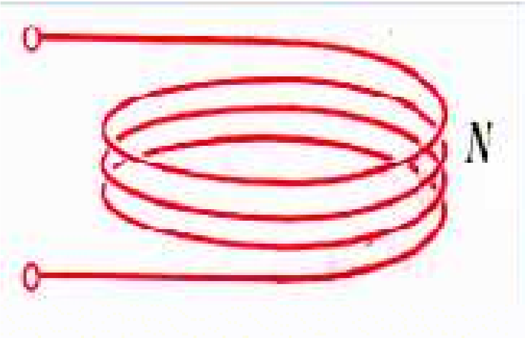
OSILASI ELEKTROMAGNETIK
&
ARUS BOLAK-BALIK

Last Time
Induktansi Diri

Induktansi Diri

Menghitung: $$L = N\Phi / I$$

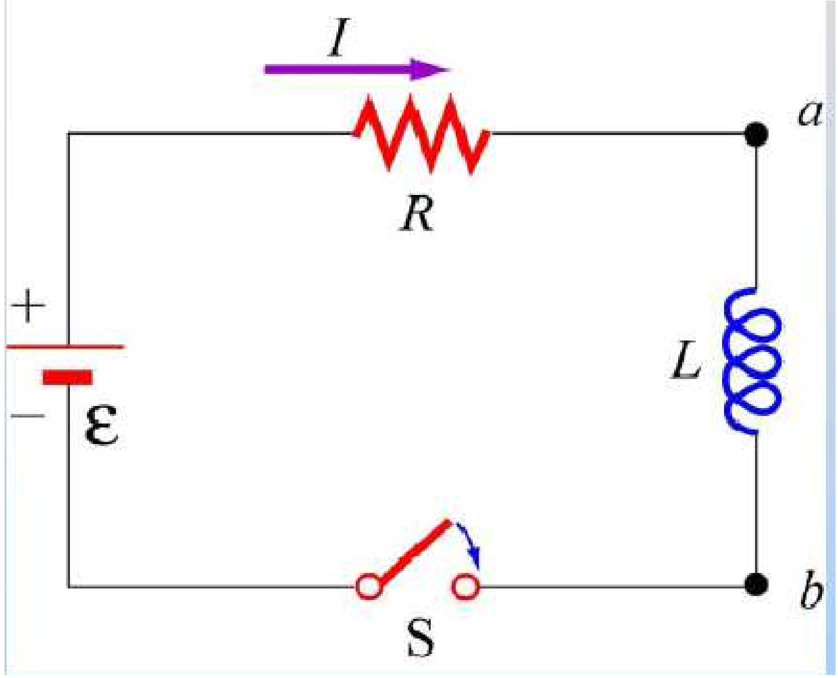


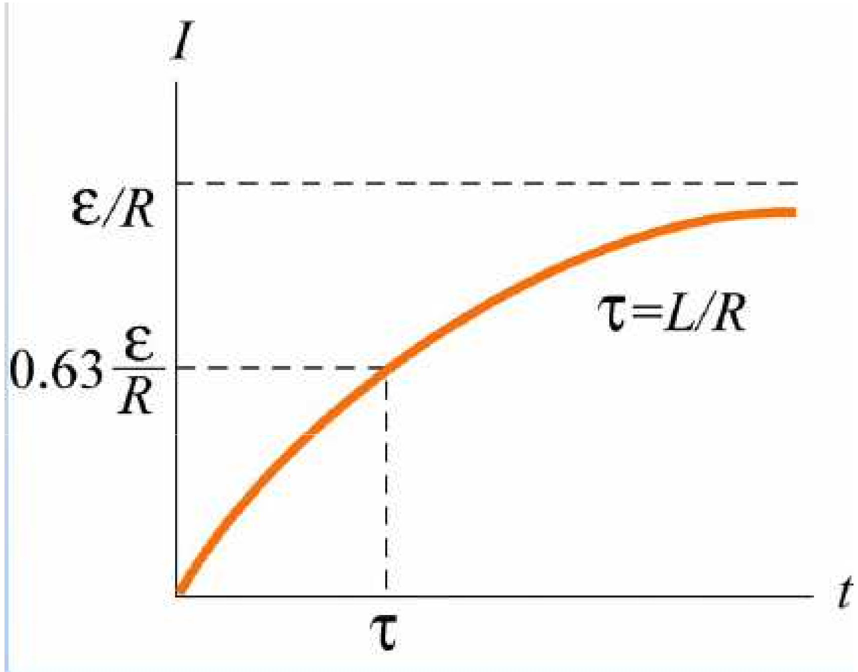
1. Asumsikan arus I mengalir
2. Hitung B akibat adanya I tersebut
3. Hitung fluks akibat adanya B tersebut
4. Hitung induktansi dirinya

Efek: GGL balik: $$\mathcal{E} \equiv -L\frac{dI}{dt}$$

Induktor tidak menyukai perubahan, tetapi menyukai keadaan stabil (steady). Kebalikan Kapasitor!

Rangkaian LR



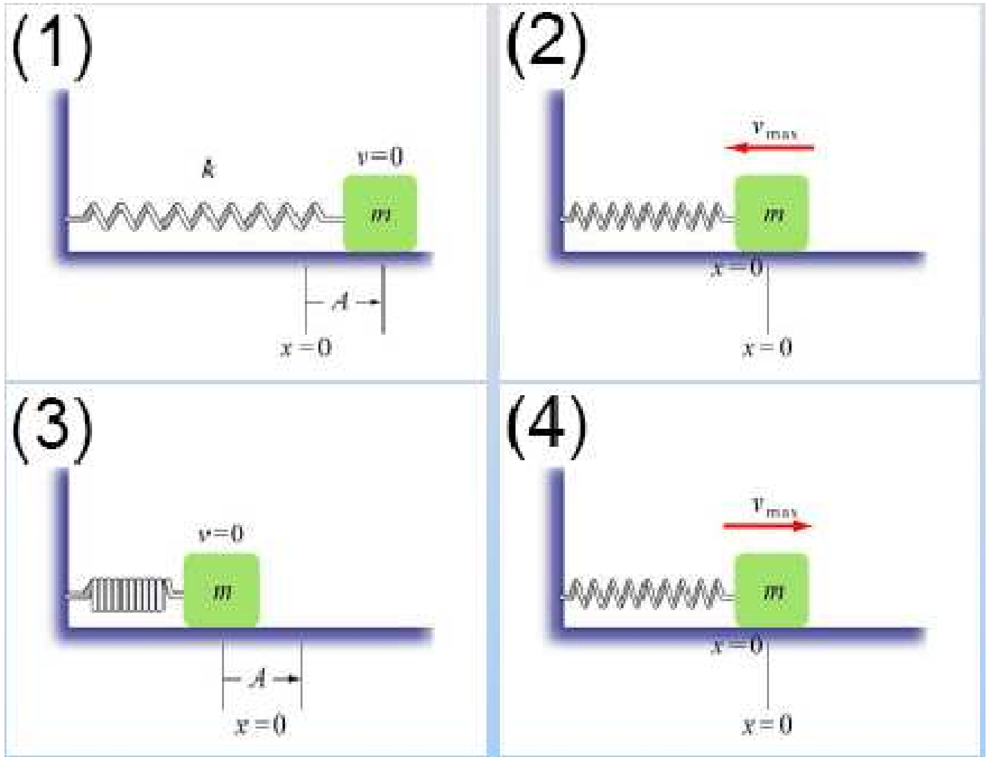


$t=0^{+}$: Arus mencoba untuk berubah. Induktor bekerja sekeras mungkin untuk menghentikannya

$t=\infty$: Arus stabil (steady). Induktor tidak berpengaruh.

Rangkaian LC
Massa pada sebuah Pegas:
Gerak Harmonik Sederhana

Massa pada Pegas



Gerakan seperti apa?

$$F = -kx = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

GHS

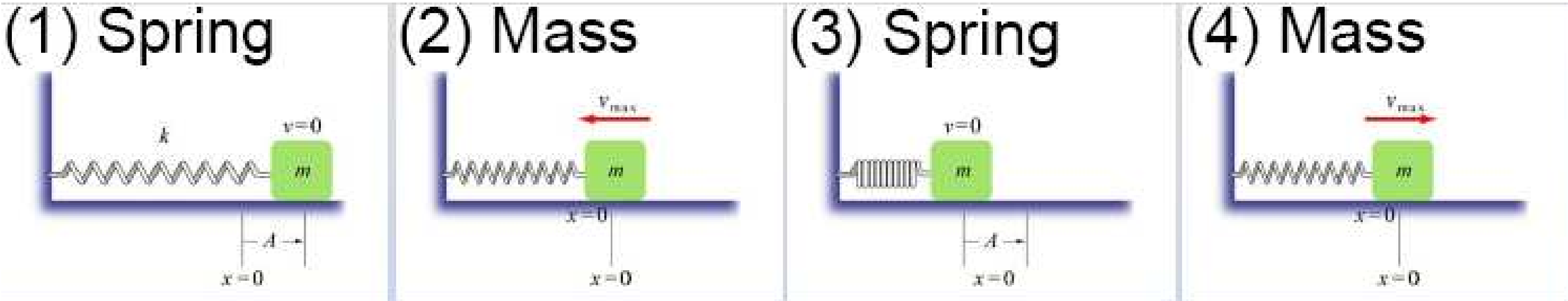
$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

x_0: Amplitudo gerak
Φ: Fase

$$\omega_0 = \sqrt{\frac{k}{m}} =$$

Frekuensi Sudut

Massa pada Pegas : Energi



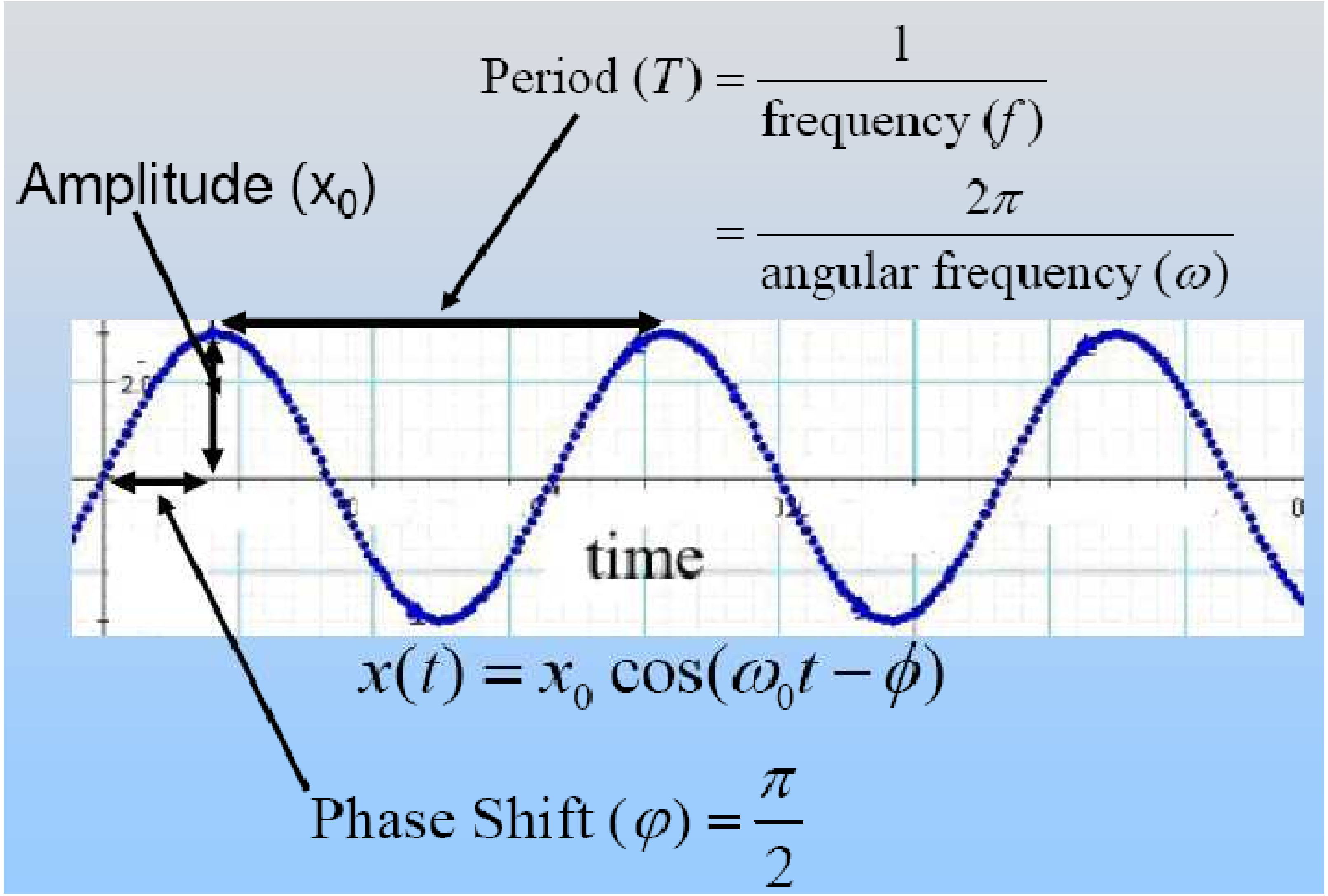
$$x(t) = x_0 \cos(\omega_0 t + \phi) \qquad x'(t) = -\omega_0 x_0 \sin(\omega_0 t + \phi)$$

Energi: 2 bagian: (Massa) Kinetik dan (Pegas) Potensial

$$K = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2 = \frac{1}{2} k x_0^{\,2} \sin^2(\omega_0 t + \phi)$$

$$U_s = \frac{1}{2} k x^2 = \frac{1}{2} k x_0^{\,2} \cos^2(\omega_0 t + \phi)$$

Gerak Harmonik Sederhana



Analog Listrik: Rangkaian LC

Analog: LC Circuit

Massa tidak menyukai percepatan

Energi Kinetik diasosiasikan dengan gerak

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}; \quad E = \frac{1}{2}mv^2$$

Induktor tidak menyukai perubahan arus

Energi diasosiasikan dengan arus

$$\varepsilon = -L\frac{dI}{dt} = -L\frac{d^2q}{dt^2}; \quad E = \frac{1}{2}LI^2$$

Analog: Rangkaian LC

Pegas tidak suka ditekan/ditarik

Energi Potensial diasosiasikan dengan tarikan

$$F = -kx; \quad E = \frac{1}{2}kx^2$$

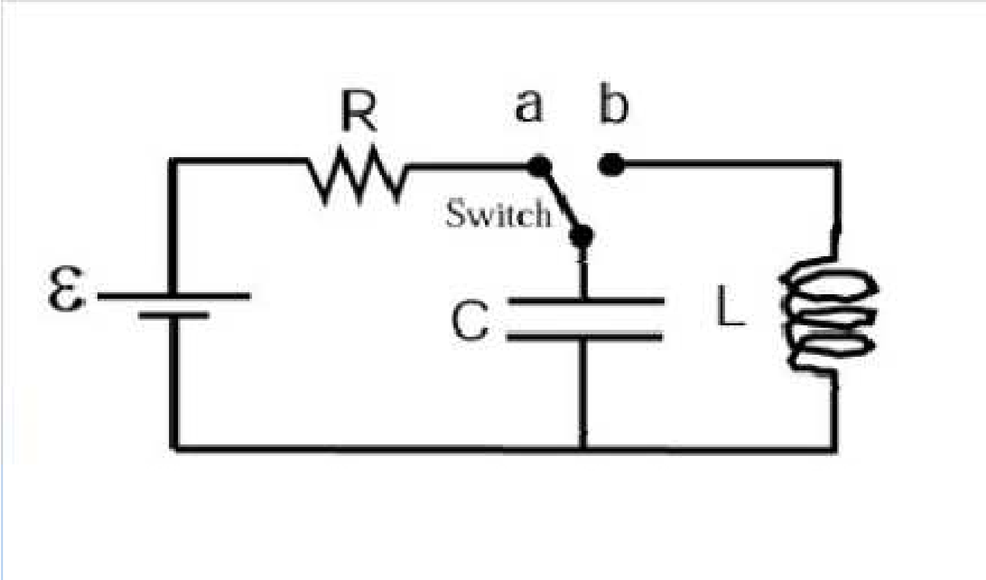
Kapasitor tidak suka dimuati (+ atau -)

Energi diasosiasikan dengan muatan yang tersimpan

$$\varepsilon = \frac{1}{C}q; \quad E = \frac{1}{2}\frac{1}{C}q^2$$

$$F \rightarrow \varepsilon; \; x \rightarrow q; \; v \rightarrow I; \; m \rightarrow L; \; k \rightarrow C^{-1}$$

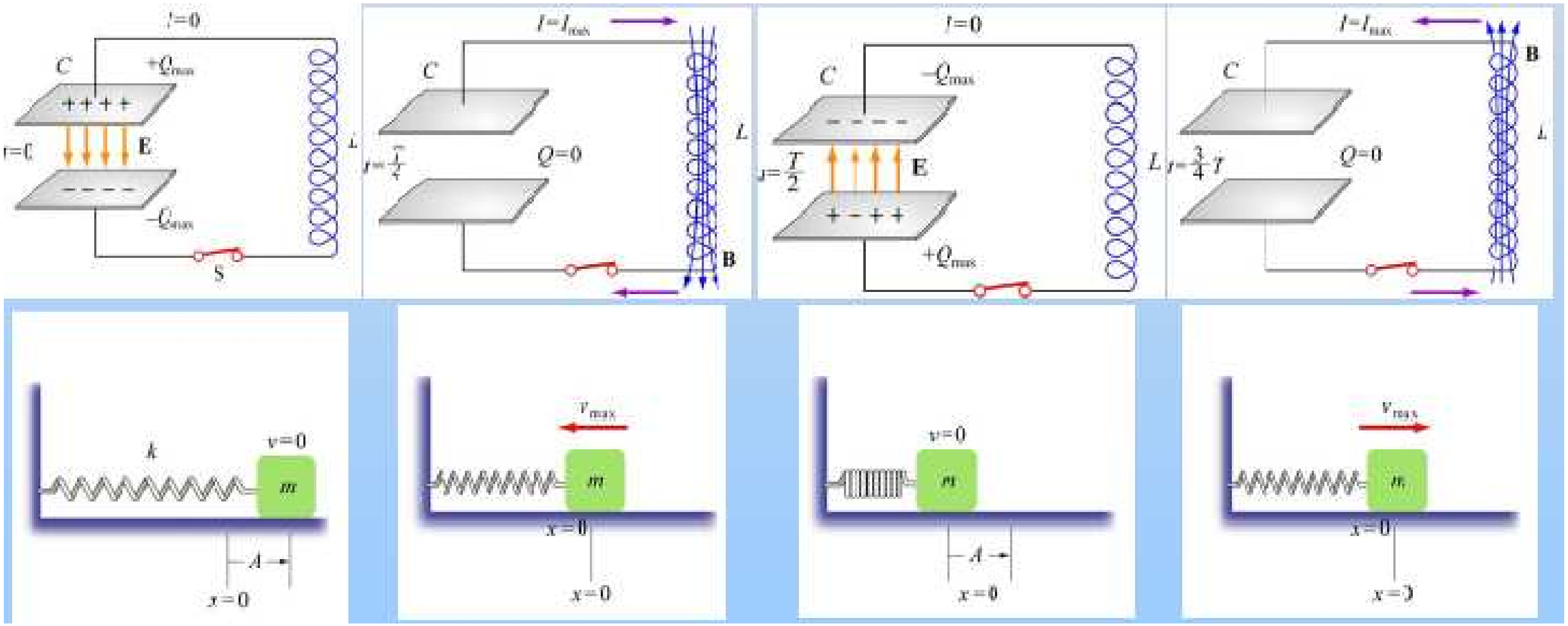
RangkaianLC



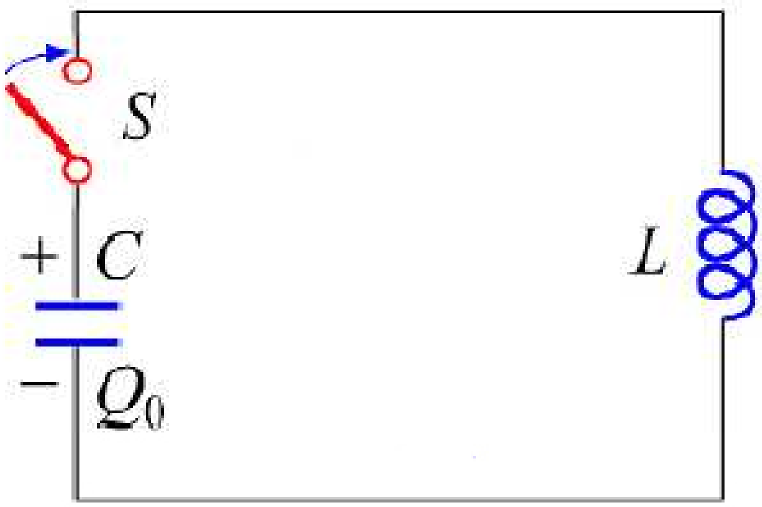
1. Susun rangkaian dengan komponen kapasitor, induktor, resistor, dan batre seperti di atas.
2. Biarkan kapasitor terisi penuh.
3. Pindahkan saklar dari a ke b
4. Apa yang terjadi?

Rangkaian LC

Akan mengalami gerak harmonik sederhana, seperti massa pada pegas, dengan pertukaran antar muatan pada kapasitor (Pegas) dan arus pada induktor (Massa)



Rangkaian LC



$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \quad ; \quad I = -\frac{dQ}{dt}$$

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

Gerak Harmonik Sederhana

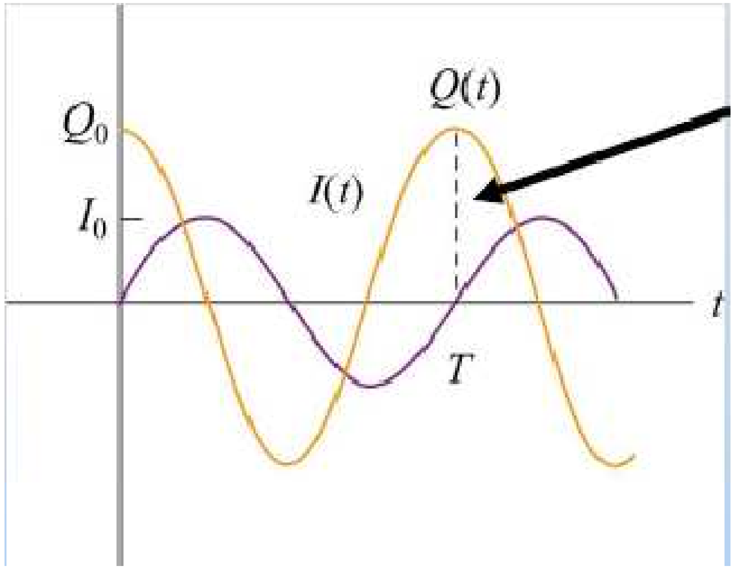
$$Q(t) = Q_0 \cos(\omega_0 t + \phi)$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

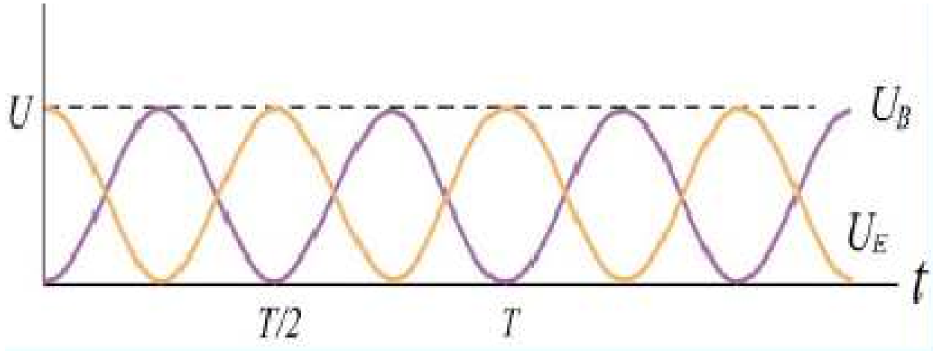
Q_0: Amplitudo Osilasi Muatan
Φ: fase

Animasi 10.1

Osilasi LC : Energi



Informasi beda fase



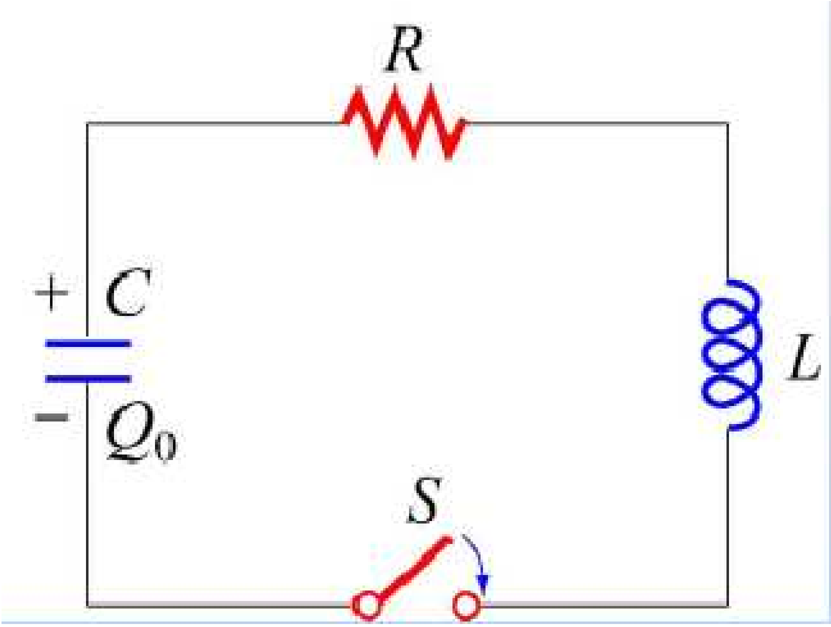
$$U_E = \frac{Q^2}{2C} = \left(\frac{Q_0^{\ 2}}{2C}\right)\cos^2 \omega_0 t \qquad U_B = \frac{1}{2}LI^2 = \frac{1}{2}LI_0^{\ 2}\sin^2 \omega_0 t = \left(\frac{Q_0^{\ 2}}{2C}\right)\sin^2 \omega_0 t$$

$$U = U_E + U_B = \frac{Q^2}{2C} + \frac{1}{2}LI^2 = \frac{Q_0^{\ 2}}{2C}$$

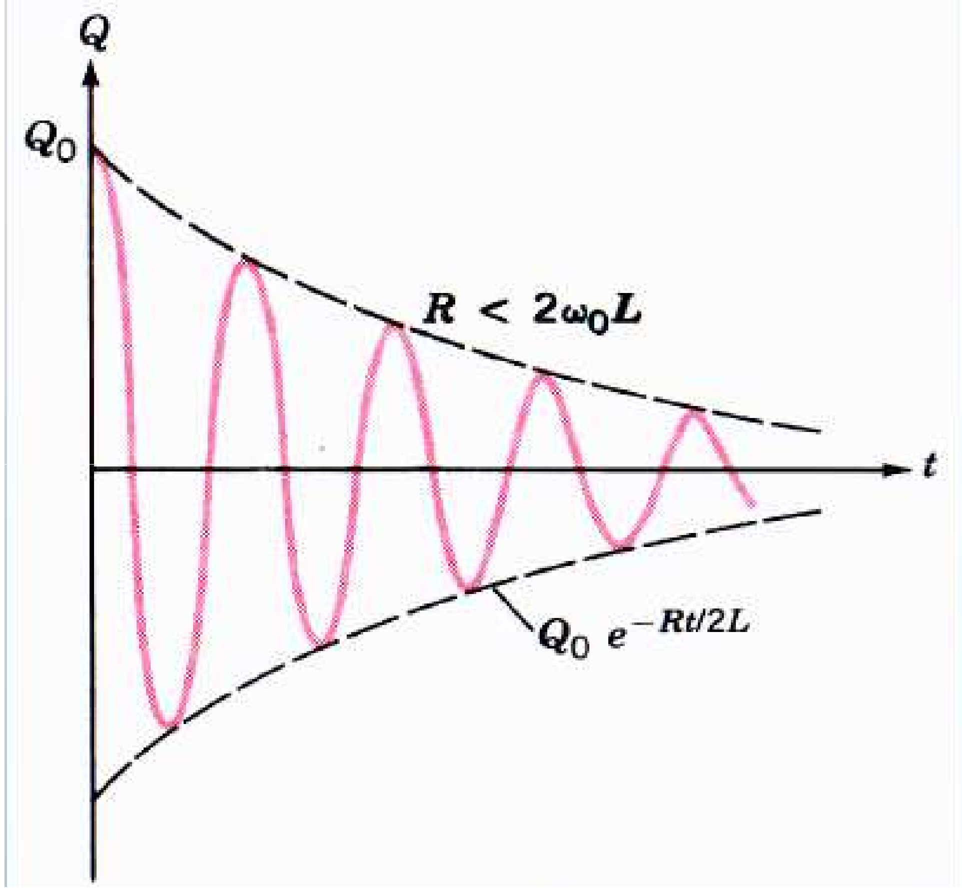
Energi Total Kekal !!

Penambahan Redaman: Rangkaian RLC

Osilasi LC Teredam



Resistor mendisipasi energi dan sistem berosilasi menurun terhadap waktu



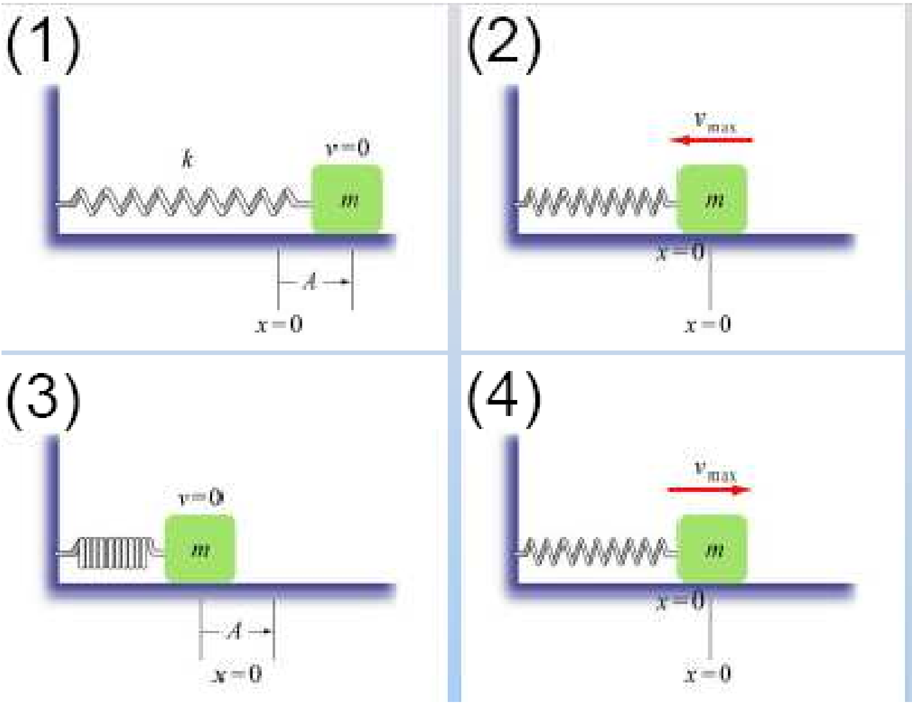
Animasi 10.2

Juga, frekuensi menurun:

$$\omega' = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$$

Massa pada Pegas:
Gerak Harmonik Sederhana
Tinjauan kedua

Massa pada Pegas



Kita pecahkan:

$$F = -kx = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

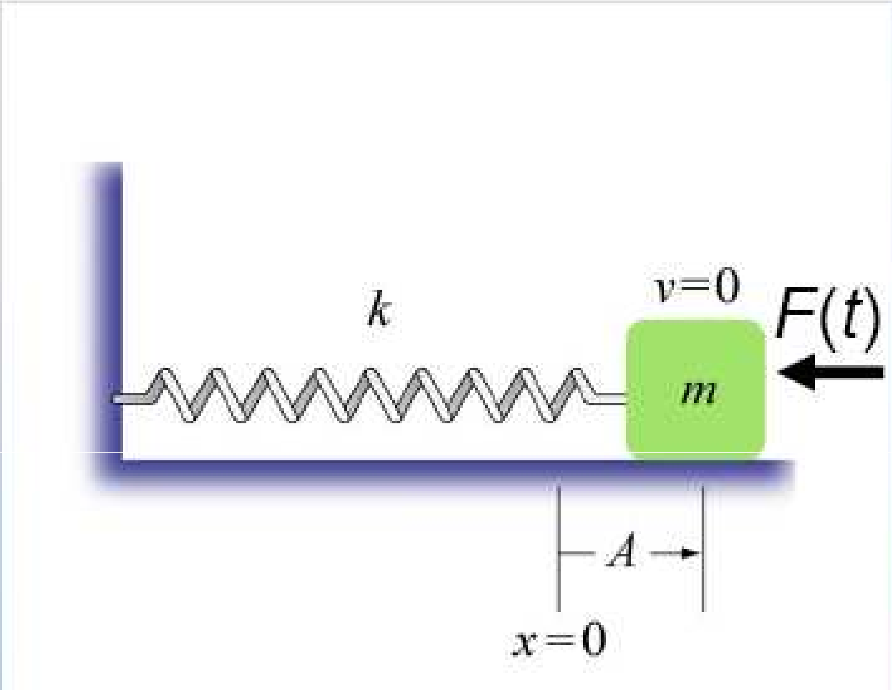
Gerak Harmonik Sederhana

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

Bergerak dengan frekuensi alami

Sekarang, bagaimana jika dindingnya digerakkan?
Atau massanya ditekan?

Pengendali Massa pada Pegas



Gerak Harmonik Sederhana

$$x(t) = x_{\max} \cos(\omega t + \phi)$$

Bergerak dengan frekuensi pengendali

Kita peroleh:

$$F = F(t) - kx = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = F(t)$$

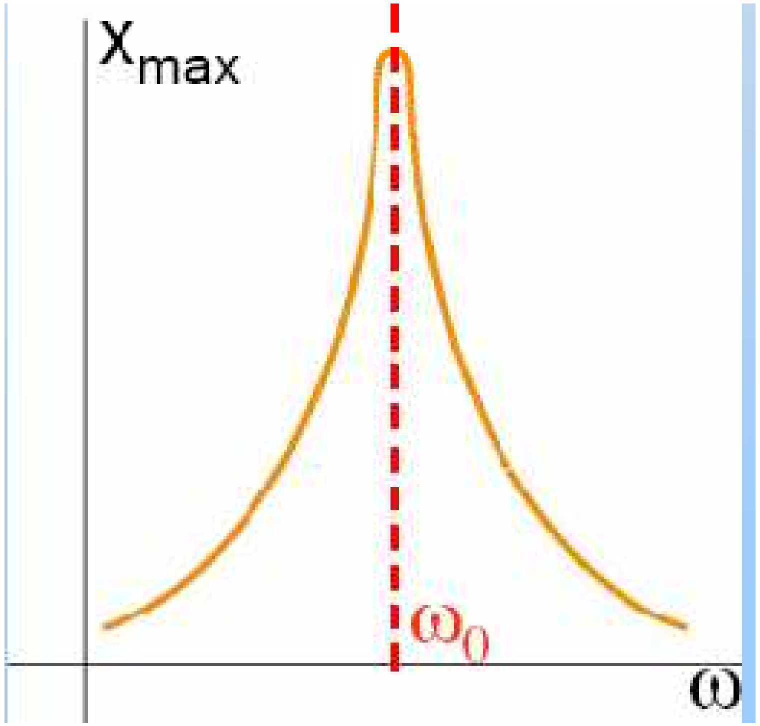
Asumsi gaya harmonik:

$$F(t) = F_0 \cos(\omega t)$$

Resonansi

$$x(t) = x_{max} \cos(\omega t + \phi)$$

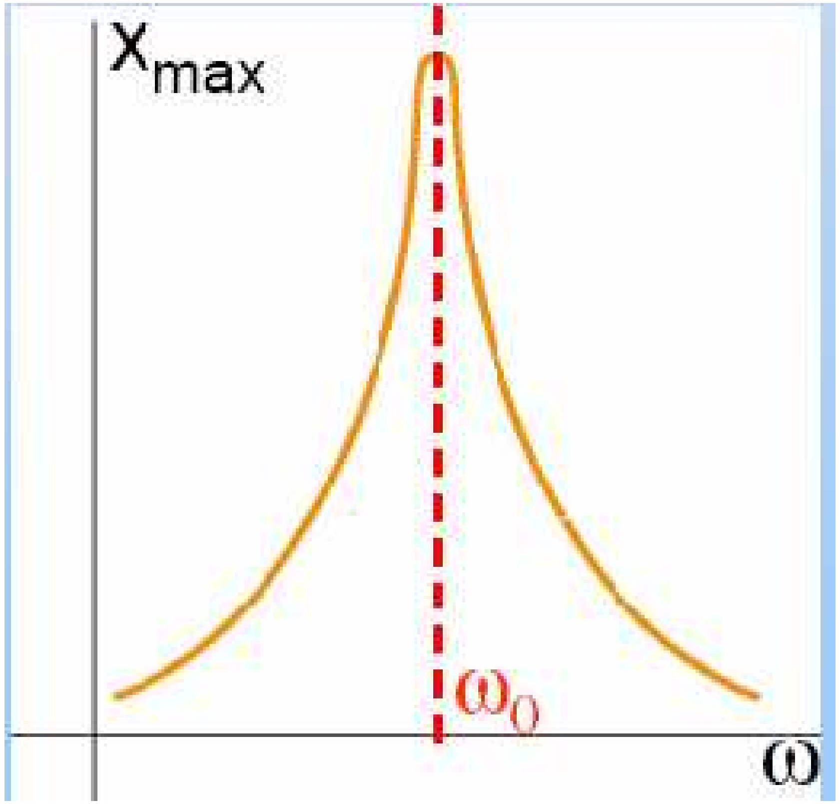
Amplitudo, x_{max}, bergantung pada seberapa dekat frekuensi pengendali dengan frekuensi alami



Resonansi

$$x(t) = x_{max} \cos(\omega t + \phi)$$

x_{max} bergantung pada frekuensi



Banyak sistem mempunyai sifat seperti ini:
Ayunan
Instrumen musik
…

Analog: Rangkaian RLC

Recall:
Induktor seperti massa (punya inersia)
Kapasitor seperti pegas (menyimpan/melepas energi)
Batteries supply external force (EMF)

Muatan pada kapasitor seperti posisi,
Arus seperti kecepatan

Sekarang, move to “frekuensi yang bergantung batrei”
Power Supplai AC / Generator AC

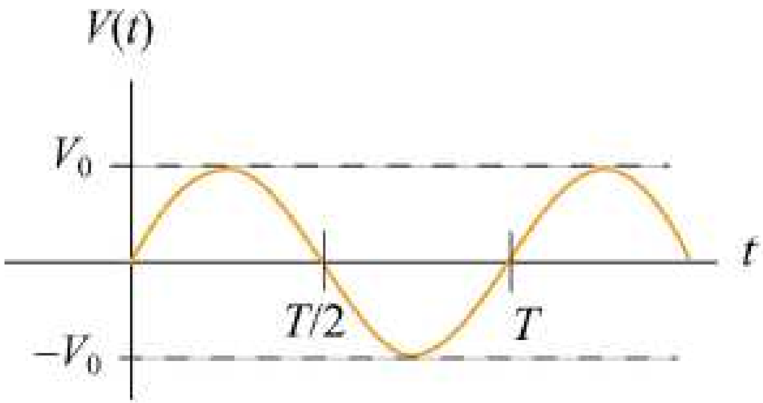
Rangkaian AC

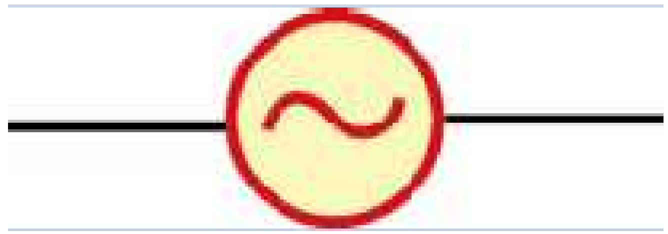
Rangkaian AC (Alternating-Current)

• direct current (dc) – arus mengalir satu arah (battery)
• alternating current (ac) – arus berosilasi

• Sumber tegangan sinusoidal

$$V(t) = V_0 \sin \omega t$$

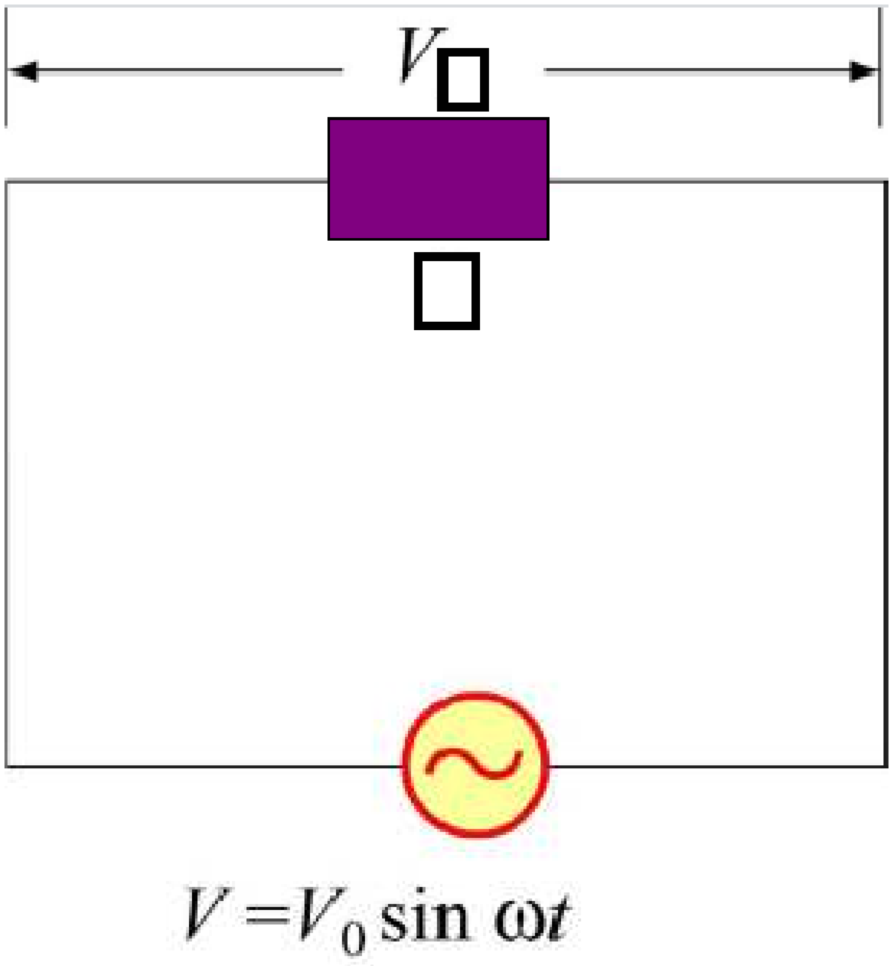




$\omega = 2\pi f$: frekuensi sudut

V_0 : amplitudo tegangan/voltase

Rangkaian AC : Elemen Tunggal

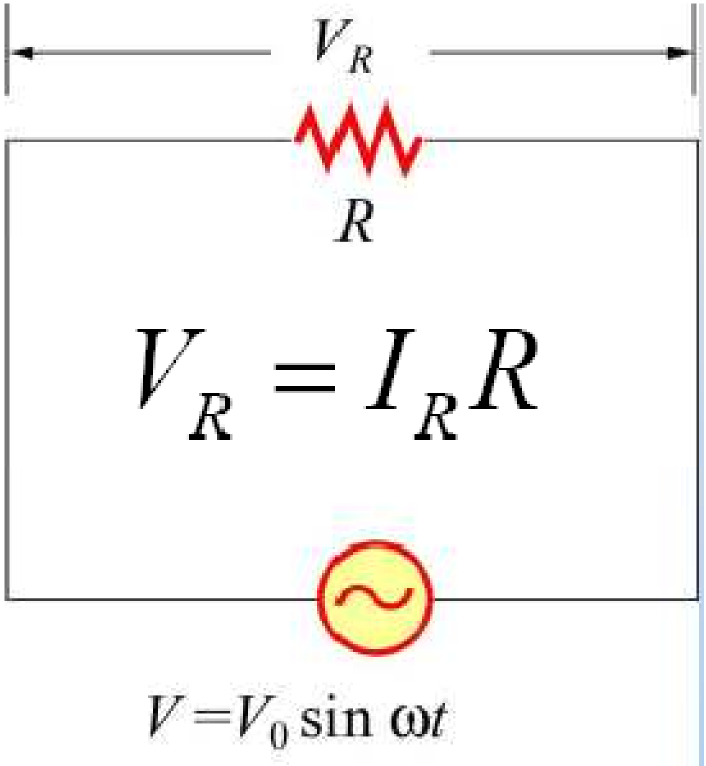


$$V_{\square} = V$$
$$= V_0 \sin \omega t$$

$$I(t) = I_0 \sin(\omega t - \phi)$$

Pertanyaan:
1. Berapa I_0?
2. Berapa Φ ?

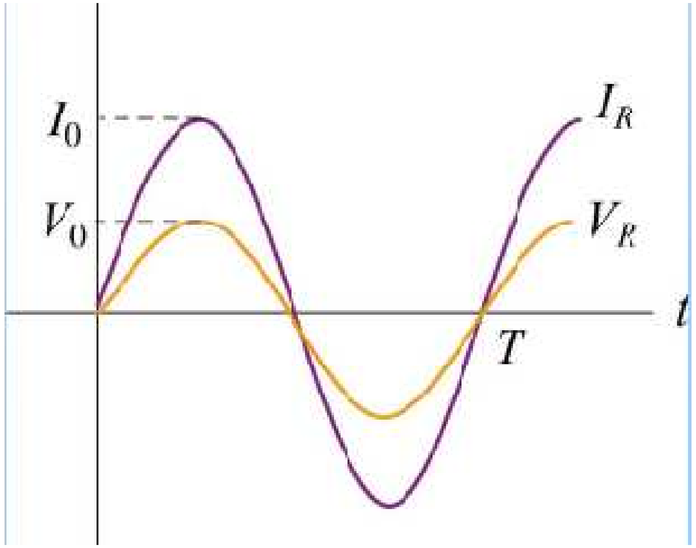
Rangkaian AC : Resistor



$$I_R = \frac{V_R}{R} = \frac{V_0}{R} \sin \omega t$$
$$= I_0 \sin(\omega t - 0)$$

$$I_0 = \frac{V_0}{R}$$
$$\varphi = 0$$

I_R dan V_R **sefase**



Animasi 10.5

$$\int \sin^2 \omega t \, dt = \int \frac{1-\cos 2\omega t}{2} dt = \frac{1}{2}\int 1-\cos 2\omega t \, dt = \frac{1}{2}\left(t - \frac{1}{2\omega}\sin 2\omega t \right) =$$

Rangkaian AC : Resistor

Arus rata-rata:

$$\langle I_R(t) \rangle = \frac{1}{T}\int_0^T I_R(t)dt = \frac{1}{T}\int_0^T I_{R0}\sin \omega t \; dt = \frac{I_{R0}}{T}\int_0^T \sin\frac{2\pi t}{T} \; dt = 0$$

Arus kuadrat rata-rata:

$$\langle I_R^2(t) \rangle = \frac{1}{T}\int_0^T I_R^2(t)dt = \frac{1}{T}\int_0^T I_{R0}^2 \sin^2 \omega t \; dt = I_{R0}^2 \frac{1}{T}\int_0^T \sin^2\left(\frac{2\pi t}{T}\right) \; dt = \frac{1}{2} I_{R0}^2$$

Arus akar kuadrat rata-rata ($I_{\rm rms}$):

$$I_{\rm rms} = \sqrt{\langle I_R^2(t) \rangle} = \frac{I_{R0}}{\sqrt{2}}$$

Tegangan akar kuadrat rata-rata ($V_{\rm rms}$):

$$V_{\rm rms} = \sqrt{\langle V_R^2(t) \rangle} = \frac{V_{R0}}{\sqrt{2}}$$

Rangkaian AC : Resistor

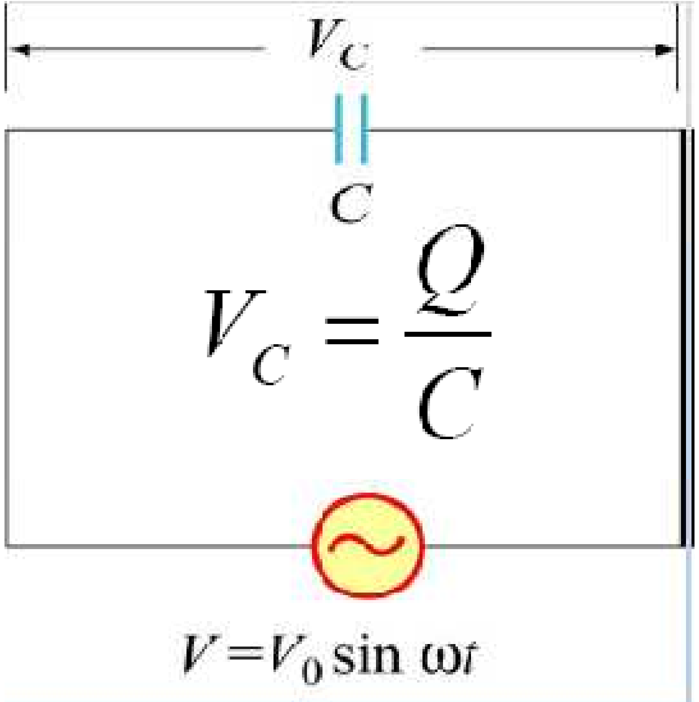
Daya yang didisipasi pada resistor:

$$P_R(t) = I_R(t)V_R(t) = I_R^2(t)R$$

Daya rata-rata yang didisipasi pada resistor:

$$\left\langle P_R(t) \right\rangle = \left\langle I_R^2(t)R \right\rangle = \frac{1}{2} I_{R0}^2 R = I_{\text{rms}}^2 R = I_{\text{rms}} V_{\text{rms}} = \frac{V_{\text{rms}}^2}{R}$$

Rangkaian AC : Kapasitor

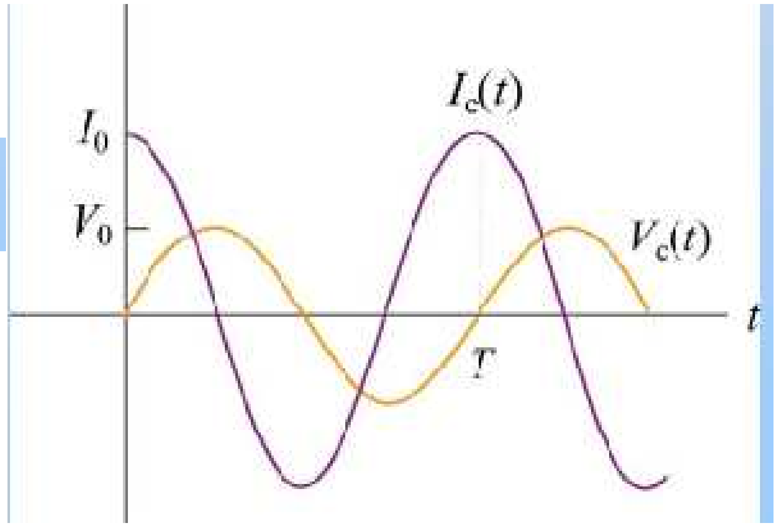


$$I_C(t) = \frac{dQ}{dt}$$
$$= \omega C V_0 \cos \omega t$$
$$= I_0 \sin(\omega t - -\pi/2)$$

$$I_0 = \omega C V_0$$
$$\varphi = -\pi/2$$

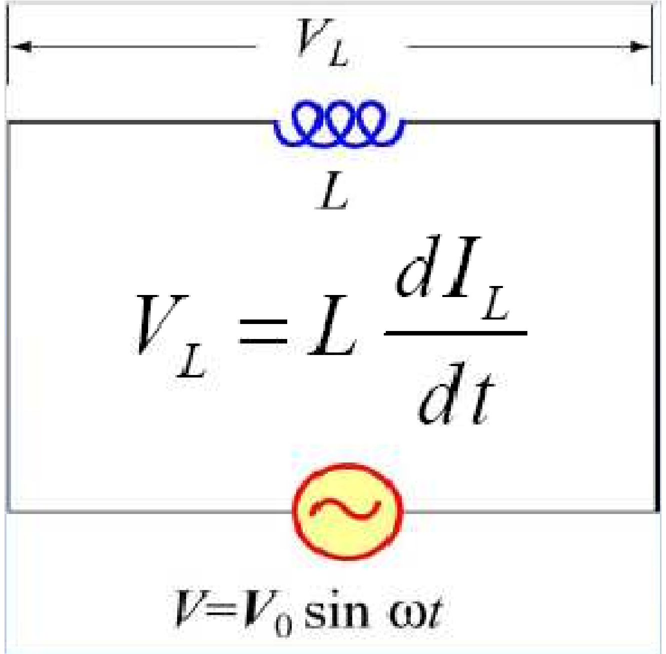
I_C **mendahului** V_C sebesar π/2

$$Q(t) = CV_C = CV_0 \sin \omega t$$



Animasi 10.3

Rangkaian AC : Induktor



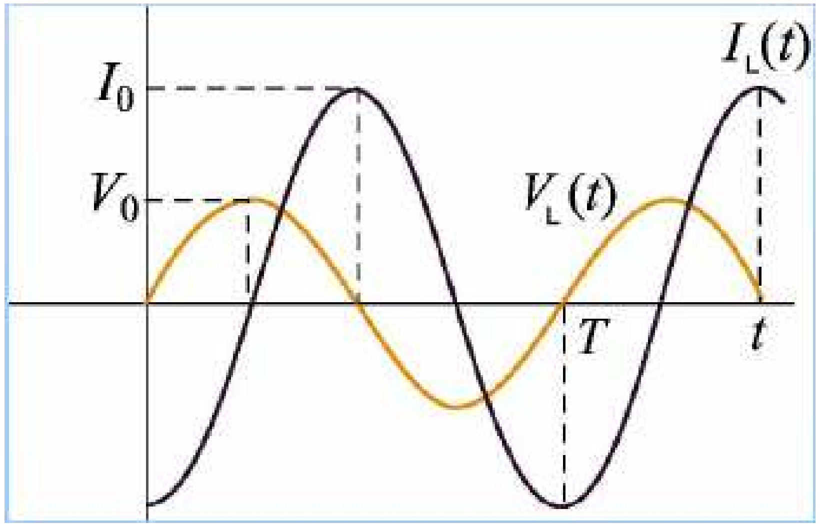
$$V_L = L \frac{dI_L}{dt}$$

$$I_L(t) = {}^{V_0}\!/_{L} \int \sin \omega t \; dt$$
$$= -{}^{V_0}\!/_{\omega L} \cos \omega t$$
$$= I_0 \sin\left(\omega t - {}^{\pi}\!/_{2}\right)$$

$$I_0 = \frac{V_0}{\omega L}$$
$$\varphi = \frac{\pi}{2}$$

I_L **tertinggal oleh** V_L sebesar π/2

$$\frac{dI_L}{dt} = \frac{V_L}{L} = \frac{V_0}{L} \sin \omega t$$



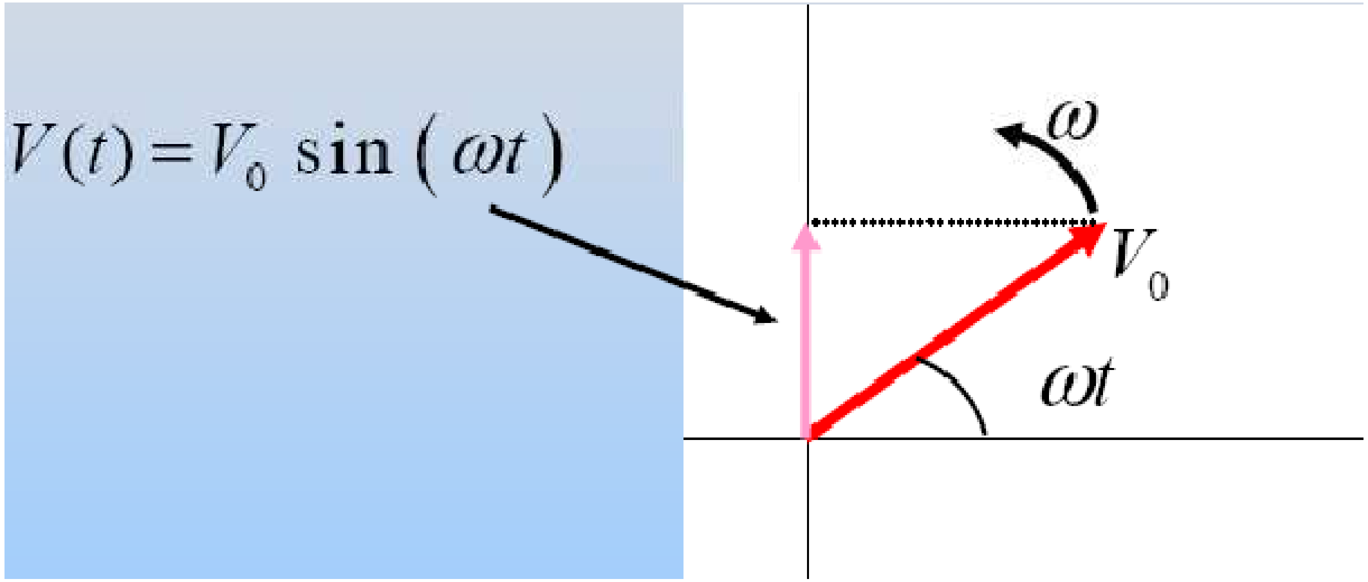
Animasi 10.4

Rangkaian AC : Summary

Element	I_0	Current vs. Voltage	Resistance Reactance Impedance
Resistor	$\frac{V_{0R}}{R}$	In Phase	$R = R$
Capacitor	$\omega C V_{0C}$	Leads	$X_C = \frac{1}{\omega C}$
Inductor	$\frac{V_{0L}}{\omega L}$	Lags	$X_L = \omega L$

Diagram Fasor

Cara menyatakan besar & fase:

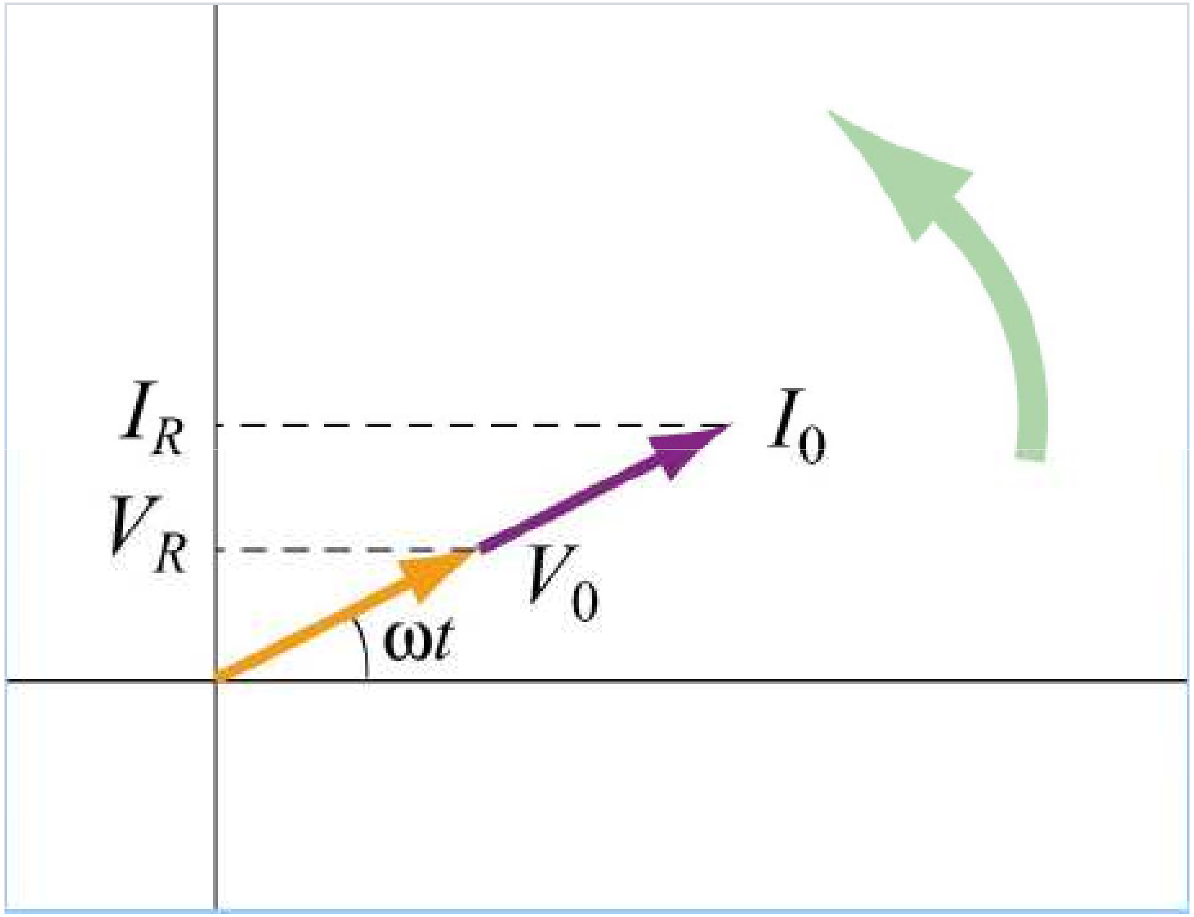


Note:

(1) Ketika fasor (vektor merah) berotasi,proyeksinya (vektor ping) berosilasi

(2) Berlaku pada keduanya (arus dan tegangan)

Diagram Fasor : Resistor



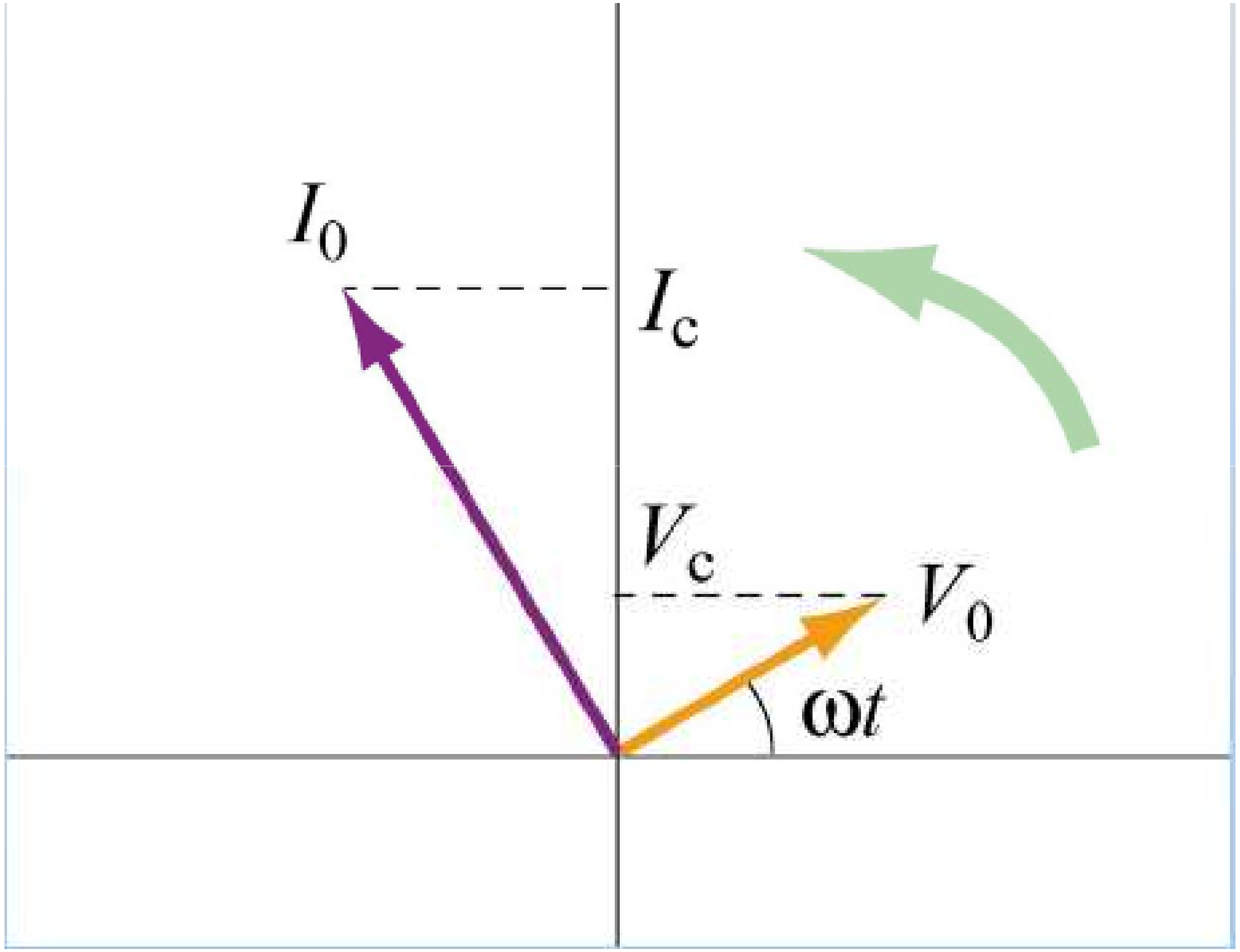
$$V_0 = I_0 R$$

$$\varphi = 0$$

I_R dan V_R **sefase**

Animasi 10.5

Diagram Fasor : Kapasitor

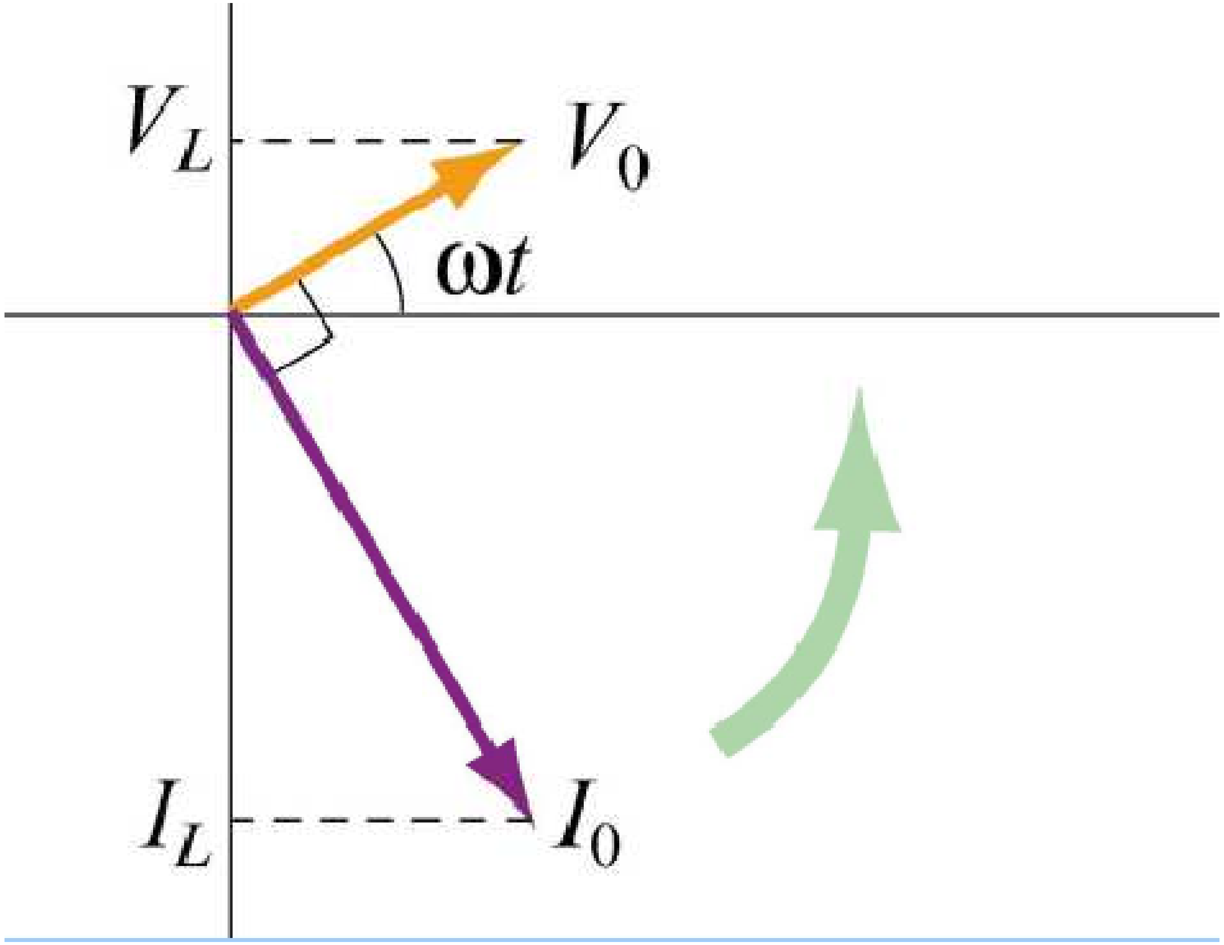


$$V_0 = I_0 X_C$$
$$= I_0 \frac{1}{\omega C}$$
$$\varphi = -\pi/2$$

Animasi 10.3

I_C **mendahului** V_C sebesar $\pi/2$

Diagram Fasor : Induktor



$$V_0 = I_0 X_L$$
$$= I_0 \omega L$$
$$\varphi = \frac{\pi}{2}$$

Animasi 10.4

I_L **tertinggal oleh** V_L sebesar $\pi/2$

Susun Bersamaan: Rangkaian RLC

Pertanyaan tentang Fase

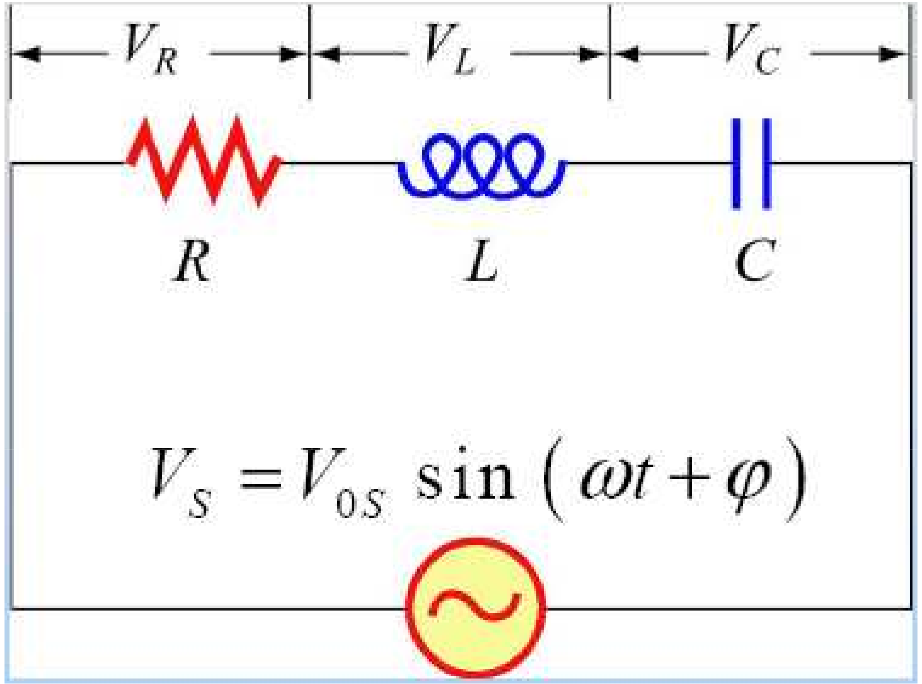
Kita mempunyai tegangan yang fasenya konstan:

$$V = V_0 \sin \omega t \qquad I(t) = I_0 \sin(\omega t - \phi)$$

Dapat ditulis:

$$V = V_0 \sin(\omega t + \phi) \qquad I(t) = I_0 \sin \omega t$$

Rangkaian RLC Seri



$$I(t) = I_0 \sin(\omega t)$$

$$V_R = V_{R0}\sin(\omega t)$$

$$V_L = V_{L0}\sin\left(\omega t + \tfrac{\pi}{2}\right)$$

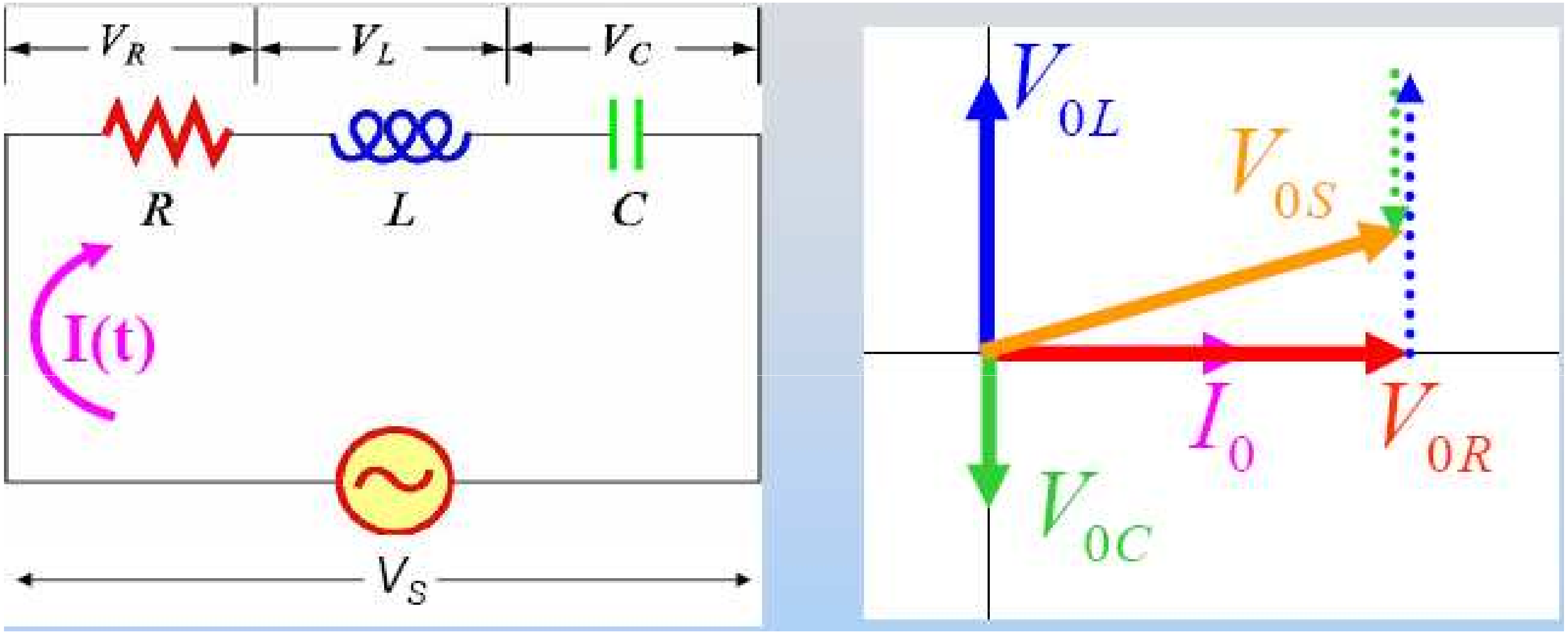
$$V_C = V_{C0}\sin\left(\omega t + \tfrac{-\pi}{2}\right)$$

Berapa I_0 (dan $V_{R0} = I_0 R,\ V_{L0} = I_0 X_L,\ V_{C0} = I_0 X_C$)?

Berapa φ ? Apakah arus mendahului atau tertinggal oleh V_s?

Pecahkan: $$V_S = V_R + V_L + V_C$$

Rangkaian RLC Seri



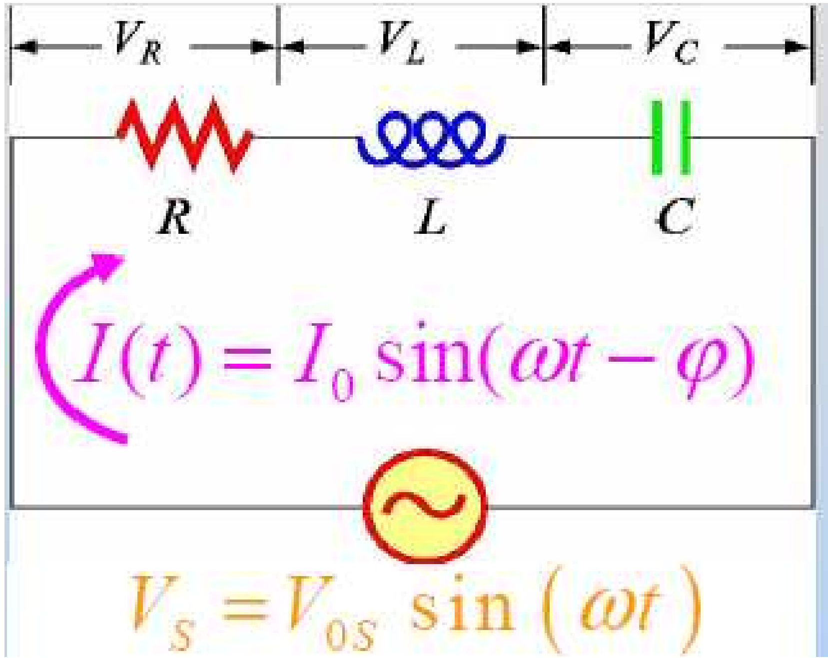
Pecahkan:

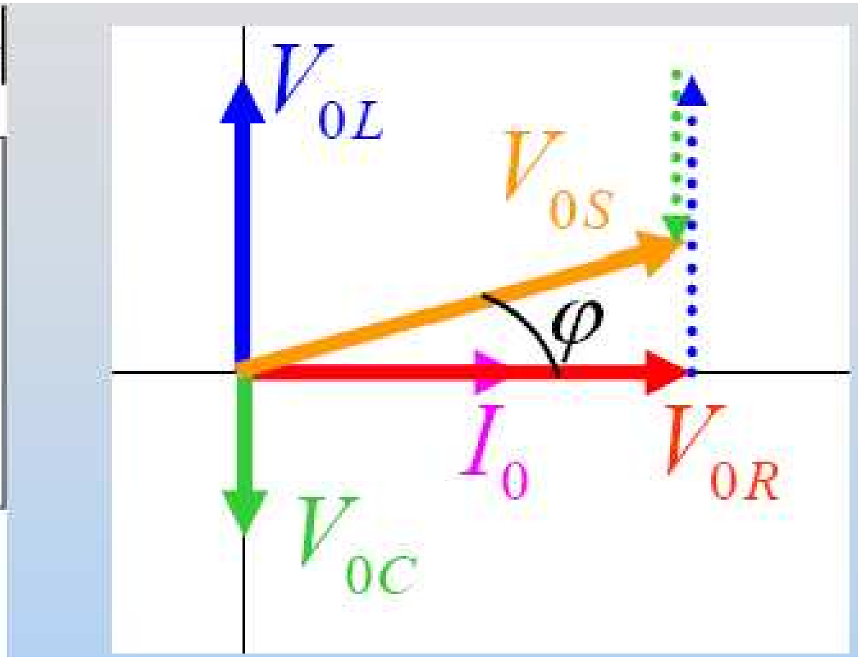
$$V_S = V_R + V_L + V_C$$

Sekarang kita hanya membaca diagram fasor!

Animasi 10.6

Rangkaian RLC Seri





$$V_{0S} = \sqrt{V_{R0}{}^2 + (V_{L0} - V_{C0})^2} = I_0\sqrt{R^2 + (X_L - X_C)^2} \equiv I_0 Z$$

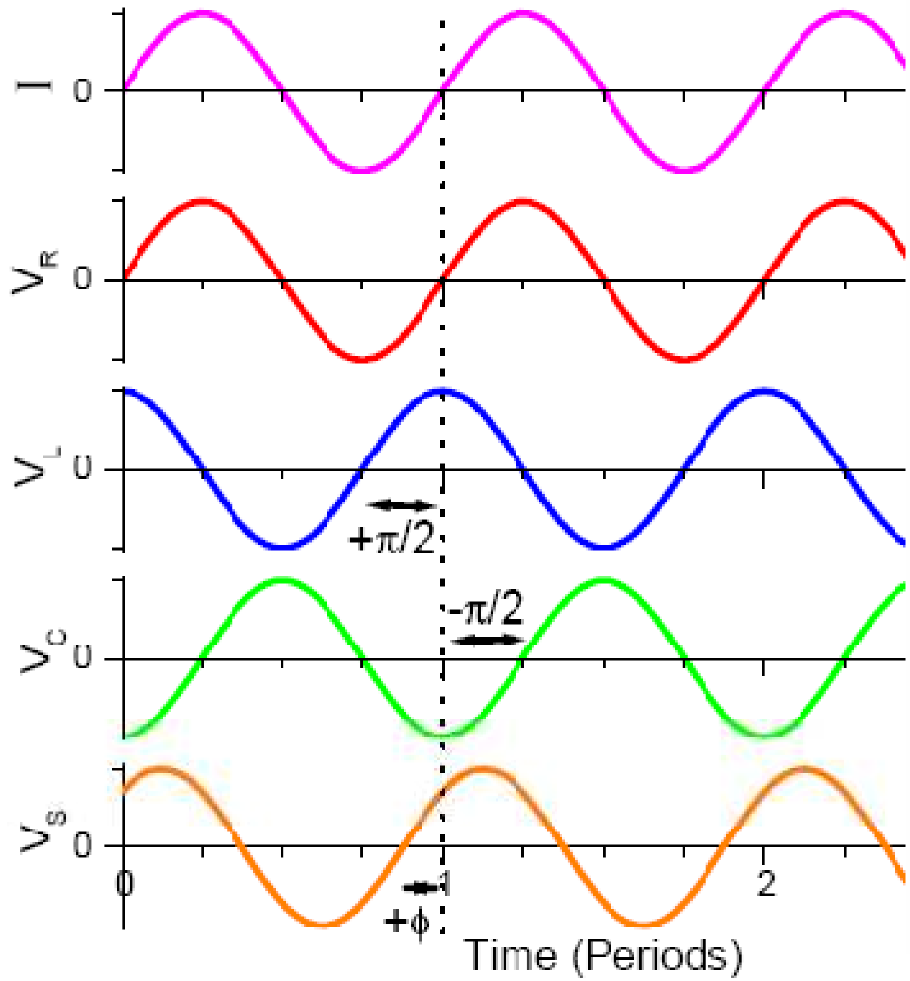
$$I_0 = \frac{V_{0S}}{Z}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Plot I, V vs Waktu



$$I(t) = I_0 \sin(\omega t)$$

$$V_R(t) = I_0 R \sin(\omega t)$$

$$V_L(t) = I_0 X_L \sin\left(\omega t + \tfrac{\pi}{2}\right)$$

$$V_L(t) = I_0 X_C \sin\left(\omega t - \tfrac{\pi}{2}\right)$$

$$V_S(t) = V_{S0} \sin(\omega t + \varphi)$$

Daya pada Rangkaian RLC Seri

Daya setiap saat yang disalurkan generator AC:

$$P(t) = I(t)V(t) = \frac{V_0}{Z}\sin(\omega t - \phi)\cdot V_0 \sin \omega t = \frac{V_0^2}{Z}\sin(\omega t - \phi)\sin \omega t$$

$$= \frac{V_0^2}{Z}\left(\sin^2 \omega t \cos\phi - \sin \omega t \cos \omega t \sin\phi\right)$$

Daya rata-rata:

$$\langle P(t)\rangle = \frac{1}{T}\int_0^T \frac{V_0^2}{Z}\sin^2 \omega t \cos\phi \; dt - \frac{1}{T}\int_0^T \frac{V_0^2}{Z}\sin \omega t \cos \omega t \sin\phi \; dt$$

$$= \frac{V_0^2}{Z}\cos\phi\langle\sin^2 \omega t\rangle - \frac{V_0^2}{Z}\sin\phi\langle\sin \omega t \cos \omega t\rangle$$

$$= \frac{1}{2}\frac{V_0^2}{Z}\cos\phi$$

$$\langle P(t)\rangle = \frac{1}{2}\frac{V_0^2}{Z}\cos\phi = \frac{V_{rms}^2}{Z}\cos\phi = I_{rms}V_{rms}\cos\phi$$

Faktor daya $\cos\phi = \frac{R}{Z}$

Rangkaian RLC : Resonansi

Resonansi

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} \;; \quad X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$

Pada frekuensi rendah, C mendominasi ($X_C >> X_L$):
Pada frekuensi tinggi, L mendominasi ($X_L >> X_C$):
Pada frekuensi sedang, terjadi **resonansi**

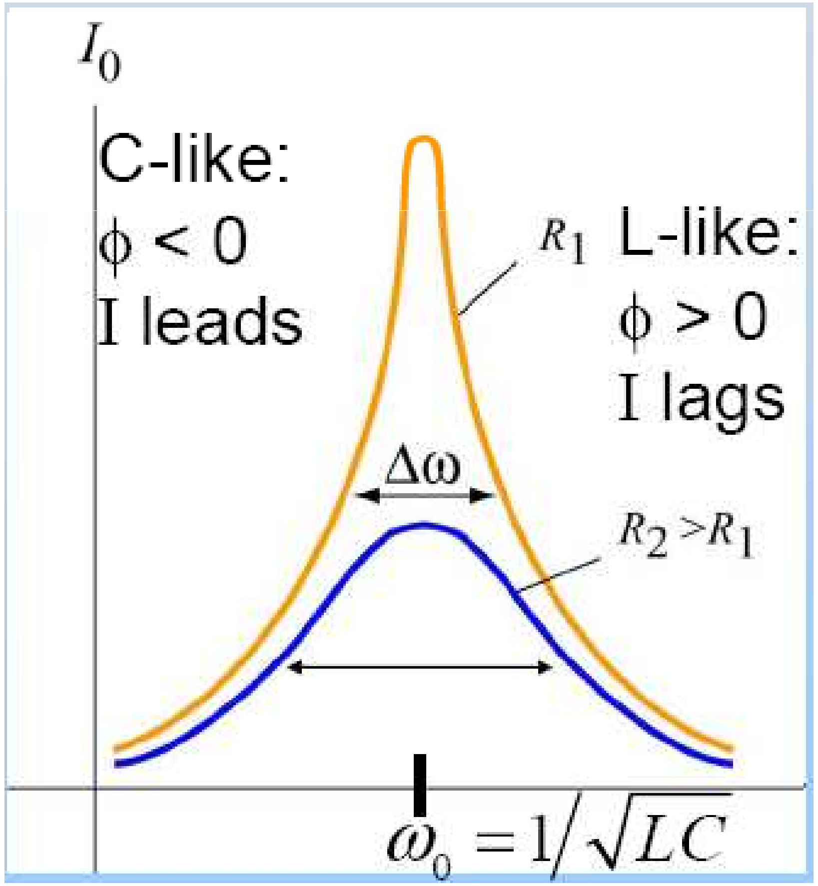
I_0 mencapai maksimum ketika

$$X_L = X_C$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Resonansi

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} \; ; \quad X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$



$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Animasi 10.7