GLE 594: Introduction to Applied Geophysics

**Electrical Resistivity Methods** 

Fall 2006

#### Earth Properties and Basic Theory





40

0 20

80 160

Ohmmeter

240 320

Reading Today : 265-290 Next Lecture : 290-322

## Introduction

- Link resistivity (i.e., the ability of the earth to prevent the conduction of an electric current) to the subsurface structure.
- Useful because resistivity of earth materials varies by around 10 orders of magnitude.
- Developed by Conrad Schlumberger (France) and Frank Wenner (US) in the early 20<sup>th</sup> century.
- Uses: archeology, environmental, mineral exploration, and groundwater investigations



Conrad Schlumberger

#### **Electricity Basics**

Voltage V - Electrical potential energy per unit charge [volts] Current *i* - amount of charge per unit time [amperes]



Resistivity R is just a proportionality constant [ohms] that relates current I to voltage V.

However, no units of length in this form of Ohm's law.

# Resistivity

Resistance includes length and area We want resisitivity  $\rho$  [ohm m] because:

- It is a material property
- No geometry included

$$R = \rho \frac{L}{A} [ohm] \stackrel{\uparrow \text{ length }}{\to} \stackrel{\uparrow \text{ resistance}}{\uparrow_{\text{area}}} \stackrel{\land \text{ resistance}}{\to} \quad \downarrow_{\text{resistance}}$$





Conductivity  $\sigma$  [siemens/m] or [mhos/m]:

$$\sigma = \frac{1}{\rho} [\text{mhos m}]$$

It is the ability of the electrical charge to move through the material

#### More general form of Ohm's law

$$i = \frac{1}{R}V = \frac{A}{\rho}\frac{V}{l}$$

When looking at a solid, Ohm's law can be written as:

$$i = \frac{A}{\rho} \frac{\Delta V}{\Delta l}$$

And in 3-D we use vectors:  $I = \frac{A}{\rho} gradV$ 

# Earth as a Circuit

Soils and rocks can be conceptually modeled as a circuit made of a <u>resistor</u>, capacitor, inductor and battery:



Resistor R: Capacitor C: Inductor L: Battery B:

dissipator of applied energy as heat storage of energy as separation of charges self voltage associated to electromagnetic methods electrokinetics and self-potentials

### Electrical Conductivity in Geomaterials

• Non-conductive minerals



	Equation	Comments	
Electrolyte	$\sigma_{el}\left[\frac{mS}{m}\right] = 0.15 \text{TDS}\left[\frac{mg}{L}\right]$	TDS: total dissolved salts	
Soil (Archie's law)	$\boldsymbol{\sigma}_{soil} = a_1 \boldsymbol{\sigma}_{el} \mathbf{S}_r^n \boldsymbol{\phi}^m$	a <sub>1</sub> ≈0.4-2.0; m~1.3-2.5; n~2	
Soil (clays)	$\sigma_{soil} = n\sigma_{el} + (1-n)\Theta\rho_g S_s$	$\Theta \approx 10^{-9} \text{ S}$ (for Kaolinite)	

#### **Electrical Conductivity of Geomaterials**

• Non-conductive Minerals



(Attia and Fratta 2006)

#### **Current Source on Surface**

Electric potential at distance away from current source on surface given as  $V(r)=\rho I/2\pi r$ . How?



**Boundary conditions:** 

1) As  $r \Rightarrow \infty$ ,  $V \Rightarrow 0$ .

- 2) V is continuous across any boundary
- 3) Tangential E continuous across any boundary
- 4) Normal i continuous across any boundary.
- 5) Surface leads to no vertical current crossing earth-air interface.

# Current Flow in a Homogeneous and Isotropic Medium



ρ

Voltage decreases as the inverse of the distance from the current source.



Current flow

Equipotential surfaces

Shape of constant voltages are hemispheres for a single point source

$$V_{\rm D} = \int_{\rm D}^{\infty} dV = \frac{i\rho}{2\pi} \int_{\rm D}^{\infty} \frac{dr}{r^2} = \frac{i\rho}{2\pi} (-1) \frac{1}{r} \Big|_{\rm D}^{\infty}$$
$$= \frac{i\rho}{2\pi} (-1) \left(\frac{1}{\infty} - \frac{1}{D}\right) = \frac{i\rho}{2\pi D}$$

#### Two Current Electrodes: Source and Sink

• Why run an electrode to infinity when we can use it?



#### Measurements

You cannot measure potential at single point unless the other end of our volt meter is at infinity. It is easier to measure the *potential difference* ( $\Delta V$ ). This lead to use of four electrode array for each measurement.



where  $2\pi G$  is the *Geometrical Factor* of the array.

### **Apparent Resistivity**

Previous expression can be rearranged in terms of resistivity:

$$\rho = 2\pi G \frac{\Delta V}{i}$$

This can be done even when medium is inhomogeneous. The result is then referred to as *Apparent Resistivity*  $\rho_a$ .



**Definition:** Resistivity of a fictitious homogenous subsurface that would yield the same voltages as the earth over which measurements were actually made.

#### **Geometric Factors**



(Sharma 1997)

#### Array advantages and disadvantages

Array	Advantages	Disadvantages	
Wenner	1. Easy to calculate $\rho_a$ in the field	1. All electrodes moved each sounding	
	2. Less demand on instrument sensitivity	2. Sensitive to local shallow variations	
		3. Long cables for large depths	
Schlumberger	<ol> <li>Fewer electrodes to move each sounding</li> <li>Needs shorter potential cables</li> </ol>	<ol> <li>Can be confusing in the field</li> <li>Requires more sensitive equipment</li> <li>Long Current cables</li> </ol>	
Dipole-Dipole	1. Cables can be shorter for deep soundings	<ol> <li>Requires large current</li> <li>Requires sensitive instruments</li> </ol>	

### **Governing Equation**

Continuity: what goes in must comes out



## **Governing Equation**

Applying Ohm's Law:

$$j_x = -\frac{1}{\rho} \frac{\partial V}{\partial x}; j_y = -\frac{1}{\rho} \frac{\partial V}{\partial y}; j_z = -\frac{1}{\rho} \frac{\partial V}{\partial z}$$

Homogeneous and isotropic medium

$$\frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\rho} \frac{\partial V}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\rho} \frac{\partial V}{\partial z} \right) = 0$$

or using

$$x = r \cos \theta$$
,  $y = r \sin \theta$ , and  $x^2 + y^2 = r^2$ 

$$\begin{vmatrix} \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \\ \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0 \end{vmatrix} \nabla^2 V = 0 \implies \text{Laplace's equation}$$

## **Governing Equation - Solution**

- The Laplace's equation is a homogeneous, partial second order differential equation
- Solution:
  - Exact solutions: only for simple geometries
  - Graphical solutions: Flow nets, master charts
  - Numerical solutions: finite difference and finite elements solutions
  - Physical analogies (electrical, hydraulic and heat flow)

# Analogous to Heat and Fluid Flow

Any solution you know for one of these flows works for the others with the analogous boundary and initial conditions.

VARIABLE	GROUNDWATER	ELECTRICITY	HEAT
Potential	Head, <i>h</i> [cm]	Voltage, V [Volts]	Temperature, T [°C]
Quantity transported	Volume discharge rate [cm <sup>3</sup> s <sup>-1</sup> ]	Electrical charge [Coulomb]	Heat [calorie]
Physical property of medium	Hydraulic conductivity, K [cm s <sup>-1</sup> ]	Electrical conductivity, $\sigma$ [mhos m <sup>-1</sup> ]	Thermal conductivity, K [cal cm <sup>-1</sup> s <sup>-1</sup> °C <sup>-1</sup> ]
Relation between potential and flow fieldDarcy's law $\mathbf{q} = -K \operatorname{grad} h$ where $\mathbf{q}$ is specific discharge $[\operatorname{cm s}^{-1}]$		Ohm's law $\mathbf{i} = -\sigma \operatorname{grad} V$ where $\mathbf{i}$ is electrical current [Amperes]	Fourier's law $\mathbf{q} = -K \operatorname{grad} T$ where $\mathbf{q}$ is heat flow $[\operatorname{cal} \operatorname{cm}^{-2} \operatorname{s}^{-1}]$
Storage quantity	Specific storage, $S_s$ [cm <sup>-1</sup> ]	Capacitance, C [microfarad]	Heat capacity, $C_v$ [cal cm <sup>-3</sup> °C <sup>-1</sup> ]

(Wang and Anderson 1982)

#### **Current Distribution**

#### Homogeneous medium

fraction total current

2

3



### Geo-electric Layering

- Goal of the resistivity survey is to determine thickness and resistivity of near surface layers.
- Often the earth can be simplified within the region of our measurement as consisting of a series of horizontal beds that are infinite in extent.

Longitudinal conductance (one layer): Transverse resistance (one layer): Longitudinal resistivity (one layer): Transverse resistivity (one layer):

Longitudinal conductance (for n-layers): Transverse resistance (for n-layers):



$$S_{L} = h/\rho = h \sigma [S]$$
  

$$\Gamma = h \rho [\Omega]$$
  

$$\rho_{L} = h/S_{L} [\Omega m]$$
  

$$\rho_{T} = T/h [\Omega m]$$

 $S_{L} = \Sigma(h_{i}/\rho_{i}) [S]$  $T = \Sigma(h_{i}\rho_{i}) [\Omega]$ 

#### Voltage and Flow in Layers

Tangent Law: The electrical current lines are bent at a boundary



If  $\rho_2 < \rho_1$  then the current lines will be refracted away from the normal If  $\rho_2 > \rho_1$  then the current lines will be refracted closer to the normal

# Resistivity Pattern in a One-Layer System



(Burger et al. 2005)

#### Voltage and Flow in Layers

Method of electrical image



Note: S' is the mirror image of S

Voltages at points P and Q:

$$V_{\rm P} = \frac{i\rho_1}{4\pi} \left(\frac{1}{r_1} + \frac{k}{r_2}\right)$$

$$V_{Q} = \frac{i\rho_{2}}{4\pi} \left(\frac{1-k}{r_{3}}\right)$$

Where k is the "reflection coefficient" is:

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

# Solving the differential equation for two layers and a source and sink

**Governing Equation**  $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$ 



**Boundary Conditions** 

#### $\rho_2$

 $1.i_{z} = 0|_{z=0}$ 2.  $V_1 = V_2$  at h 3.  $\frac{1}{\rho_1} \frac{\partial V_1}{\partial z} = \frac{1}{\rho_2} \frac{\partial V_2}{\partial z}$  at h 4.  $V = \frac{i\rho_1}{2\pi(r^2 + z^2)^{\frac{1}{2}}}$  at r = 0, z = 0 Particular solution

No current at surface Voltage is continuous

Normal current density is continous

#### Layer Calculations



• It obviously gets much more difficult with more layers.

#### **Current Distribution**



### **Vertical Electric Sounding**

- When trying to probe how resistivity changes with depth, need multiple measurements that each give a different depth sensitivity.
- This is accomplished through *resistivity sounding* where greater electrode separation gives greater depth sensitivity.



(Sharma 1997)

### **VES Data Plotting Convention**

- Plot apparent resistivity as a function of the log of some measure of electrode separation.
  - Wenner *a spacing*
  - Schlumberger AB/2
  - Dipole-Dipole *n spacing*
- Asymptotes:
  - Short spacings  $<< h_1, \rho_a = \rho_1$ .
  - Long spacings >> total thickness of overlying layers,
     ρ<sub>a</sub>=ρ<sub>n</sub>
- To get  $\rho_a = \rho_{true}$  for intermediate layers, layer must be thick relative to depth.



(Sharma 1997)

# Solutions for a Wenner Array for two layers



 $k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ 

# Solutions for a Wenner Array for two layers

- Simple for two layer case.
  - Plot data at same scales as master curves.
  - Overlie shallow-layer resistivity asymptote with '1' on master curves.
  - Determine depth to layer, and resistivity of lower layer by comparing scaled master-curve values to data values
- Gets rapidly more difficult as more layers added.



Fig. 6.16 Example of the interpretation of a field curve (dashed line with crosses) by matching it with a set of master two-layer resistivity curves (for explanation see text).

(Sharma 1997)

# Equivalence: several models produce the same results

- Ambiguity in physics of 1D interpretation such that different layered models basically yield the same response.
- Different Scenarios:
  - Conductive layers between two resistors, where longitudinal conductance ( $\sigma$  h) is the same.
  - Resistive layer between two conductors with same transverse resistance ( $\rho$  h).

## Equivalence: several models produce the same results



- Although ER cannot determine unique parameters, can determine range of values.
- Also exists in 2D and 3D, but much more difficult to quantify. In these multidimensional cases simply referred to as *non-uniqueness*.

# Suppression

- Principle of *suppression:* Thin layers of small resistivity contrast with respect to background will be missed.
- Thin layers of greater resistivity contrast will be detectable, but equivalence limits resolution of boundary depths, etc.



# Horizontal Profiling

- Used for rapid location/delineation of lateral variations in resistivity.
- Usually involves moving an electrode array of constant separation horizontally along surface.





(Reynolds 1997)

• Can be difficult to interpret directly from resulting curve due to formation of 'cusps'. Also, different arrays produce somewhat different results.

### Profile Cusps – Wenner Array



- i. Current lines converge toward boundary, decrease potential gradient at potential electrodes
- ii. C2 at boundary
- iii. Current density increases adjacent to boundary in low resistivity unit, causes potential gradient between potential electrodes to rise
- iv. Both potential electrodes are in the low resistivity unit so potential gradient between them falls dramatically
- v. C1 enters the low resisitivy unit
- vi. Current flow becomes dipolar

(Reynolds 1997)

## **Combined Sounding and Profiling**

- Increase electrode separation as well as make measurements at multiple locations along the horizontal axis.
- Provides data for two dimensional interpretation of subsurface.
- Data often plotted in *pseudo-section* for qualitative analysis.

#### Wenner Pseudo-Section



Wenner: h=a/2 Schlumberger: h=L/3 Dipole Dipole: h=n a

#### **Pseudo-Sections**



- Can sometimes be used to qualitatively assess geology
- <u>Warning</u>: Can also prove to be very difficult to interpret directly, with different arrays yielding very different results.



# Measurement Systems

#### Transmitter

- Power Supply
  - DC

- •Ammeter
- •Metal electrodes
- AC (more common)

#### Receiver

- Voltmeter
- Metal Electrodes



#### **Fully Automated Systems**



Electrodes: 30 Station Spacing: 1/2 Dipole N-spacings: 12 per diagonal (n=0.5, 1, 1.5, 2, ...) Data points per Spread: 234

(Source: Zonge)

## Field Considerations for DC Resistivity

- Good electrode contact with the earth
  - Wet electrode location
  - Add NaCl solution or bentonite.
- Surveys should be conducted along a straight line whenever possible
- Try to stay away from cultural features whenever possible (power lines, pipes, grounded metal fences, pumps, etc)



- Instrument noise
- Cultural Features
- Telluric Currents naturally occurring earth currents.
  - Self potentials generally caused by either geochemical reactions or greater than normal subsurface fluid flow.
  - Magnetotelluric Currents Electromagnetically induced by naturally occurring or man-made magnetic fields.
  - In some cases, it may be unavoidable

- Geologic Noise
  - <u>Near surface variations</u>: Can dominate response thus masking signature of deeper targets.



- Geologic Noise
  - <u>Topography:</u> Currents will be *focused* under valleys, and *dispersed* under hills, thus causing perturbations in measured voltages.



- Small heterogeneities produce cusps
- Long linear features (rivers, wires) may produce current leakage.





(Reynolds 1997)

(C)

Current leakage

Electrode spacing (m)

100

#### **Generalized Profile Interpretation**

• Looking for changes in apparent resistivity that will enhance your understanding of what you already know about the geology.



**Fig. 6.19** Horizontal resistivity profile across a shear zone and limestone fault block in **Illinois**; Wenner configuration. (After Hubbert, 1932.)

## Qualitative 2D Profile Interpretation

- Sometimes pseudo-sections can be interpreted qualitatively directly if
  - Good data quality
  - Simplified geology
- This is the exception rather than the norm



# Computerized Interpretation: Forward Modeling

- Using mathematical expressions that describe the physics to calculate the data that would result from a given combination of geoelectric model and electrode configuration.
- Generally a *linear* process.
- Forward modeling produces *unique* results.



# Computerized Interpretation: Inverse Modeling

- Going the opposite direction. We measure data and know the array configuration, and through *inversion* wish to determine a geoelectric model that would produce data similar to those measured.
- Problems:
  - Generally problem is non-linear.
  - Problem is non-unique. Thus must add constraints of some sort to provide a reasonable answer.
  - Danger in over-interpreting the results.



## Inverse Modeling

#### • Benefits:

-Automatic, it helps to remove user bias. However sometimes the user's bias is needed to produce a decent model

-Automatically removes differences associated with different electrode collection schemes.



### Interpretation Issues - Resolution

- General 2D and 3D surveys
  - Basically, spatial resolution falls off as you get further away from the surface. Need larger bodies/higher contrasts at greater depths to be detectable
  - <u>Near surface very well resolved</u> min. horizontal resolution equal to minimum electrode separation
  - Depth resolution is array dependent
  - REMEMBER Over-interpretation is very easy given the nonuniqueness of physics as well as the inverse problem. Thus need to be careful.

#### Locating Water Bearing Fracture Zones in Bedrock





#### Mapping the Limits of a Municipal Landfill



Courtesy of SAIC. An Employee-Owned Company GEOPHYSICAL SERVICES Middletown, Pennsylvania



Tel: +1 (512) 335-3338 Fax: +1 (512) 258-9958 E-mail sales@agiusa.com Web site http://www.agiusa.com

#### Mapping of Stratigraphy Sand and Gravel Lenses in Clay Environment



Two parallel profiles, 25' apart. Sand and gravel solution channels show higher resistivity than the silty clay or the shale. Note how the two profiles show almost the same layering since they are only 25' (ca 8 m) apart.



#### **Plume Mapping Using High Resolution Resistivity**



 Objective:
 The objective was to map the extent of a pollution plume

 Survey date:
 1996

 Instrument:
 Sting R1 using manual cables

 Method:
 Pole-pole array

 Spacing:
 5 - 200 feet

 Units:
 Feet and Ohmmeter







Tel: +1 (512) 335-3338 Fax: +1 (512) 258-9958 E-mail sales@agiusa.com Web site http://www.agiusa.com

### Hydrologic/Contaminant Studies



Source: Alumbaugh and co-workers

#### Hydrologic/Contaminant Studies

