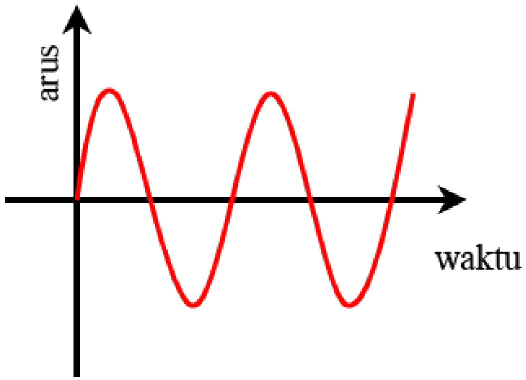
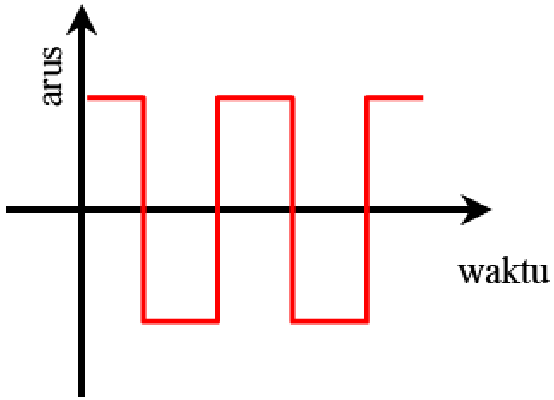
ARUS BOLAK-BALIK

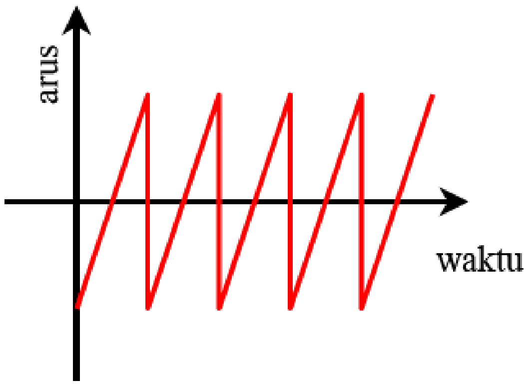
Arus bolak-balik..??? Arus yang arahnya berubah-ubah secara bergantian



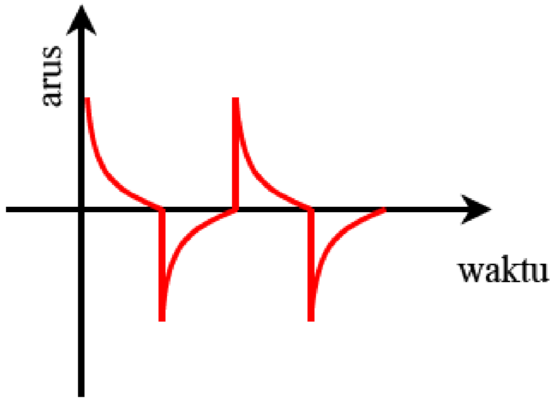
Arus bolak-balik berubah secara sinusoidal



Arus bolak-balik berubah secara persegi



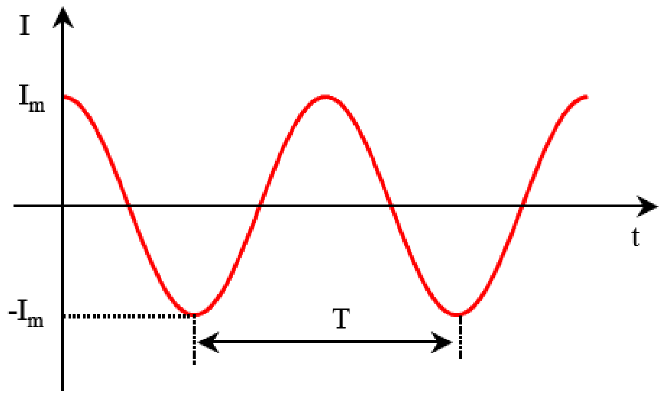
Arus bolak-balik berubah secara persegi

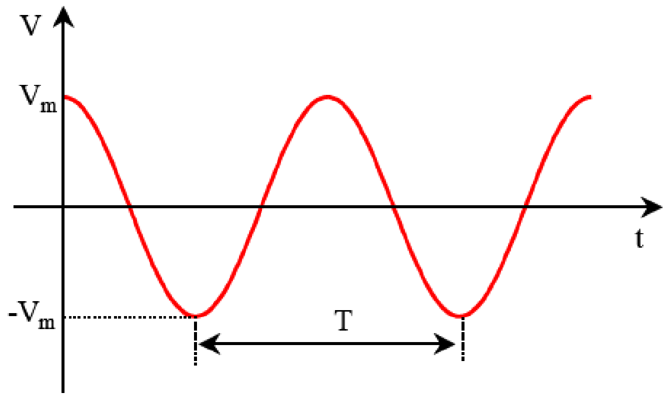


Arus bolak-balik berubah secara transien

Bentuk Arus bolak-balik yang paling sederhana = Arus sinusoidal

Kebergantungan Arus terhadap waktu





Vm = Amplitudo tegangan

fungsi Cosinus

$$I = I_m \cos\left(\frac{2\pi}{T}t + \varphi_o\right)$$

Im = Arus maksimum (Amplitudo arus)

T = Periode Arus

t = waktu

$$\varphi_o = fase\ mula - mula\ (saat\ t = 0)$$

Jika arus tersebut melalui sebuah hambatan, tegangan antara ujung hambatan memenuhi **hukum ohm**

$$V = IR = I_m \cos\left(\frac{2\pi}{T}t + \varphi_o\right)R$$

$$V = V_m \cos\left(\frac{2\pi}{T}t + \varphi_o\right)$$

Sejumlah alat ukur dirancang hanya dapat mengukur nilai rata-rata suatu besaran

Jika ada alat ukur tegangan rata-rata:

1. Berapa tegangan rata-rata yang dihasilkan arus bolak-balik???
2. Berapa arus rata-rata yang dihasilkan arus bolak-balik???

Tegangan rata-rata:

$$V_r = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^{\tau} V dt$$

Untuk fungsi sinusoidal, perata-rataan menghasilkan hasil yang sama dengan perata-rataan selama satu periode

$$V_r = \frac{1}{T}\int_0^{T} V dt$$

$$V = V_m \cos\left(\frac{2\pi}{T}t + \varphi_o\right)$$

$$V_r = \frac{1}{T}\int_0^{T} V_m \cos\left(\frac{2\pi}{T}t + \varphi_o\right) dt$$

$$V_r = \frac{V_m}{T}\int_0^{T} \cos\left(\frac{2\pi}{T}t + \varphi_o\right) dt$$

$$V_r = \frac{V_m}{T}\int_0^T \cos\left(\frac{2\pi}{T}t + \varphi_o\right)dt$$

Misal : $\frac{2\pi}{T}t + \varphi_o = x$

$$\frac{2\pi}{T}dt = dx \qquad dt = \frac{T}{2\pi}dx$$

$$V_r = \frac{V_m}{T}\int_0^T \cos x \frac{T}{2\pi}dx = \frac{V_m}{2\pi}\int_0^T \cos x\, dx = \frac{V_m}{2\pi}\sin x$$

$$V_r = \frac{V_m}{2\pi}\sin\left(\frac{2\pi}{T}t + \varphi_0\right)\Big|_0^T = \frac{V_m}{2\pi}\left[\sin\left(\frac{2\pi}{T}T + \varphi_0\right)\right] - \left[\sin\left(\frac{2\pi}{T}0 + \varphi_0\right)\right]$$

$$V_r = \frac{V_m}{2\pi}\left[\sin(2\pi + \varphi_0) - \sin(0 + \varphi_0)\right] = \frac{V_m}{2\pi}\left[\sin(\varphi_0) - \sin(\varphi_0)\right] = 0$$

$$\sin(2\pi + \varphi_0) = \sin(\varphi_0)$$

Arus rata-rata arus bolak-balikk = nol

Selama setengah periode , tegangan & arus memiliki nilai positif dan setengah periode berikutnya memiliki nilai negatif

Untuk arus bolak-balik, nilai rata-rata tidak memberikan informasi lengkap tentang arus dan tegangan → **rms = root mean square**

↓

Tegangan & arus rms :

$$V_{rms} = \sqrt{V_r}$$

$$I_{rms} = \sqrt{I_r}$$

Bagaimana hubungan V_{rms} dan V_m ?

$$V = V_m \cos\left(\frac{2\pi}{T}t + \varphi_o\right)$$

$$V^2 = V_m{}^2 \cos^2\left(\frac{2\pi}{T}t + \varphi_o\right)$$

$$V^2 = V_m{}^2 \left\langle \cos^2\left(\frac{2\pi}{T}t + \varphi_o\right)\right\rangle$$

$$V^2 = V_m{}^2 \frac{1}{T}\cos^2\left(\frac{2\pi}{T}t + \varphi_o\right)dt$$

Misal : $\frac{2\pi}{T}t + \varphi_o = x$

$$\frac{2\pi}{T}dt = dx \qquad dt = \frac{T}{2\pi}dx$$

$$V^2 = V_m{}^2 \frac{1}{T}\int \cos^2 \frac{T}{2\pi}dx = \frac{V_m{}^2}{2\pi}\int \cos^2 x\,dx$$

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

Atau $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

Maka:

$$\int \cos 2x = \int \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right) dx = \int \frac{1}{2} dx + \int \frac{1}{2} \cos 2x dx = \frac{1}{2} x + \frac{1}{2} \sin 2x$$

$$\int \cos 2x = \frac{1}{2} \left(\frac{2\pi}{T} t + \varphi_o \right) + \frac{1}{2} \sin 2 \left(\frac{2\pi}{T} t + \varphi_o \right)$$

$$V^2 = \frac{V_m^{\ 2}}{2\pi} \left[\frac{1}{2} \left(\frac{2\pi}{T} t + \varphi_0 \right) + \frac{1}{2} \sin 2 \left(\frac{2\pi}{T} t + \varphi_0 \right) \right] \int_0^T$$

$$V^2 = \frac{V_m^{\ 2}}{2\pi} \left\{ \left[\frac{1}{2} \left(\frac{2\pi}{T} t + \varphi_0 \right) + \frac{1}{2} \sin 2 \left(\frac{2\pi}{T} t + \varphi_0 \right) \right] - \left[\frac{1}{2} \left(\frac{2\pi}{T} 0 + \varphi_0 \right) + \frac{1}{2} \sin 2 \left(\frac{2\pi}{T} 0 + \varphi_0 \right) \right] \right\}$$

$$V^2 = \frac{V_m^{\ 2}}{2\pi} \left\{ \left[\frac{1}{2} (2\pi + \varphi_0) + \frac{1}{2} \sin 2 (2\pi + \varphi_0) \right] - \left[\frac{1}{2} (0 + \varphi_0) + \frac{1}{2} \sin 2 (0 + \varphi_0) \right] \right\}$$

$$V^2 = \frac{V_m^{\ 2}}{2\pi} \left\{ \left[\pi + \frac{\varphi_0}{2} + \frac{1}{2} \sin (4\pi + 2\varphi_0) \right] - \left[\frac{\varphi_0}{2} + \frac{1}{2} \sin (2\varphi_0) \right] \right\}$$

Sifat ingat : sifat periodisitas fungsi sinus: $\sin(4\pi + 2\varphi_0) = \sin 2\varphi_0$

$$V^2 = \frac{V_m{}^2}{2\pi}\left\{\left[\pi + \frac{\varphi_0}{2} + \frac{1}{2}\sin\left(2\varphi_0\right)\right] - \left[\frac{\varphi_0}{2} + \frac{1}{2}\sin\left(2\varphi_0\right)\right]\right\}$$

$$V^2 = \frac{V_m{}^2}{2\pi}\pi = \frac{V_m{}^2}{2}$$

Tegangan rata-rata: $V_{rms} = \sqrt{V^2} = \sqrt{\frac{V_m{}^2}{2}} = \frac{V_m}{\sqrt{2}}$

Daya & Daya Rata-rata

$$P = VI = \frac{V^2}{R}$$

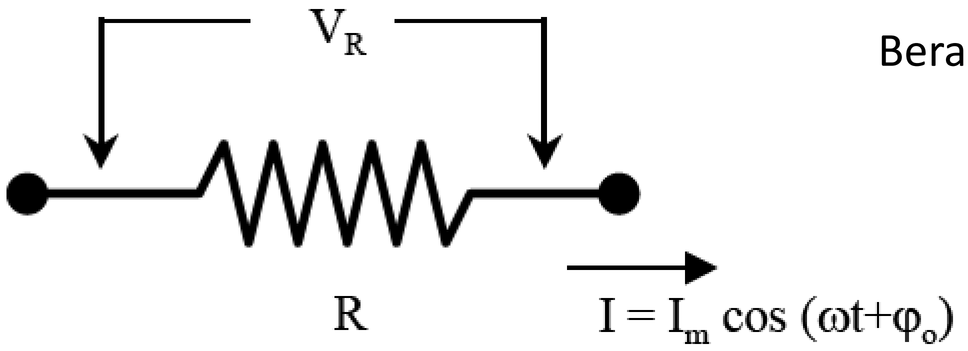
Tegangan antara 2 ujung hambatan: $V = V_m \cos\left(\frac{2\pi}{T} t + \varphi_o\right)$

Arus yang mengalir pada hambatan: $I = \frac{V_m}{R} \cos\left(\frac{2\pi}{T} t + \varphi_o\right)$

Disipasi daya pada hambatan : $P_r = V_r I = \frac{V_r^{\,2}}{R}$ atau $P_r = V_r I = \frac{V_{rms}^{\,2}}{R}$

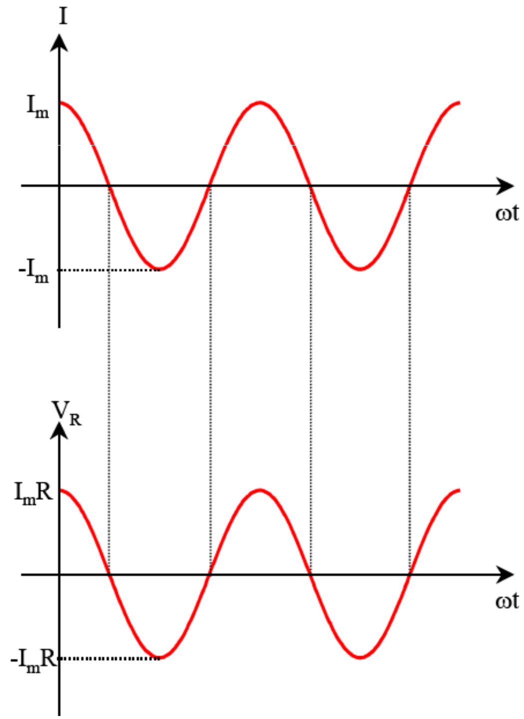
Problem:

Misal arus bolak-balik mengalir melalui suatu hambatan



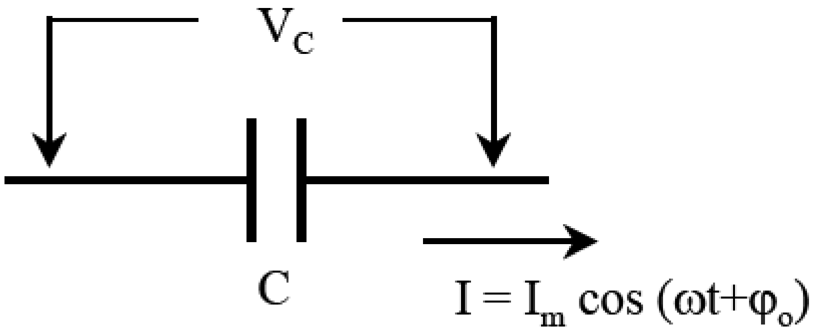
Berapa tegangan antara 2 ujung hambatan?

$$V_R = IR = I_m R \cos\left(\omega t + \varphi_0\right)$$



Problem: Tegangan antara 2 ujung kapasitor

Misal kapasitor dengan kapasitansi C dialiri arus bolak-balik



Berapa tegangan antara 2 ujung kapasitor?

$$V_C = \frac{Q}{C}$$

$$Q = \int I dt = \int I_m \cos(\omega t + \varphi_0) dt = I_m \int \cos(\omega t + \varphi_0) dt = \frac{I_m}{\omega} \sin(\omega t + \varphi_0)$$

$$V_C = \frac{I_m}{\omega C} \sin(\omega t + \varphi_0)$$

$V_C = I_m X_C \sin(\omega t + \varphi_0)$ dengan $X_C = \frac{1}{\omega C}$ **Reaktansi Kapasitif**

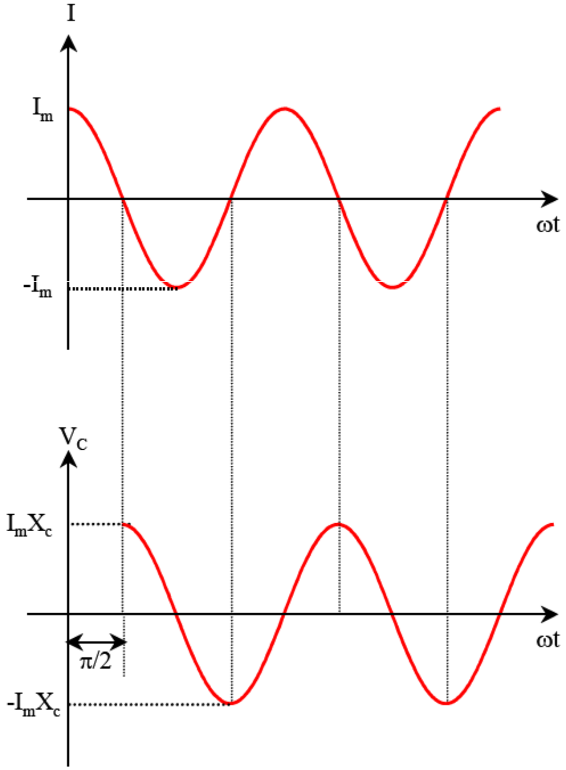
$$X_C = \frac{1}{\omega C}$$

Reaktansi Kapasitif

jika frekuensi arus (ω) >> → X_L <<

jika frekuensi arus →∞ ,XL →0 (terhubung singkat)

jika frekuensi arus →0, XL → ∞ (saklar terbuka)



TEGANGAN ANTARA 2 UJUNG KAPASITOR:

$$V_C = I_m X_C \sin(\omega t + \varphi_0)$$

Aturan trigonometri

$$\sin(\omega t + \varphi_0) = \cos\left(\omega t + \varphi_0 - \frac{\pi}{2}\right)$$

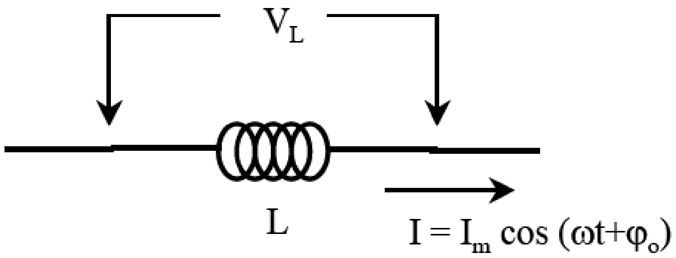
$$V_C = I_m X_C \cos\left(\omega t + \varphi_0 - \frac{\pi}{2}\right)$$

Tegangan antara 2 ujung kapasitor muncul lebih lambat daripada arus

Tegangan antara 2 ujung kapasitor mengikuti arus dengan keterlambatan fasa π/2

Problem: Tegangan antara 2 ujung Induktor

Misal kapasitor dengan kapasitansi C dialiri arus bolak-balik



Berapa tegangan antara 2 ujung induktor?

Tegangan antara 2 ujung induktor:

$$V_L = L\frac{dI}{dt} \Rightarrow V_L = L\frac{d}{dt}\left(I_m \cos\left(\omega t+\varphi_0\right)\right) = -\omega L I_m \sin\left(\omega t+\varphi_0\right)$$

Reaktansi Induktif $\boxed{\omega L = X_L}$

$$V_L = -I_m X_L \sin\left(\omega t+\varphi_0\right)$$

jika frekuensi arus >> → X_L >>

jika frekuensi arus →∞ ,XL →∞ (saklar terbuka)

jika frekuensi arus →0, XL →0 (terhubung singkat)

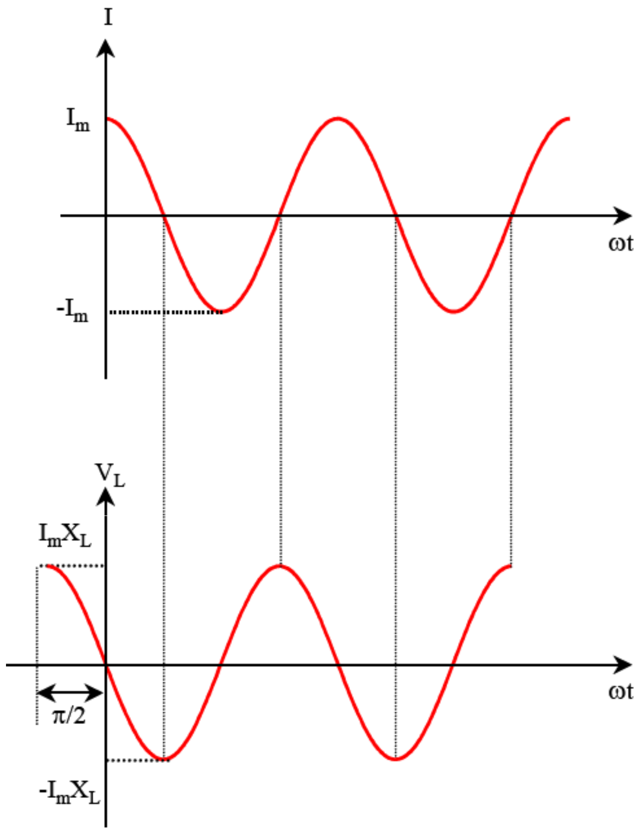
$$V_L = -I_m X_L \sin(\omega t + \varphi_0)$$

Aturan trigonometri

$$-\sin(\omega t + \varphi_0) = \cos\left(\omega t + \varphi_0 + \frac{\pi}{2}\right)$$

Tegangan antara 2 ujung induktor:

$$V_L = I_m X_L \cos\left(\omega t + \varphi_0 + \frac{\pi}{2}\right)$$



Tegangan antara 2 ujung induktor mendahului arus dengan fasa π/2