Microgeometry Analysis of Two Dimensional-Random Sierspinski Carpets (RSCs)

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Abstract

Microgeometry of two dimensional Random Sierspinski Carpets (RSCs) is analyzed for providing a quantitative means for understanding the dependence of physical properties on the pore structure. Six models of 2D-RSCs with the same porosity and fractal dimension but three kind pore size distributions are investigated. We obtain estimates of the porosity (ϕ), specific surface area (s) and hydraulic diameter (D_H) of the models from the concept of two point correlation function and the entropy length (L^*) from concept of local porosity distribution and local geometry entropies. Estimated porosity (ϕ), specific surface area (s) and hydraulic diameter (D_H) of the models generally agree with expected results. Both local porosity distribution density function and entropy function for wider spectrum of pore sizes is more fluctuated than the other two. The entropy lengths (L^*) are 15 and 14 for two models which consist four different pore sizes, respectively, 14 and 16 for two models which consist two different pore sizes, respectively and 10 for the last two models which consist only one pore size.

1. Introduction

One of the most important problems in studies of porous and heterogeneous media is the specification of the random microstructure, which is needed to predict macroscopic physical properties. A complete specification of the random microstructure is both impractical and unnecessary. It is therefore important to have general statistical description of the microstructure available. Such a description should meet four criteria [1]: (a) it should be well-defined in terms of geometrical quantities, (b) it should involve only experimentally accessible parameters, (c) it should be economical size and (d) it should be usable and exact or appropriate solutions of the underlying equation of motion. Currently there are only two statistical methodologies available which fulfill all four requirements; these are correlation functions [1-3] and local geometry distributions [1-2,4-6]

The two point correlation function developed by Blair [3] is of interest because it provides a measure of several important parameters of the microstructure in a very compact form and its usefulness is not limited by any assumption about particle shape.

There are two main reasons for considering local porosity or geometry distribution. First, the distribution is potential to distinguish between different microstructure [2,5] and second, the basic idea underlying local porosity theory is to consider the fluctuations of 'local porosities' or local volume fractions inside mesoscopic regions (measurement cells)[1]. The size of these regions becomes a parameter controlling the transition from microscales to macroscales. The length scale dependent local porosity distributions are used to calculate length scale dependent effective transport coefficients.[1]

In this paper, we analyze microgeometry of two dimensional Random Sierspinski Carpets (RSCs) for providing a quantitative means for understanding the dependence of physical properties on the pore structure. Six models of 2D-RSCs with the same porosity and fractal dimension but three kind pore size distributions are investigated. We obtain estimates of the porosity (ϕ), specific surface area (s) and hydraulic diameter (D_H) of the models from the concept of two point correlation function and the entropy length (L^*) from concept of local porosity distribution and local geometry entropies.

2. Microgeometry Analysis

2.1. Two Point Correlation Function

A binary image of the cross section through porous media can be idealized as a two-phase medium consisting of pore and phase. We can define indicator function f for any position x in the material

$$f(x) = \begin{cases} 1 \text{ for pore} \\ 0 \text{ for phase (grain)} \end{cases} \qquad (1)$$

Porosity (ϕ) can be estimated, if we sum of f over the area of the image of any cross section. The sum is known as the one point correlation function S_1 [3]:

$$S_1(r) = \langle f(x) \rangle = \phi \qquad (2)$$

Meanwhile, two point correlation function (S_2) define as the probability that two point separated by a distance r will both be in the pore space [3]:

$$S_2(r) = \langle f(x) f(x+r) \rangle \qquad (3)$$

The other properties that can be estimated from two point correlation function is specific surface area (s) defined as the ratio of the total surface area of the pore-phase interface to the total volume of the porous media. The slope near the origin is proportional to the specific surface area (s) of the media [3]:

$$S_2'(0) = -\frac{s}{4} \qquad (4)$$

Thus, we can write line tangent to the S_2 curve at r=0:

$$S_2(r) = S_2'(0) r + \phi \qquad (5)$$

Since, the two point correlation functions are fluctuated around ϕ^2, Blair *et.al* [3] suspect that there will be an important property estimated from intersect between the line tangent to the S_2 and ϕ^2:

$$\phi^2 = S_2'(0) r_c + \phi$$

$$r_c = \frac{\phi(\phi-1)}{S_2'(0)} = \frac{4\phi}{s}(\phi-1) \qquad (6)$$

Since, Blair illustrate the application of S_2 based on idealized sphere packs, he define r_c as an effective pore diameter, because r_c is related to the hydraulic diameter D_H, which is defined as [7]

$$D_H = \frac{4\phi}{s} \qquad (7)$$

If we look closely to the derivation of r_c, it is not limited by any assumption about particle shape. Since the pore shape of our model is a collection of squares, it's not appropriate to define r_c as effective pore diameter. We interpret that r_c and $\frac{4\phi}{s}$ are corresponded to side length (L) of the pore.

In capillary tube model, the hydraulic diameter is [7]

$$D_H = \frac{4A}{L_A} \qquad (8)$$

where A and L_A are area and periphery of pores, respectively. Since the pore shape in this case is a collection of squares, so D_H is associated with average side length of the pores of the models ($\bar{L}$).

2.2. Local Porosity Distribution

For a stochastic porous medium the one-cell porosity density function is defined for each measurement cell [1]:

$$\mu(\phi;K_j)=\left\langle \delta(\phi-\phi(K_j))\right\rangle \tag{9}$$

where K_j is an element of the partitioning of the sample space and $\phi(K_j)$ is local porosity inside a measurement cell K_j defined as

$$\phi(K_j)=\frac{1}{M}\sum_{x\in K_j} f(x) \tag{10}$$

The average local porosity define as $\bar{\phi}=\int_0^1 \phi\mu(\phi)d\phi$,

therefore for a homogenous porous medium the definitions (8) and (9) yield [1]:

$$\overline{\phi(K_j)}=\int_0^1 \phi\mu(\phi;K_j)d\phi=\langle\phi\rangle \tag{11}$$

One interesting possibility is to optimize an entropy or the geometrical content contained in $\mu(\phi;L)$ or equivalently to minimize the entropy function [1]

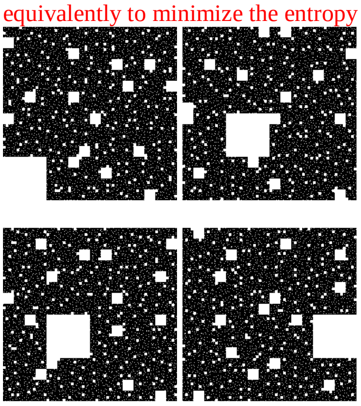
$$I(L)=\int_0^1 \mu(\phi;L)\log\mu(\phi;L)d\phi \tag{12}$$

relative to the conventional a priory uniform distribution. The entropy length L* is then determined through the condition [1]

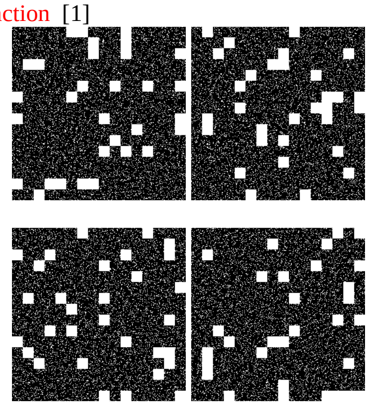
$$\left.\frac{d\mu(\phi;L)}{dL}\right|_{L=L^*}=0 \tag{13}$$

3. Description of the models

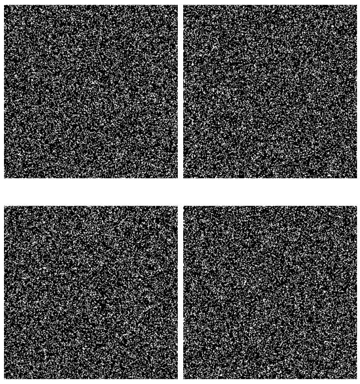
The twelve fractal models of porous media shown in figure 1 are generated by Random Sierpinski Carpets (RSCs). White indicates the pore space. Model (a) through (d) have the same scale factor 4 and generate at initial porosity 0.062 and at fourth iteration. The next four models (Model (e) through (h)) have scale factor 16 and generate at initial porosity 0.1216 and at second iteration. And the last four models generate at initial porosity 0.2275 and at first iteration. The number of iteration shows the number of pore size. The distributions of pore size for the models are shown in figure 1. The resolution of all images is 256x256 pixels. All images has the same porosity (ϕ=0.2275) and fractal dimension of phase (D=1.95).



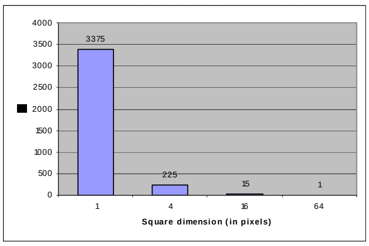
Model (a),(b),(c) and (d)



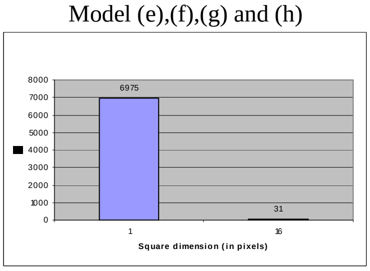
Model (e),(f),(g) and (h)



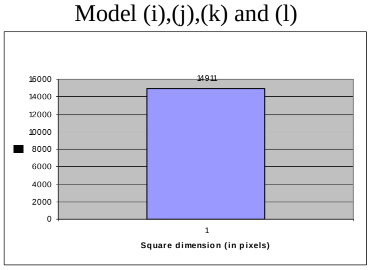
Model (i),(j),(k) and (l)



Pore size distribution of model (a) through (d)



Pore size distribution of model (e) through (h)



Pore size distribution of model (i) through (l)

Figure 1. Twelve models of Random Sierspinski Carpets with their associated pore size distribution.

4. Result and Discussion

Figure 2 displays the two point correlation functions as a function of r for model (a),(e) and (f). Each 4 models which have the same pore size distribution, have similar two point correlation function. For large r, the two point correlation functions are fluctuated around ϕ^2. The two point correlation function $S_2(r)$ for images which has the same pore size distribution are nearly indistinguishable at small r, but distinguishable at large r except for model (e) through (f). Figure 2 show that, $S_2(r)$ can distinguish models that have different kind of pore size distributions. If we look closely to figure 2, we can identify the curve bends of $S_2(r)$ at small r. In these cases the number of bends is equal to number of pore size. These bends are easily observed because of the square-shape of pores. For rounder-shape of pores it will result smoother two point correlation functions.

As we mention earlier that, two point correlation function is very useful for characterize microgeometry and estimate several important properties such as porosity, specific surface area (s) and hydraulic diameter (D_H). Table 1 list pore parameters estimated from two point correlation function for all models of figure 1. The porosities estimated using the correlation functions agree with the calculated porosities. The estimated mean pore diameter (r_c) generally agree with average (mean) side length of pores ($\bar{L}$). The deviation is large for wider spectrum of pore size distribution. Even though model (a) through (h) have wider spectrum of pore size distribution, but the number of larger pore sizes are less extremely than smaller ones, resulting average of side length ($\bar{L}$) of the models are near 1. Since model (e) and (f) have a collection of pores which uniform in size, the estimated mean pore diameter (r_c) has almost the same value with average of side length ($\bar{L}$). The hydraulic diameters (D_H) for all models are larger than estimated pore diameters (r_c). it is consequences of equation (6). Blair *et.al*[3] used higher magnification images for determining the image specific-surface area because determining $S_2(0)$ from high magnification images is not accurate due to poor statistical sampling of the total pore space, resulting diameter hydraulic (D_H) of their samples generally smaller than estimated mean pore diameter (r_c).

Although each four models with the same pore size distributions have different spatial distributions, their graphical of two point correlation functions are having almost the same trend. The estimated parameters such as porosity, specific surface area, mean pore diameter (r_c) and hydraulic diameter (D_H) generally almost have the same value. From this facts, we infer that the spatial distribution of porous media in these cases have no significant influence to parameters such as specific surface area mean pore diameter (r_c) and hydraulic diameter (D_H).

Local porosity distributions $\mu(\phi;L)$ are calculated using equation (9) for several measurement cells with side length L = 5 and L=70 pixels. Each four models which have the same pore size distribution, have also similar local porosity distribution. Figure 3 shows that local porosity distributions are generally concentrated at origin and at 1 for small L and around $\bar{\phi}$ for large L. From equation (11), we found that all image of figure 1 is found to be homogeneous. Local porosity distribution of the models (model (a) to (h)) which have wider spectrum of pore size distribution generally more fluctuated than the models which have only one pore size (model (i) through (l)).

Figure 4 shows entropy functions as a function of the length of the measurement cell. For each four models which have the same pore size distribution almost have the same trend of entropy function but not similar. The entropy lengths (L*) of the first four models are found at 15, 14, 15 and 14, respectively, the next four models are found at 16, 14, 15 and 14, respectively and the last four models are found all at 10. The entropy length of the last four models are the same due to their uniform pore size and are smaller than the other models due to their smaller pore size.

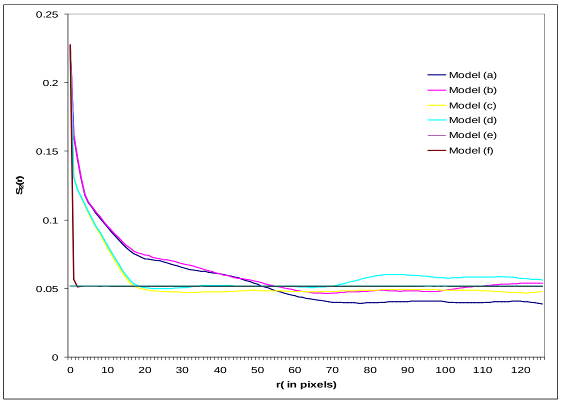
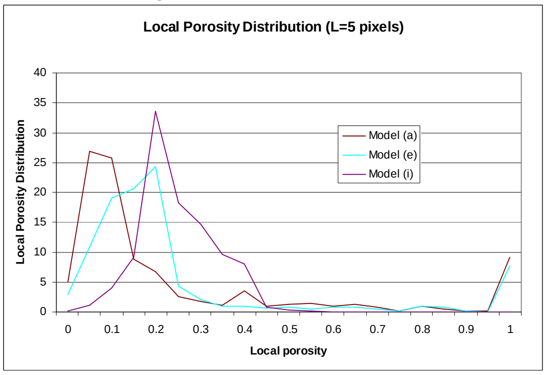


Figure 2. Two point correlation function for three represented models of their associated pore size distribution at figure 1.



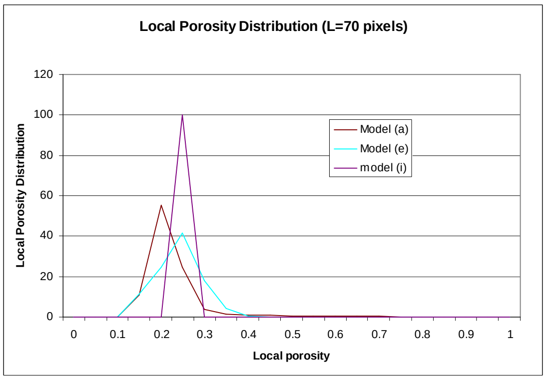


Figure 3. Local porosity density function $\mu(\phi,L)$ of for three represented models of their associated pore size distribution at figure 1 for measurement cells of side length L=5 and 70 pixels, respectively

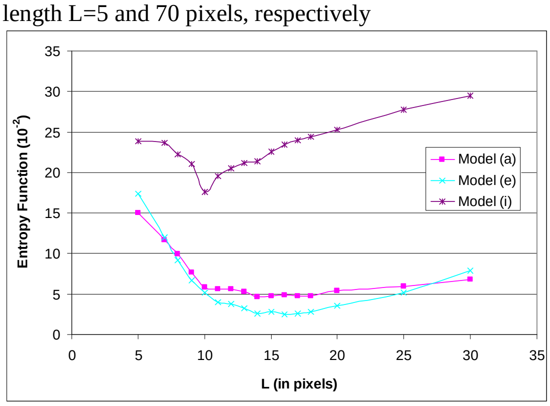


Figure 4. Entropy function S(L) as a function of the length of the measurement cell for three represented models of their associated pore size distribution.

Table 1. Pore parameters estimated from two point correlation function for twelve RSC models

Model	ϕ_{real}	ϕ_{TPCF}	s (in pixel^{-1})	r_c (in pixels)	$D_H=4\phi/s$ (in pixels)	$\overline{L}$ (in pixels)
(a)	0.2275	0.2271	0.262	2.682	3.476	1.266
(b)	0.2275	0.2279	0.262	2.685	3.471	1.266
(c)	0.2275	0.2268	0.262	2.675	3.470	1.266
(d)	0.2275	0.2270	0.261	2.687	3.483	1.266
(e)	0.2275	0.2275	0.387	1.817	2.353	1.066
(f)	0.2275	0.2274	0.387	1.817	2.360	1.066
(g)	0.2275	0.2275	0.390	1.804	2.333	1.066
(h)	0.2275	0.2275	0.387	1.814	2.348	1.066
(i)	0.2275	0.2275	0.684	1.028	1.330	1
(j)	0.2275	0.2273	0.683	1.029	1.332	1
(k)	0.2275	0.2275	0.680	1.027	1.337	1
(l)	0.2275	0.2275	0.683	1.028	1.332	1

6. Conclusion

We analyzed microgeometry of two dimensional Random Sierspinski Carpets (RSCs). The six models of 2D-RSCs with the same porosity and fractal dimension but three kind pore size distributions have investigated. We obtain estimates of the porosity (ϕ), specific surface area (s) and hydraulic diameter (D_H) of the models from the concept of two point correlation function and the entropy length (L^*) from concept of local porosity distribution and local geometry entropies.

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5. Acknowledgements
We would like

6. References

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