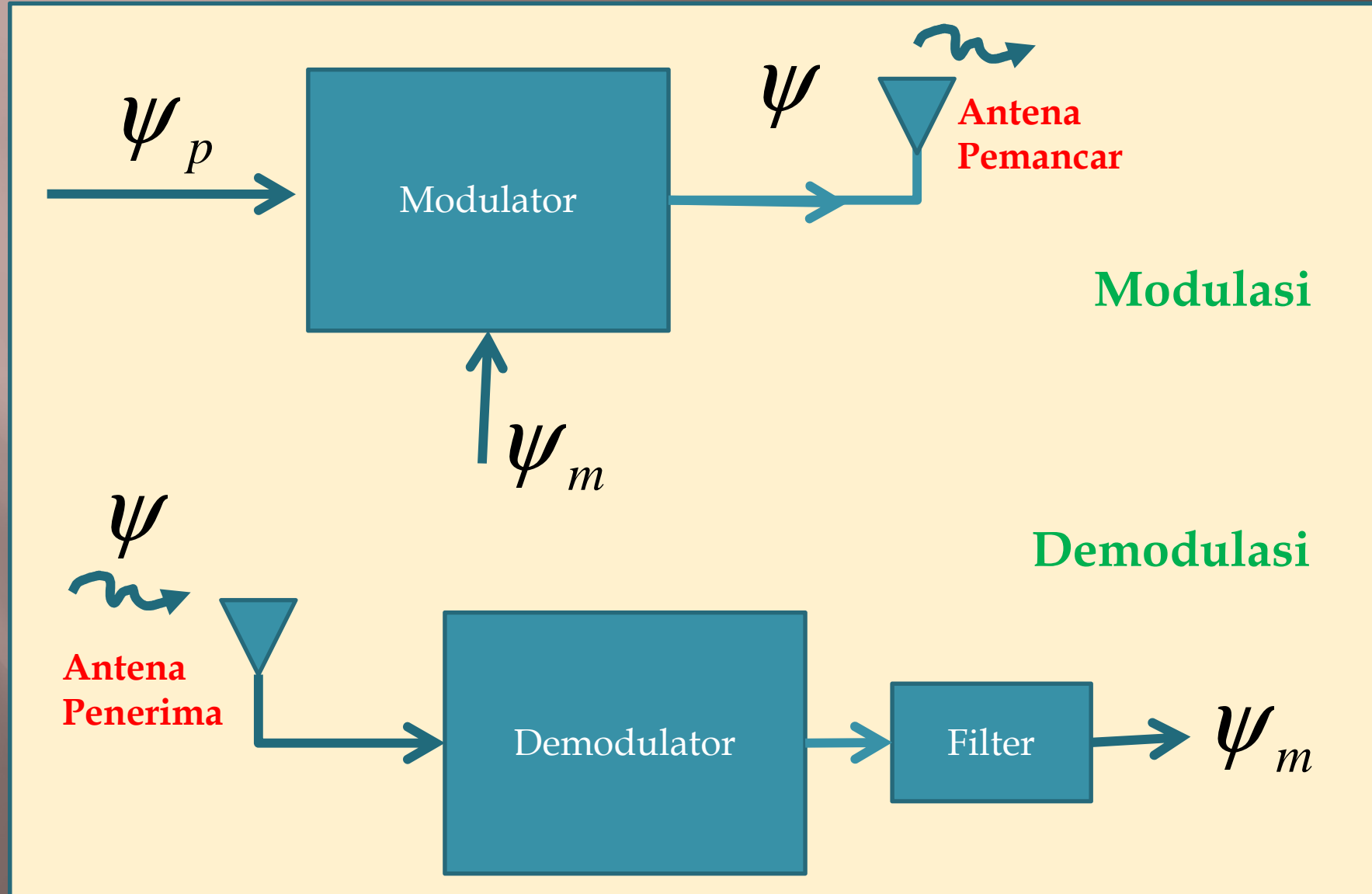
Transformasi Fourier dan Fungsi Delta Dirac

Modulasi Double Side Band (DSB)

# A. Pendahuluan

- ▣ Modulasi → proses perubahan karakteristik suatu gelombang menurut pola gelombang lain, dengan cara menumpangkan (memboncengkan) suatu gelombang pada gelombang lainnya.
- ▣ Dalam teknik komunikasi, gelombang atau sinyal pita dasar (base band) dikirimkan dengan modulasi gelombang pembawa yang berfrekuensi tinggi.
- ▣ sinyal pita dasar → gelombang informasi atau gelombang modulasi →  $\psi_m$
- ▣ Gelombang pembawa →  $\psi_p$

# Teknik modulasi dan demodulasi secara umum



# Transformasi Fourier dan Fungsi Delta Dirac

- ▣ **Transformasi Fourier:** operasi yang menghubungkan kelakuan suatu fungsi dalam dua domain yang berkonjugasi.

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt \quad \psi(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\omega) e^{i\omega t} d\omega$$

$$g(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(x) e^{-ikx} dx \quad \psi(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(k) e^{ikx} dk$$

- ▣ **Fungsi Delta Dirac**

$$\delta(\omega - \omega_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-it(\omega - \omega_0)} dt \quad \delta(t - t_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega(t - t_0)} d\omega$$

$$\delta(x - x_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ik(x - x_0)} dk \quad \delta(k - k_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ix(k - k_0)} dx$$

$$\delta(x - x_0) = 0 \text{ untuk } x \neq x_0$$

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$$\delta(x - x_0) = \infty \text{ untuk } x = x_0$$

# Modulasi DSB

Hasil modulasi secara umum dapat kita ungkapkan dengan :

$$\psi(t) = \psi_p(t)\psi_m(t)$$

$$\psi(t) = \psi_{po} \cos(\omega_p t) \psi_{mo} \cos(\omega_m t)$$

$$\psi(t) = \frac{1}{2} \psi_{po} \psi_{mo} [\cos(\omega_p - \omega_m)t + \cos(\omega_p + \omega_m)t]$$



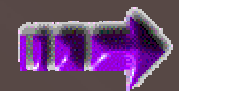
Operasi di atas disebut dengan *mixing*, hasilnya berupa dua komponen gelombang (side band), masing-masing berfrekuensi  $\omega_p + \omega_m$  dan  $\omega_p - \omega_m$ .

$\omega_p + \omega_m$  : pita sisi atas (upper side band)

$\omega_p - \omega_m$  : pita sisi bawah (lower side band)

Tampak secara eksplisit bahwa akibat modulasi, terjadi translasi frekuensi gelombang modulasi dari  $\omega_m$  menjadi  $\omega_p \pm \omega_m$ .

Gelombang pembawa dalam domain frekuensi :



$$g_p(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_p(t) e^{-i\omega t} dt$$

$$g_p(\omega) = \frac{1}{2\pi} \psi_{po} \int_{-\infty}^{\infty} \cos(\omega_p t) e^{-i\omega t} dt$$

$$g_p(\omega) = \frac{1}{4\pi} \psi_{po} \int_{-\infty}^{\infty} (e^{i\omega_p t} + e^{-i\omega_p t}) e^{-i\omega t} dt$$

$$g_p(\omega) = \frac{1}{4\pi} \psi_{po} \int_{-\infty}^{\infty} [e^{-i(\omega - \omega_p)t} + e^{-i(\omega + \omega_p)t}] dt$$

$$g_p(\omega) = \frac{1}{2} \psi_{po} [\delta(\omega - \omega_p) + \delta(\omega + \omega_p)]$$



Untuk gelombang modulasi :

$$g_m(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_m(t) e^{-i\omega t} dt$$

$$g_m(\omega) = \frac{1}{2\pi} \psi_{m0} \int_{-\infty}^{\infty} \cos(\omega_m t) e^{-i\omega t} dt$$

$$g_m(\omega) = \frac{1}{4\pi} \psi_{m0} \int_{-\infty}^{\infty} (e^{i\omega_m t} + e^{-i\omega_m t}) e^{-i\omega t} dt$$

$$g_m(\omega) = \frac{1}{4\pi} \psi_{m0} \int_{-\infty}^{\infty} [e^{-i(\omega - \omega_m)t} + e^{-i(\omega + \omega_m)t}] dt$$

$$g_m(\omega) = \frac{1}{2} \psi_{m0} [\delta(\omega - \omega_m) + \delta(\omega + \omega_m)]$$





Hasil modulasinya dalam domain frekuensi :

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt$$

$$g(\omega) = \frac{1}{2\pi} \frac{1}{2} \psi_{mo} \psi_{po} \int_{-\infty}^{\infty} (\cos(\omega_p - \omega_m)t + \cos(\omega_p + \omega_m)t) e^{-i\omega t} dt$$

$$g(\omega) = \frac{1}{4\pi} \psi_{mo} \psi_{po} \int_{-\infty}^{\infty} \left[ \frac{e^{i(\omega_p - \omega_m)t} + e^{-i(\omega_p - \omega_m)t}}{2} + \frac{e^{i(\omega_p + \omega_m)t} + e^{-i(\omega_p + \omega_m)t}}{2} \right] e^{-i\omega t} dt$$



$$g(\omega) = \frac{1}{4} \psi_{mo} \psi_{po} \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ e^{-i(\omega - (\omega_p - \omega_m))t} + e^{-i(\omega + (\omega_p - \omega_m))t} \right. \\ \left. + e^{-i(\omega - (\omega_p + \omega_m))t} + e^{-i(\omega + (\omega_p + \omega_m))t} \right] dt$$

$$g(\omega) = \frac{1}{4} \psi_{po} \psi_{mo} \left[ \delta(\omega - (\omega_p - \omega_m)) + \delta(\omega + (\omega_p - \omega_m)) \right. \\ \left. + \delta(\omega - (\omega_p + \omega_m)) + \delta(\omega + (\omega_p + \omega_m)) \right]$$

$$g(\omega) = \frac{1}{2} \psi_{po} \left[ g_m(\omega - \omega_p) + g_m(\omega + \omega_p) \right]$$

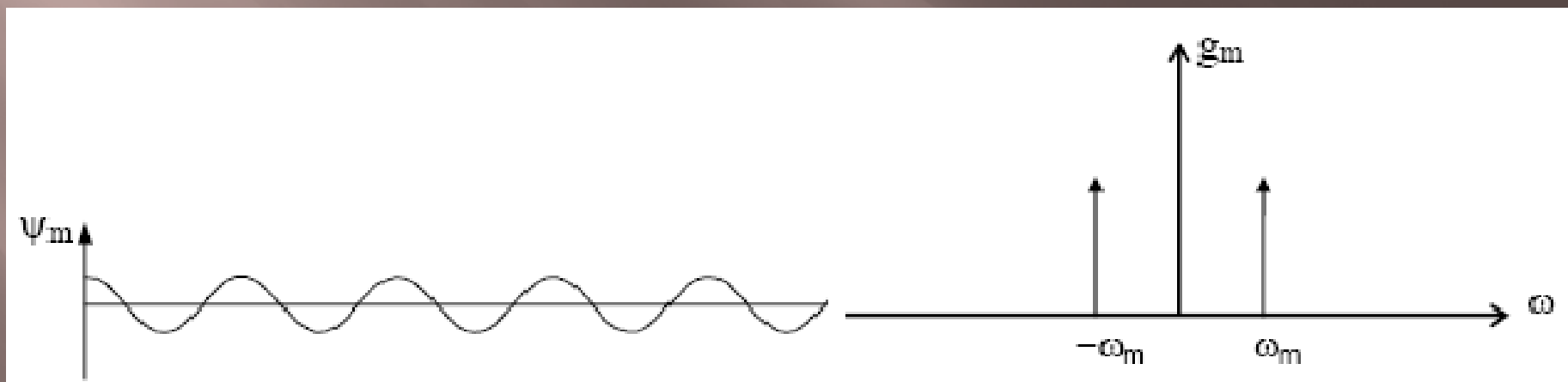


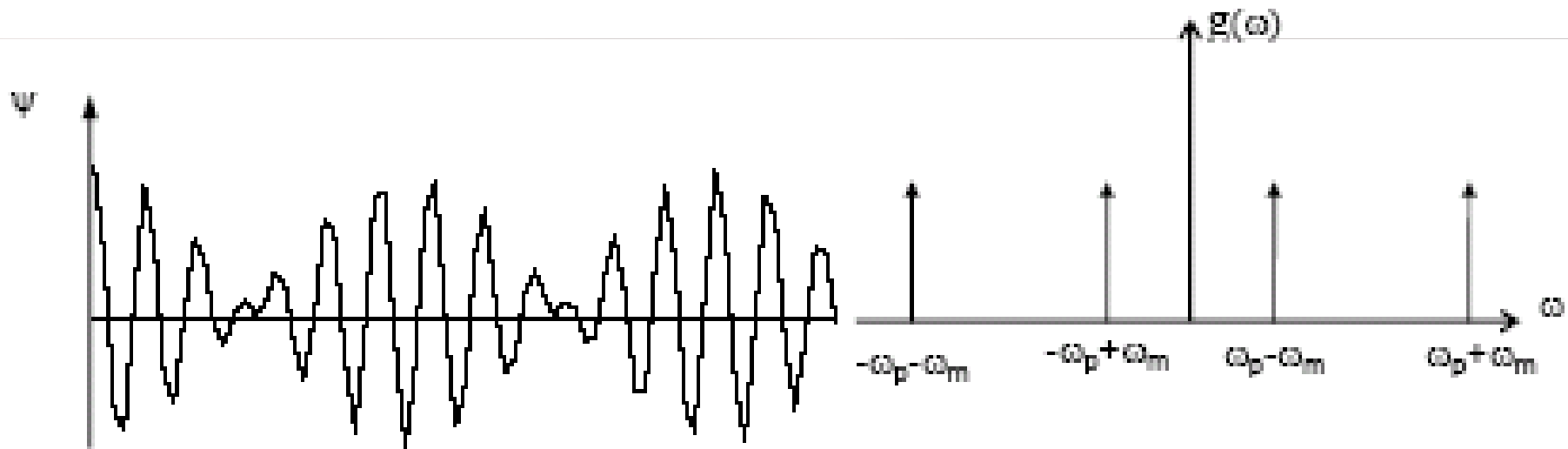
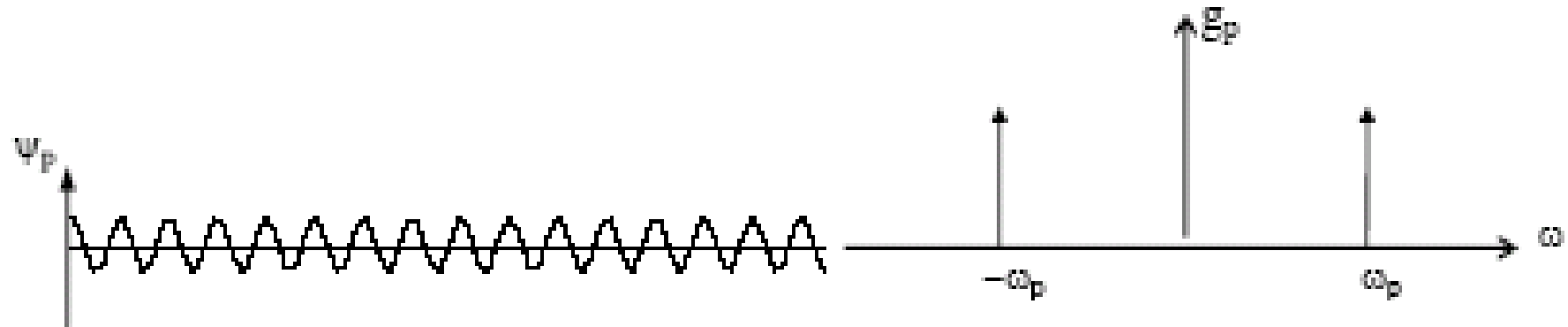
Lebar pita (bandwidth) B :

B = upper side - lower side

$$= 2 \omega_m$$

Grafik dalam domain waktu dan domain frekuensi untuk modulasi DSB ini diperlihatkan seperti pada gambar 5.6.





Daya rata-rata yang diteruskan :

$$\bar{p} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi(t)]^2 dt$$

$$\bar{p} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \{\psi_m(t)\}^2 \psi_{po}^2 \cos^2(\omega_p t) dt$$

$$\bar{p} = \lim_{T \rightarrow \infty} \frac{1}{T} \frac{\psi_{po}^2}{2} \left[ \int_{-\frac{T}{2}}^{\frac{T}{2}} \{\psi_m(t)\}^2 dt + \int_{-\frac{T}{2}}^{\frac{T}{2}} \{\psi_m(t)\}^2 \cos(2\omega_p t) dt \right]$$

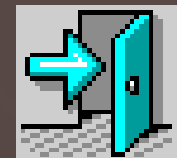


Untuk  $\omega_p \gg \omega_m$  suku ke dua ruas kanan persamaan ini sama dengan nol, maka daya rata-rata :

$$\bar{P} = \bar{P}_p \bar{P}_m$$

$$\bar{P}_p = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \psi_{po}^2 \cos^2(\omega_p t) dt = \frac{1}{2} \psi_{po}^2$$

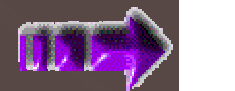
$$\bar{P}_m = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t)]^2 dt$$



## Demodulasi DSB

Demodulasi diartikan sebagai operasi untuk memperoleh kembali sinyal modulasi  $\psi_m(t)$  dari gelombang hasil modulasi  $\psi(t)$ . Demodulasi DSB dilakukan dengan dua tahap sebagai berikut :

- a. Gelombang hasil modulasi dikalikan dengan osilator lokal yang sinkron dengan gelombang pembawa  $\psi_p(t)$ . Osilator lokal:  $2 \cos(\omega_p t)$ .



$$\psi'(t) = \psi(t)2 \cos(\omega_p t)$$

$$\psi'(t) = \psi_{po} \psi_m(t) [1 + \cos(2\omega_p t)]$$

$$\psi'(t) = \psi_{po} \psi_m(t) + \psi_{po} \psi_{mo} \cos(\omega_m t) \cos(2\omega_p t)$$

$$\psi'(t) = \psi_{po} \psi_m(t) + \frac{1}{2} \psi_{po} \psi_{mo} [\cos(\omega_m t - 2\omega_p t) + \cos(\omega_m t + 2\omega_p t)]$$





Dalam domain frekuensi :

$$g'(\omega) = \frac{1}{2\pi} \psi_{po} \psi_{mo} g_m(\omega)$$

$$g'(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi'(t) e^{-i\omega t} dt$$

$$g'(\omega) = \frac{1}{2\pi} \left[ \int_{-\infty}^{\infty} \psi_{po} \psi_m(t) e^{-i\omega t} dt + \frac{1}{2} \psi_{po} \psi_{mo} \int_{-\infty}^{\infty} [\cos(\omega_m t - 2\omega_p t) + \cos(\omega_m t + 2\omega_p t)] e^{-i\omega t} dt \right]$$



$$g'(\omega) = \frac{1}{2\pi} \left[ \int_{-\infty}^{\infty} \psi_{po} \psi_{mo} \cos(\omega_m t) e^{-i\omega t} dt + \frac{1}{2} \psi_{po} \psi_{mo} \int_{-\infty}^{\infty} [\cos(\omega_m t - 2\omega_p t) + \cos(\omega_m t + 2\omega_p t)] e^{-i\omega t} dt \right]$$

$$g'(\omega) = \frac{\psi_{po} \psi_{mo}}{2\pi} \left[ \int_{-\infty}^{\infty} \left[ \frac{e^{-\omega_m t} + e^{\omega_m t}}{2} \right] e^{-i\omega t} dt + \frac{1}{4\pi} \psi_{po} \psi_{mo} \int_{-\infty}^{\infty} \left[ \frac{e^{i(\omega_m - 2\omega_p)t} + e^{-i(\omega_m - 2\omega_p)t}}{2} \right] e^{-i\omega t} dt \right. \\ \left. + \frac{1}{4\pi} \psi_{po} \psi_{mo} \int_{-\infty}^{\infty} \left[ \frac{e^{i(\omega_m + 2\omega_p)t} + e^{-i(\omega_m + 2\omega_p)t}}{2} \right] e^{-i\omega t} dt \right]$$



$$g'(\omega) = \frac{\psi_{po}\psi_{mo}}{4\pi} \left[ \int_{-\infty}^{\infty} [e^{-i(\omega-\omega_m)t} + e^{-i(\omega+\omega_m)t}] dt + \frac{1}{8\pi} \psi_{po}\psi_{mo} \int_{-\infty}^{\infty} [e^{-i(\omega-(\omega_m-2\omega_p))t} + e^{-i(\omega+(\omega_m-2\omega_p))t}] dt \right. \\ \left. + \frac{1}{8\pi} \psi_{po}\psi_{mo} \int_{-\infty}^{\infty} [e^{-i(\omega-(\omega_m+2\omega_p))t} + e^{-i(\omega+(\omega_m+2\omega_p))t}] dt \right]$$

$$g'(\omega) = \frac{\psi_{po}\psi_{mo}}{2} [\delta(\omega-\omega_m) + \delta(\omega+\omega_m)] + \frac{\psi_{po}\psi_{mo}}{4} [\delta(\omega-\omega_m+2\omega_p) + \delta(\omega+\omega_m-2\omega_p)] \\ + \frac{\psi_{po}\psi_{mo}}{4} [\delta(\omega-\omega_m-2\omega_p) + \delta(\omega+\omega_m+2\omega_p)]$$

Kemudian, dapat dituliskan dalam bentuk :

$$g'(\omega) = \psi_{po}g_m(\omega) + \frac{\psi_{po}}{2} [g_m(\omega+2\omega_p) + g_m(\omega-2\omega_p)]$$



b. Karena  $\omega_p \gg \omega_m$ , maka  $\omega_m \ll 2\omega_p - \omega_m$ ; berarti sinyal  $\psi_{po}$   $g_m(\omega)$  dapat dipisahkan dengan tapis lolos rendah (lowpass filter) dengan frekuensi pancung (cut off)  $\omega_{co}$ .

$$\omega_m < \omega_{co} < 2\omega_p - \omega_m$$