CHAPTER V

ASTROPHYSICS

Outline:

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- *Stellar apparent magnitude and color*
- *Stellar distance*
- *Stellar absolute magnitude*
- *Stellar mass-radius-temperature*
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⊙ *Stellar Evolution*
- *Energy generation*
- *Evolution off the main sequence (low and high mass stars)*

V.1 Property of the Stars

V.1.1 Stellar apparent magnitude and color

When we look at the sky, we note that some stars appear brighter than others. All we know at first glance is that stars *appear* to have different brightnesses.

We would like to have some way of quantifying the observed brightnesses of stars. When we speak loosely of brightness, we are really talking about the *energy flux, f*, which is the energy per unit area per unit time received from the star.

Ancient astronomers made naked eye estimates of brightness. Hipparchus, the Greek astronomer, and later Ptolemy, a Greek living in Alexandria, Egypt, around 150 BC, divided stars into six classes of brightness. These classes were called *magnitudes*. This was an ordinal arrangement, with first-magnitude stars being the brightest and sixth-magnitude stars being the faintest.

When quantitative measurements were made, it was found that each jump of one magnitude corresponded to a fixed *flux ratio*, not a flux difference. Because of this, the magnitude scale is essentially a logarithmic one. This is not too surprising, since the eye is approximately logarithmic in its response to light. This type of response allows us to see in very low and very high light levels. It was found that a difference of five magnitudes corresponds to a factor of 100 in brightness. In setting up the magnitude scale, this relation is defined to be exact.

Let b_1 and b_2 be the observed brightnesses of two stars, and let m_1 and m_2 be the corresponding magnitudes. The statement that a five-magnitude difference gives a flux ratio of 100 corresponds to

$$\frac{b_1}{b_2} = 100^{\frac{(m_2 - m_1)}{5}} \quad \ldots\ldots\ldots.(5.1)$$

Equation (5.1) gives brightness ratios in powers of 100, but we usually work in powers of ten. To convert this we write 100 as 10^2, so equation (5.1) becomes

$$\frac{b_1}{b_2} = 10^{\frac{(m_2 - m_1)}{2,5}} \quad \ldots\ldots\ldots.(5.2)$$

If we want to calculate a magnitude difference for a given brightness ratio, we take the logarithm (base 10) of both sides, giving

$$m_2 - m_1 = 2.5\log_{10}\frac{b_1}{b_2} \quad \text{.........(5.3)}$$

Objects brighter than magnitude 1 can have magnitude 0 or even negative magnitudes.

Example:

<u>Problem</u> *The largest ground-based telescopes extend our range from 6 to 26 mag. What is the brightness ratio?*

<u>Answer</u> *Using equation (5.2), we obtain*

$$b_1/b_2 = 10^{(26-6)/2.5}$$

$$b_1/b_2 = 10^8$$

The color of a star can tell us about the star's temperature. However, we need a way of quantifying a color, rather than just saying something is red, green or blue. For example, if we compare two blue stars, how do we decide which one is bluer? In other words, which one is hotter?

We define two standard wavelength ranges, centered at λ_1 and λ_2 and take the ratio of the observed brightnesses, $b(\lambda_1)/b(\lambda_2)$. We then convert that brightness ratio into a magnitude difference (using equation (5.3)), giving

$$m_2 - m_1 = 2.5\log_{10}\left[\frac{b(\lambda_1)}{b(\lambda_2)}\right] \quad \text{.........(5.4)}$$

We define the quantity $m_2 - m_1$ as the color, measured in magnitudes, corresponding to the wavelength pair, λ_1 and λ_2. For definiteness, let's assume that $\lambda_2 > \lambda_1$. As we increase the temperature, $b(\lambda_1)/b(\lambda_2)$ increases. This means that the quantity $m_2 - m_1$ decreases, since the magnitude scale runs backwards. If we know that an object is radiating exactly like a blackbody, we need only take the ratio of brightnesses at any two wavelengths to determine the temperature.

We don't really measure the intensity of radiation at a wavelength. Instead, we measure the amount of energy received in some wavelength interval. We can control that wavelength interval by using a filter that only passes light in that wavelength range. When we use a filter, we are actually measuring the integral of $I(\lambda, T)$, that is the energy per second per wavelength interval, over some wavelength range. Actually, the situation is more complicated. The transmission of any real filter is not 100% over the selected range.

The wavelength ranges of the various filters are shown in **Table 5.1**. The most commonly discussed filters are U (for ultraviolet) B (for blue) and V (for visible, meaning the center of the visible part of the spectrum). More recently, R (for red), and I (for infrared) have been added (There are actually a couple of filters in different parts of the infrared.).

Filter	Peak wavelength (nm)	Width (nm)[a]
U	350	70
B	435	100
V	555	80
R	680	150
I	800	150

[a]Full width at half maximum.

Table 5.1 Filter system.

For example, the $B - V$ color is defined by $B - V = 2.5 \log_{10} [I(\lambda_V) / I(\lambda_B)] +$ constant, where $I(\lambda_V)$ and $I(\lambda_B)$ are the intensities averaged over the filter ranges (the constant is adjusted so that $B - V$ is zero for a particular temperature star, designated A0). As the temperature of an object increases, the ratio of blue to visible increases. This means that the $B - V$ color decreases (again because the magnitude scale runs backwards.). So we can conclude that for two particular stars, if one star has a value of $B - V < 0$ and the other star with $B - V > 0$, this means that star with $B - V < 0$ is hotter than star with $B - V > 0$.

V.1.2 Stellar distance

So far we have discussed how bright stars appear as seen from Earth. However, the apparent brightness depends on two quantities: *the intrinsic luminosity of the star* and *its distance from us*. Two identical stars at different distances will have different apparent brightnesses. If we want to understand how stars work, we must know their total luminosities. This requires correcting the apparent brightness for the distance to the star.

If we have a star of luminosity L, we can calculate the observed energy flux at a distance d. If no radiation is absorbed along the way, all the energy per second leaving the surface of the star will cross a sphere at a distance d in the same time. It will just be spread over a larger area. Therefore, the energy per second reaching d is still L, but it is spread over an area of $4\pi d^2$ so the energy flux, f, is

$$\mathbf{f} = \frac{\mathbf{L}}{4\pi\mathbf{d}^2} \quad \text{..........(5.5)}$$

Unfortunately, distances to astronomical objects are generally hard to determine. There is a direct method for determining distances to nearby stars. It is called *trigonometric parallax.* The situation is illustrated in **Figure 5.1**. We note the position of the star against the background of distant stars, and then six months later we note the angle by which the position has shifted. If we take half of the value of this angle, we have the *parallax angle, p*.

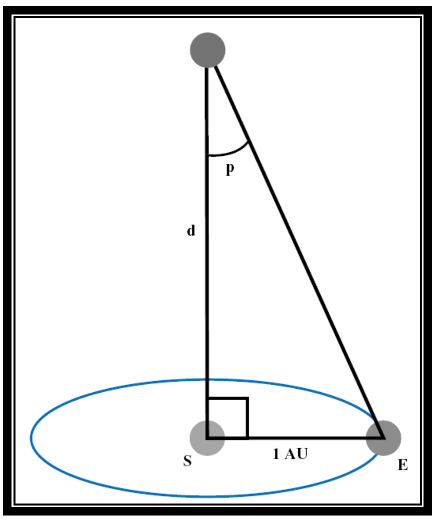

Figure 5.1 Geometry for parallax measurements.The figure is not to scale. In reality the distance to the star, d, is much greater than 1 AU, so the parallax angle, p, would normally be very small.

From the right triangle, we can see that

$$\tan p = \frac{1AU}{d} \quad \text{.........}(5.6)$$

Since p is small, $\tan(p) \approx p$ (rad), which is the value of p, measured in radians. Equation (5.6) then gives us

$$p(rad) = \frac{1AU}{d} \quad \text{.........}(5.7)$$

We define the *parsec* (abbreviated pc) as the distance of a star that produces a parallax angle p of 1 arc sec. We also convert radian to arc seconds since it is more convenient. Knowing that 1 radian equals to 206265 arc seconds (symbolized ") and 1 pc equals to 206265 AU,

$$d(pc) = \frac{1}{p('')} \quad \text{.........}(5.8)$$

With current ground-based equipment, we can measure parallax to within a few hundredths of an arc second. Parallax measurements are therefore useful for the few thousand nearest stars. They are a starting point for a very complex system of determining distances to astronomical objects.

Example:

Problem *The nearest star (Proxima Centauri) has a parallax p = 0.76 arc sec. Find its distance from Earth in parsecs!*

Answer *We use equation (5.8) to give*

$$d = 1/0.76$$
$$d = 1.32 \text{ pc}$$

V.1.3 Stellar absolute magnitude

The magnitudes discussed in Section V.1.1, based on observed energy fluxes, are called *apparent magnitudes*. In order to compare intrinsic luminosities of stars, we define a system of *absolute magnitudes*. The absolute magnitude of a star is that magnitude that it would appear to have as viewed from a standard distance, d_0. This standard distance is chosen to be 10 pc. From this definition, it can be seen that if a star is actually at a distance of 10 pc, the absolute and apparent magnitudes will be the same.

To see how this system works, consider two identical stars, one at a distance d and the other at the standard distance d_0. We let m be the apparent magnitude of the star at distance d, and M that of the star at distance d_0 (Of course, M will be the absolute magnitude for both stars.). The energy flux falls off inversely as the square of the distance, therefore the ratio of the flux of the star at d to that from the star at d_0 is $(d_0/d)^2$. Equation (5.3) then gives us

$$\mathbf{m - M = 2.5\log_{10}\left(\frac{d}{d_0}\right)^2} \quad \text{.........(5.9)}$$

Using the fact that $\log(x^2) = 2\log(x)$ gives

$$\mathbf{m = M + 5\log_{10}\left(\frac{d}{10pc}\right)} \quad \text{.........(5.10)}$$

The quantity $5\log_{10}(d/10\ \text{pc})$, which is equal to $(m - M)$, is called the *distance modulus* of the star. It indicates the amount (in magnitudes) by which distance has dimmed the starlight. If you know any two of the quantities (m, M or d) you can use equation (5.10) to find the third. For any star that we can observe, we can always measure m, its apparent magnitude. Therefore, we are generally faced with knowing M and finding d or knowing d and finding M.

Example:

Problem *A star is at a distance of 100 pc, and its apparent magnitude is +5. What is its absolute magnitude?*

Answer *We use equation (5.10) to find*

$$M = m - 5\log(d/10\ pc)$$
$$M = 5 - 5\log(100\ pc/10\ pc)$$
$$M = 5 - 5\log(10)$$
$$M = 5 - 5$$
$$M = 0$$

We should note that changing the distance of a star changes its apparent magnitude, but it does not change any of its colors. Because colors are defined to be differences in magnitudes, each is changed by the distance modulus. For example, using equation (5.10)

$$\mathbf{m_V = M_V + 5\log\left(\frac{d}{10pc}\right)}$$

$$\mathbf{m_B = M_B + 5\log\left(\frac{d}{10pc}\right)}$$

Taking the difference between filter B and V gives

$$\mathbf{m_B - m_V = M_B - M_V}$$

Therefore, the distance modulus never appears in the colors.

When we talk about determining an absolute magnitude, we are really only determining it over some wavelength range, corresponding to the wavelength range of the observations. We would like to have an absolute magnitude that corresponds to the total luminosity of the star. This magnitude is called the *bolometric magnitude* of the star. For any type of star, we can define a number, called the *bolometric correction* (abbreviated BC), which relates the bolometric magnitude to the absolute visual magnitude M_V. Therefore

$$\mathbf{M_{Bol} = M_V + BC} \quad \text{..........(5.11)}$$

V.1.4 Stellar mass, radius and temperature

The best way to measure the mass of an object is to measure its gravitational influence on another object. For stars, we are fortunate to be able to measure the gravitational effects from pairs of stars, called *binary stars*. Many stars we can observe appear to have companions, the two stars orbiting their common center of mass. It appears that approximately half of all stars in our galaxy are in binary systems. By studying the orbits of binary stars, we can measure the gravitational forces that the two stars exert on each other. This allows us to determine the masses of the stars using

$$\mathbf{\left(\frac{P}{2\pi G}\right)\left(\frac{v_{1r} + v_{2r}}{\sin(i)}\right)^3 = m_1 + m_2} \quad \text{..........(5.12)}$$

In equation (5.12), **P** represents orbital period, v_{1r} and v_{2r} are radial velocity (component of velocity on observer's line of sight) of each components, **i** orbital inclination with respect to celestial plane and **m** represents stellar mass.

Example:

Problem *A binary system is observed to have a period of 10 years. The radial velocities of the two stars are determined to be v_{1r} = 10 km/s and v_{2r} = 20 km/s, respectively. Find the masses of the two stars (a) if the inclination of the orbit is 90^0, and (b) if it is 45^0.*

Answer *Using equation (5.12) we obtain*

$$\frac{m_1 + m_2}{M_\odot} = \frac{(10\text{yr})(3.16\times10^7\text{s/yr})\left[(10+20)(1\times10^3\text{m/s})\right]^3}{2\pi(6.67\times10^{-11}\text{Nm}^2/\text{kg}^2)(2\times10^{30}\text{kg})(\sin^3 i)}$$

$$m_1 + m_2 = {10.2M_\odot}\big/{\sin^3 i}$$

If i = 90^0, $\sin^3 i$ = 1, so
$m_1 + m_2 = 10.2\ M_\odot$

We find the ratio of the masses from the ratio of the radial velocities:
$m_1 / m_2 = v_2 / v_1 = 2.0$

This means that $m_1 = 2m_2$, so
$2m_2 + m_2 = 3m_2 = 10.2\ M_\odot$, giving $m_1 = 6.8\ M_\odot$ and $m_2 = 3.4\ M_\odot$

*If i = 45^0, $1/\sin^3 i = 2.8$. The ratio of the masses does not change, since the **sin i** drops out of the ratio of the radial velocities. This means that we can just multiply each mass by 2.8 to give 19.2 $M_\odot$ and 9.5 $M_\odot$, respectively.*

As a result of studying many binary systems, astronomers have a good idea of the masses of main sequence stars (Stars in which energy is primarily produced from the fusion of hydrogen into helium in their cores.). Just as the Sun's temperature places it in the middle of the main sequence, its mass is in the middle of the range of stellar masses. The lowest mass main sequence stars have about 0.07 of a solar mass, and the most massive stars commonly encountered have about 60 solar masses. When we think of how large or small stars might have turned out to be, the observed range of stellar masses is not very large. This range is an important constraint on theories of stellar structure.

An even more stringent constraint is the relationship between mass and temperature on the main sequence. The cooler stars are less massive and the hotter stars are more massive. We have already said that the existence of the main sequence implies a certain relationship between size and temperature. This means that if a star is on the main sequence, once its mass is specified, its radius and temperature are determined. Another way of looking at this to say that a star's mass determines where on the main sequence it will fall.

Since the mass determines the radius and temperature of a main sequence star, it should not be surprising that it also determines the luminosity. The exact dependence of the luminosity on mass is called the *mass–luminosity relationship*. This relationship is also explainable from theories of stellar structure. We can summarize it by saying that the luminosity varies approximately as some power, α, of the mass. If we express luminosities in terms of solar luminosities, and masses in terms of solar masses, this means that

$$\frac{\mathbf{L}}{\mathbf{L}_{\odot}} = \left(\frac{\mathbf{M}}{\mathbf{M}_{\odot}}\right)^{\alpha} \quad \text{..........(5.13)}$$

In this section also, we will look at various methods for measuring stellar radii. The star whose size is easiest to measure is the Sun. This is actually quite useful. We have seen that the Sun is intermediate in its mass and temperature, so its radius is probably a fairly representative stellar radius. The angular radius of the Sun, $\Delta\theta$, is 16 arc minutes. The Sun is at a distance $d = 1.50 \times 10^8$ km, so its radius, $R_\odot$, is given by

$$\mathbf{R}_{\odot} = \mathbf{d} \times \tan \Delta\theta \quad \text{..........(5.14)}$$

Using the above equation, we obtain the Sun's radius 6.96×10^5 km.

The Sun is the only star whose disk subtends an angle larger than the seeing limitations of ground-based telescopes. We therefore need other techniques for determining radii. If we know the luminosity (from its absolute magnitude) and the surface temperature (from the spectral type) of a star, we can calculate its radius using equation

$$\mathbf{R} = \left(\frac{\mathbf{L}}{\mathbf{4\pi\sigma T_{eff}^4}} \right)^{\frac{1}{2}} \text{……….(5.15)}$$

Eclipsing binaries provide us with another means of determining stellar radii. This method involves analysis of the shape of the light curve and a knowledge of the orbital velocities from Doppler shift measurements (In an eclipsing binary, we don't have to worry about the inclination of the orbit.). Particularly important is the rate at which the light level decreases and increases at the beginning and end of eclipses.

We can also estimate the radii of rotating stars. If there are surface irregularities, such as hot spots or cool spots, the brightness of the star will depend on whether these spots are facing us or are turned away from us. The brightness variations give us the rotation period P. From the broadening of spectral lines, due to the Doppler shift, we can determine the rotation speed v. This speed is equal to the circumference $2\pi R$, divided by the period. Solving for the radius gives

$$\mathbf{R} = \frac{\mathbf{Pv}}{\mathbf{2\pi}} \text{……….(5.16)}$$

Sometimes the Moon passes in front of a star bright enough and close enough for detailed study. An analysis of these *lunar occultations* tells us about the radius of the star. The larger the star is, the longer it takes the light to go from maximum value to zero as the lunar edge passes in front of the star. Actually, since light is a wave, there are diffraction effects as the starlight passes the lunar limb. The light level oscillates as the star disappears. The nature of these oscillations tells us about the radius of the star.

We can understand the relationship between color and temperature by considering objects called *blackbodies*. A blackbody is a theoretical idea that closely approximates many real objects in *thermodynamic equilibrium*. An object is in thermodynamic equilibrium with its surroundings when energy is freely interchanged and a steady state is reached in which there is no *net* energy flow. That is, energy flows in and out at the same rate. A blackbody is an object that absorbs all of the radiation that strikes it.

A blackbody can also emit radiation. In fact, if a blackbody is to maintain a constant temperature, it must radiate energy at the same rate that it absorbs energy. If it radiates less energy than it absorbs, it will heat up. If it radiates more energy than it absorbs, then it will cool. However, this does not mean that the spectrum of emitted radiation must match the spectrum of absorbed radiation. Only the total energies must balance. The spectrum of emitted radiation is determined by the temperature of the blackbody. As the temperature changes, the spectrum changes. The blackbody will adjust its temperature so that its emitted spectrum contains just enough energy to balance the absorbed energy. When the temperature which allows this balance is reached, the blackbody is in equilibrium. **Figure 5.2** shows some sample blackbody spectra.

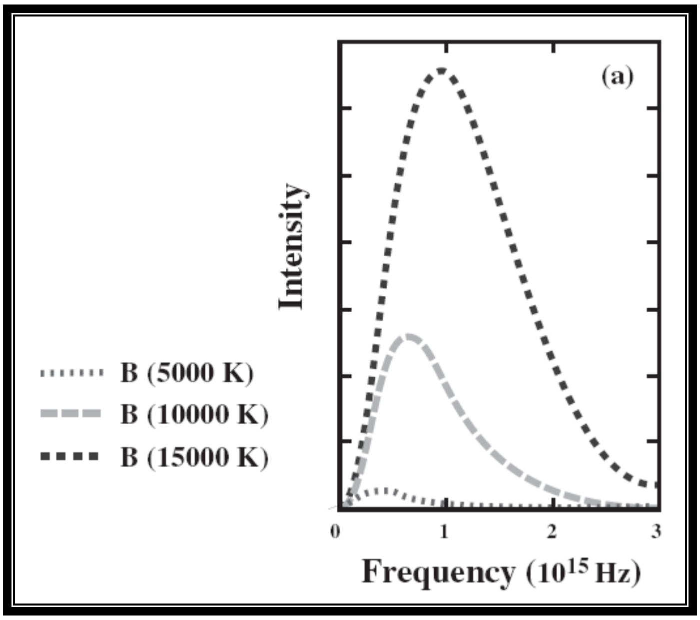

Figure 5.2 Blackbody spectra. Note the shift of the peak wavelength to higher frequency (shorter wavelength) at higher temperature. Note also that, at any frequency, a hotter blackbody gives off more radiation than a cooler one.

If we compare these spectra to those of actual stars, we see that the actual spectra are very much like blackbody spectra. Notice that in any wavelength range, a hotter blackbody gives off more energy than a cooler blackbody of the same size. We also see that as the temperature increases the peak of the spectrum shifts to shorter wavelengths. The relationship between the wavelength at which the peak occurs, λ_{max}, and temperature, T, is very simple. It is given by *Wien's displacement law:*

$$\lambda_{max} T = 2.90 \times 10^{-3}\,mK \quad \ldots\ldots\ldots(5.17)$$

In this law, we must use temperature on an absolute (Kelvin) scale. The temperature on the Kelvin scale is the temperature on the Celsius scale plus 273.1.

Example:

Problem *Find the temperature of an object whose blackbody spectrum peaks in the middle of the visible part of the spectrum, λ = 550 nm!*

Answer *We use equation (5.17) to give*

$$T = 2.9 x 10^{-3}\ mK/[(550\ nm)(1 x 10^{-9}\ m/nm)]$$

$$T = 5270\ K$$

This is close to the temperature of the Sun.

V.1.5 Stellar spectrum

We know that if we pass white light through a prism, light of different colors (wavelengths) will emerge at different angles with respect to the initial beam of light. If we pass white light through a slit before it strikes the prism (**Figure 5.3**), and then let the spread-out light fall on the screen, at each position on the screen we get the image of the slit at a particular wavelength.

Both *William Hyde Wollaston* (1804) and *Josef von Fraunhofer* (1811) used this method to examine sunlight. They found that the normal spectrum was crossed by dark lines. These lines represent wavelengths where there is less radiation than at nearby wavelengths. The lines are only dark in comparison with the nearby bright regions. The linelike appearance comes from the fact that, ateach wavelength, we are seeing the image of the slit. It is this linelike appearance that leads us to call these features *spectral lines*. If we were to make a graph of intensity vs. wavelength, we would find narrow dips superimposed on the continuum. The solar spectrum with dark lines is sometimes referred to as the *Fraunhofer spectrum*. Fraunhofer gave the strongest lines letter designations that we still use today.

The origin of these lines was a mystery for some time. In 1859, the German chemist *Gustav Robert Kirchhoff* noticed a similar phenomenon in the laboratory. He found that when a beam of white light was passed through a tube containing some gas, the spectrum showed dark lines. The gas was absorbing energy in a few specific narrow wavelength bands. In this situation, we refer to the lines as *absorption lines*. When the white light was removed, the spectrum showed bright lines, or *emission lines*, the wavelengths where absorption lines had previously appeared. The gas could emit or absorb energy only in certain wavelength bands.

Kirchhoff found that the wavelengths of the emission or absorption lines depend only on the type of gas that is used. Each element or compound has it own set of special wavelengths. If two elements which don't react chemically are mixed, the spectrum shows the lines of both elements. Thus, the emission or absorption spectrum of an element identifies that element as uniquely as fingerprints identify a person.

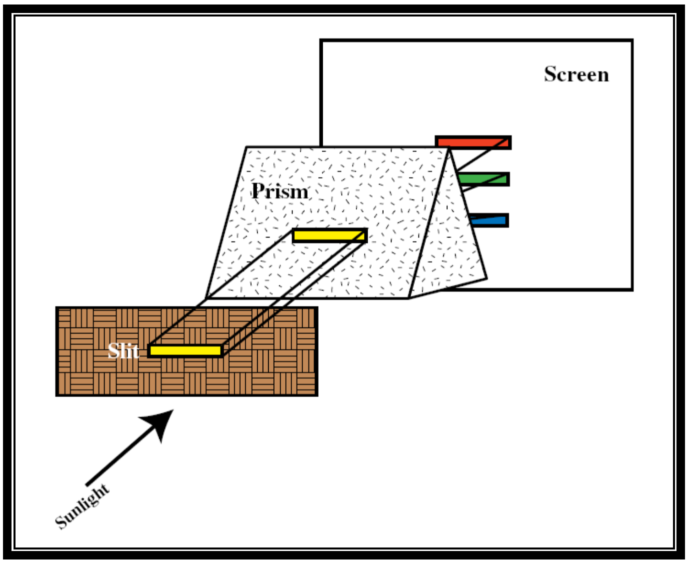

Figure 5.3 If we pass white light through a slit, the beam of light is then spread out as it passes through the prism. On the screen, we are seeing a succession of images of the slit in different colors.

When spectra were taken of stars other than the Sun, they also showed absorption spectra. Astronomers began to classify and catalog the spectra, even though they still did not understand the mechanism for producing the lines. This points out an important general technique in astronomy – studying large numbers of objects to look for general trends. In one very important study, over 200,000 stars were classified by *Annie Jump Cannon* at the Harvard College Observatory.

One set of spectral lines common to many stars was recognized as belonging to the element hydrogen. The stars were classified according to the strongest hydrogen absorption lines. In this system, A stars have the strongest hydrogen lines, B stars the next strongest, and so on. These letter designations were called *spectral classes* or *spectral types*. We now know that the different spectral types correspond to different surface temperatures. However, the sequence A, B, . . . is not a temperature-ordered sequence. For reasons we will discuss below, hydrogen lines are strongest in intermediate temperature stars.

The spectral classes we use, in order of decreasing temperature, are O, B, A, F, G, K, M. We break each of these classes into ten subclasses, identified by a number from zero to nine; for example, the sequence O7, O8, O9, B0, B1, B2, . . , B9, A0, A1, ... (For O stars the few hottest subclasses are not used.). For some of the hotter spectral types, we even use half subclasses, for example, B1.5. It was originally thought that stars became cooler as they evolved, so that the temperature sequence was really an evolutionary sequence. Therefore, the hotter spectral types were called *early* and the cooler spectral types were called *late*. We now know that these evolutionary ideas are not correct. However, the nomenclature still remains. We even talk about a B0 or B1 star being 'early B' and a B8 or B9 as being a 'late B'.

We now look at the properties of different spectral types, in order of increasing temperature. Sample spectra are shown in **Figure 5.4**.

M	*Temperatures in M stars are below 3,500 K, explaining their red color. The temperature is not high enough to produce strong H absorption, but some lines from neutral metals are seen. The stars are cool enough for simple molecules to form, and many lines are seen from molecules such as CN (cyanogen) and TiO (titanium oxide). If cool stars show strong CH lines, we designate them as C-type or 'carbon stars'. If any M star has strong ZrO (zirconium oxide) lines as opposed to TiO lines, we call it an S-type.*
K	*Temperatures range from 3,500 to 5,000 K. There are many lines from neutral metals. The H lines are stronger than in M stars but most of the H is still in the ground state.*
G	*Temperatures in the range 5,000–6,000 K. The Sun is a G2 star. The H lines are stronger than in K stars, as more atoms are in excited states. The temperature is high enough for metals with low ionization energies to be partially ionized. Two prominent lines are from Ca(II). When Fraunhofer studied the solar spectrum, he gave the strongest lines letter designations. These Ca(II) lines are the H and K lines in his sequence.*
F	*Temperatures range from 6,000 to 7,500 K. The H lines are a little stronger than in G stars. The ionized metal lines are also stronger.*
A	*Temperatures range from 7,500 to 10,000 K. These stars are white–blue in color. They have the strongest H lines. Lines of ionized metals are still present.*
B	*Temperatures are in the range 10,000–30,000 K, and the stars appear blue. The H lines are beginning to weaken because the temperatures are high enough to ionize a significant fraction of the hydrogen. The lines of neutral and singly ionized helium begin to appear.*
O	*Temperatures range from 30,000 to over 60,000 K, and the stars appear blue. The earliest spectral types that have been seen are O3 stars and there are very few O3 and O4 stars. The hydrogen lines fall off very sharply because of the high rate of ionization. The lines of singly ionized helium are still present, but there are very few lines overall in the visible part of the spectrum. There are several lines in the ultraviolet.*

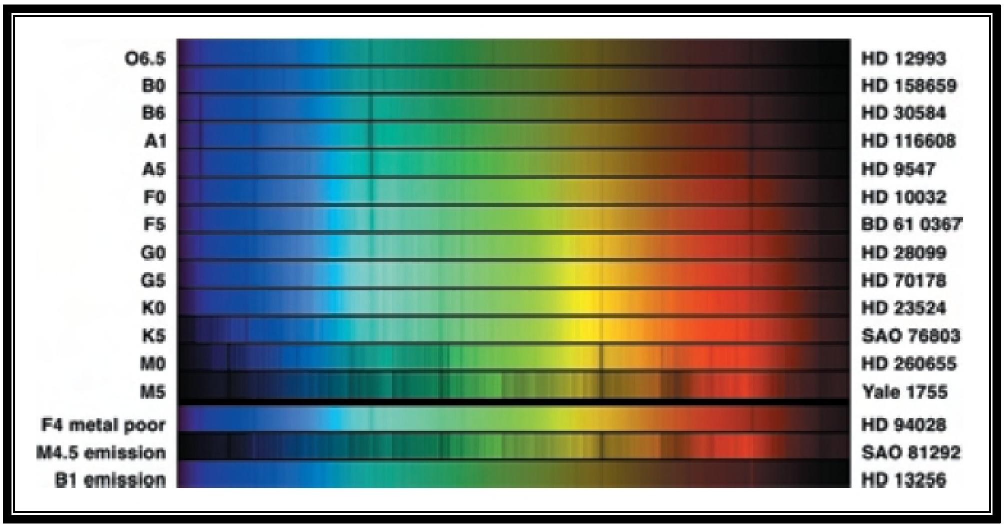

Figure 5.4 Samples of spectra from stars of different spectral types.The name of the star appears on the right of each spectrum, and the spectral type appears on the left. In each spectrum, the wavelength increases from left to right. Hotter stars are at the top.

V.2 Hertzsprung–Russell Diagram

V.2.1 Luminosity class

Even though we cannot study any one star (except for the Sun) in great detail, we can compensate somewhat by having a large number of stars to study. From statistical studies we learn about general trends. For example, if we find that brighter stars tend to be both hotter and larger, then any theory of stellar structure would have to explain that trend.

One of the earliest statistical studies was carried out in 1910 independently by the Danish astronomer *Ejnar Hertzprung*, and the American astronomer *Henry Norris Russell*. They plotted the properties of stars on a diagram in which the horizontal axis is some measure of temperature (e.g. color or spectral type) and the vertical axis is some measure of luminosity. We call such a diagram a *Hertzprung–Russell diagram,* or simply an *HR diagram*.

An HR diagram for over 40,000 nearby stars is shown in **Figure 5.5**. These stars were studied by the Hipparcos satellite, which was designed to measure trigonometric parallaxes, so distances to these stars are well known. Most of the stars are found in a narrow band, called the *main sequence*. The significance of the main sequence is that most stars of the same temperature have essentially the same luminosity, and hence essentially the same size. This close relationship between size and temperature must be a result of the laws of physics as applied to stars. It gives us hope that we can understand stellar structure by applying the known laws. It also gives us a crucial test: any theory of stellar structure must predict the existence of the main sequence.

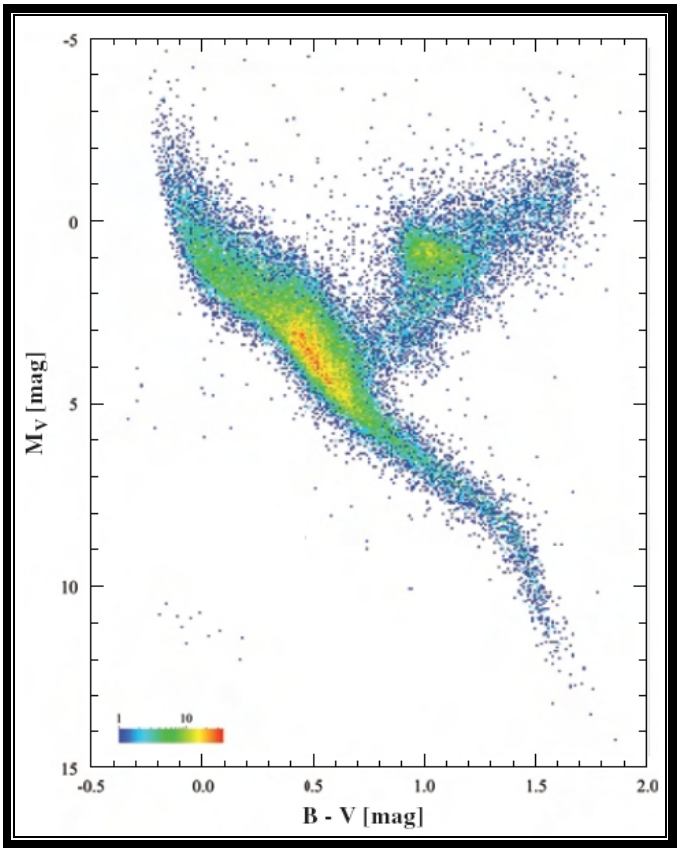

Figure 5.5 HR diagram for over 40,000 nearby stars studied by the Hipparcos satellite. In this figure, the color represents the number of stars in each category, with red being the most and blue being the least.

Not all stars appear on the main sequence. Some appear above the main sequence. This means that they are more luminous than main sequence stars of the same temperature. If two stars have the same temperature but one is more luminous, it must be larger than the other. Stars appearing above the main sequence are therefore larger than main sequence stars. We call these stars *giants*. By contrast, we call the main sequence stars *dwarfs*. We subdivide the giants into three groups: *subgiants, giants, supergiants*.

To keep track of the size of a star of a given spectral type, we append a *luminosity class* to the spectral type (see **Figure 5.6** below). The luminosity class is denoted by a roman numeral. Main sequence stars are luminosity class V. The Sun, for example, is a G2 V star. Subgiants are luminosity class IV, giants are luminosity class III. Luminosity class II stars are somewhere between giants and supergiants. Supergiants are luminosity class I. We further divide supergiants into Ia and Ib, with Ia being larger. When we look at the spectral lines from a star we can actually tell something about the size. Stars of different sizes will have different accelerations of gravity near their surface. The surface gravity affects the detailed appearance of certain spectral lines.

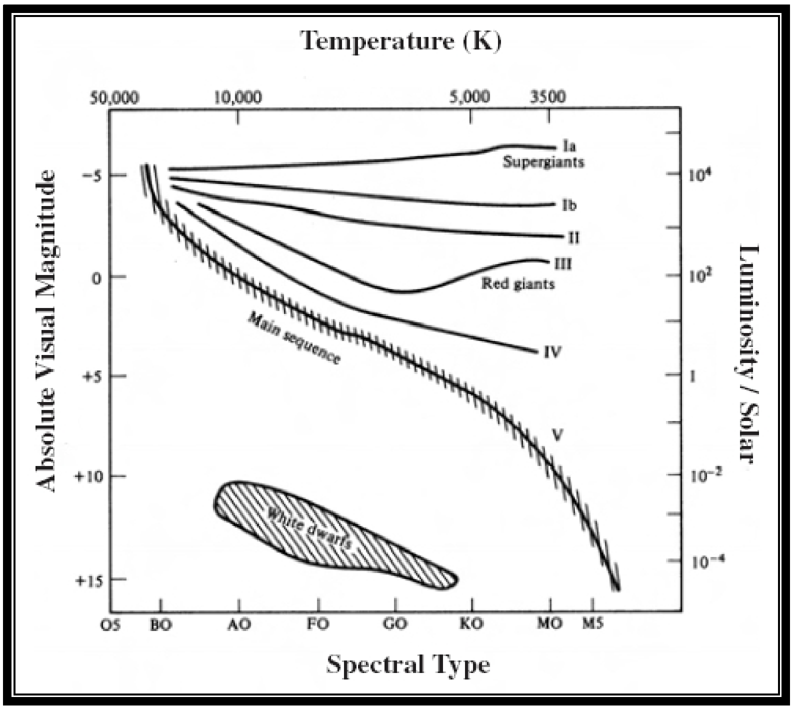

Figure 5.6 A schematic HR diagram, showing the main features of the actual diagrams. Luminosity classes are indicated by roman numerals.

There are also stars that appear below the main sequence. These stars are typically 10 magnitudes fainter than main sequence stars of the same temperature. They are clearly much smaller than main sequence stars. Since most of these are in the middle spectral types, and therefore appear white, we refer to them as *white* *dwarfs*. Do not confuse dwarfs, which are main sequence stars, with white dwarfs, which are much smaller than ordinary dwarfs.

Example:

Problem *Suppose that a white dwarf has the same spectral type as the Sun, but has an absolute magnitude that is 10 mag fainter than the Sun. What is the ratio of the radius of the white dwarf, R_{wd}, to that of the Sun, $R_{\odot}$?*

Answer *Using equation (5.2) but now for absolute magnitude, we obtain*

$$\frac{L_{wd}}{L_{\odot}} = 10^{\frac{M_{\odot}-M_{wd}}{2.5}}$$

$$\frac{L_{wd}}{L_{\odot}} = 10^{\frac{-10}{2.5}} = 10^{-4}$$

Having the same spectral type means both stars have the same temperature

From equation (5.15), we obtain

$$\frac{R_{wd}}{R_{\odot}} = \left(\frac{L_{wd}}{L_{\odot}}\right)^{\frac{1}{2}}$$

$$\frac{R_{wd}}{R_{\odot}} = \left(10^{-4}\right)^{\frac{1}{2}} = 10^{-2}$$

The radius of a white dwarf is 1% of the radius of the Sun!

II.2.2 Hertzsprung–russell diagram of stellar cluster

For any cluster for which we plot an HR diagram, we only know the apparent magnitudes, not the absolute magnitudes. If we know the absolute magnitude for one spectral type, then we can find the distance modulus for stars of that spectral type in the cluster. The distance modulus is the same for all the stars in the cluster, so we can calibrate the whole HR diagram in terms of absolute magnitudes. To obtain a reliable calibration, we would like to carry it out for many stars. We have already seen that there is a growing group of nearby stars for which trigonometric parallax can give us a good distance measurement.

Once we know the absolute magnitude for a given spectral type, we have a very useful way of determining distances. For any given star, we measure m, the apparent magnitude. We take a spectrum of the star to determine its spectral type. From the spectral type we know the absolute magnitude, M. Since we know m and M, we know the distance modulus, $m - M$, and therefore the distance. This procedure is called *spectroscopic parallax*. The word 'spectroscopic' refers to the fact that we use the star's spectrum to determine its absolute magnitude. The word 'parallax' refers to the fact that this is a distance measurement (just as trigonometric parallax was a distance measurement using triangulation).

Example:

Problem *For a B0 star (M = –3), we observe an apparent magnitude m = 10. What is the distance to the star, d?*

Answer *We use equation (5.10) to find the distance*

$$\mathbf{m - M = 5\log_{10}\left(\frac{d}{10pc}\right)}$$

Solving for d gives d = 4,000 pc

II.3 Stellar Evolution

II.3.1 Energy generation

When a star is on the main sequence, its basic source of energy is the conversion of hydrogen into helium. We start with four protons and end up with one ^{4}He nucleus. However, it is unlikely that four protons will get close enough to directly form a ^{4}He nucleus in a single reaction. There are different series of reactions that achieve this net result, and they will be discussed below.

We can calculate the energy released by converting four protons to one ^{4}He by comparing their masses. We find that

$$4m_p - m\left(^{4}He\right) = 0.007\left(4m_p\right) \quad \text{.........(5.18)}$$

This means that 0.007 of the mass of each proton is converted into energy.

If we assume that most of the mass of the Sun was originally in the form of protons, then 0.007 of the Sun's total mass is available for conversion into energy. The total energy available is therefore

$$E = 0.007 M_{\odot} c^2 \quad \text{.........(5.19)}$$

Example:

<u>Problem</u> *If only 10% of the mass of the Sun is in a region hot enough for nuclear reactions (the core), estimate the lifetime of the Sun for producing energy at its current rate from nuclear fusion!*

<u>Answer</u> *We use equation (5.19) to obtain the total energy*

$$E = 0.007\left(10\% \times 2.0\times10^{30}\,kg\right)\left(3.0\times10^{8}\,m/s\right)^2$$
$$E = 1.3\times10^{44}\,Joule$$

The lifetime is this energy divided by the luminosity:

$$t = \frac{E}{L} = \frac{1.3\times10^{44}\,J}{4.0\times10^{26}\,J/s}$$
$$t = 3.3\times10^{17}\,s = 1.0\times10^{10}\,years$$

The Sun has already lived half of this time

II.3.2 Evolution off the main sequence: low and high mass stars

We first look at stars whose mass is less than about 5 M⊙. Eventually a star will reach the point where all the hydrogen in the core has been converted to helium. For a low mass star, the central temperature will not be high enough for the helium to fuse into heavier elements. There is still a lot of hydrogen outside the core, but the temperature is not high enough for nuclear reactions to take place. The core begins to contract, converting gravitational potential energy into kinetic energy, resulting in a heating of the core. The hydrogen just outside the core is heated to the point where it can fuse to form helium, and this takes place in a shell at the outer edge of the core (see **Figure 5.7**).

We refer to this as a *hydrogen-burning shell*, where the word "burning" refers to nuclear reactions, rather then chemical burning. As the core contracts, the rate of energy generation in the shell increases. The shell can easily give off energy at a greater rate than the core did during the star's normal lifetime.

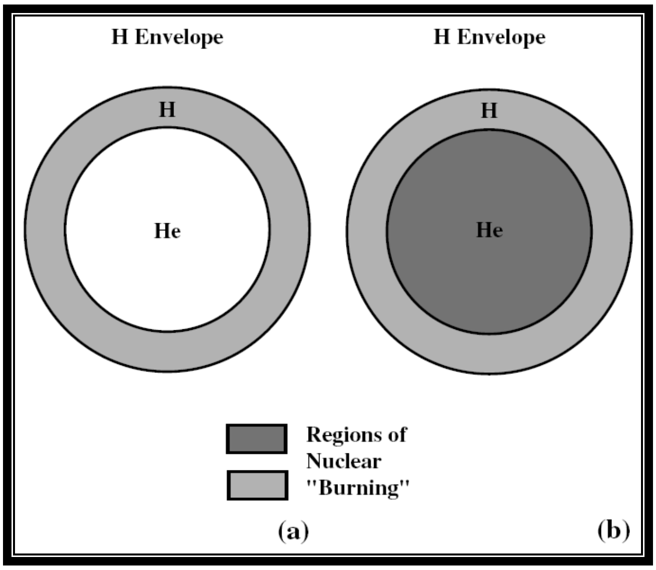

Figure 5.7 Star with an H-burning shell. (a) The temperature in the star is not hot enough to fuse the helium in the center, but is hot enough to keep the H in the shell burning. (b) In this star, the temperature is hot enough to keep both burning.

While all of this is happening in the interior, the outer layers of the star are changing. Energy transport from the core is radiative, and is limited by the rate at which photons can diffuse through the star. The outer layers of the star become hotter and expand. As the gas expands, it cools. The star's radius has increased, but its temperature has decreased, so the luminosity increases slightly. The behavior of the star's track on the HR diagram is shown in **Figure 5.8**. The track moves to the right (cooler), and the star appears as a subgiant.

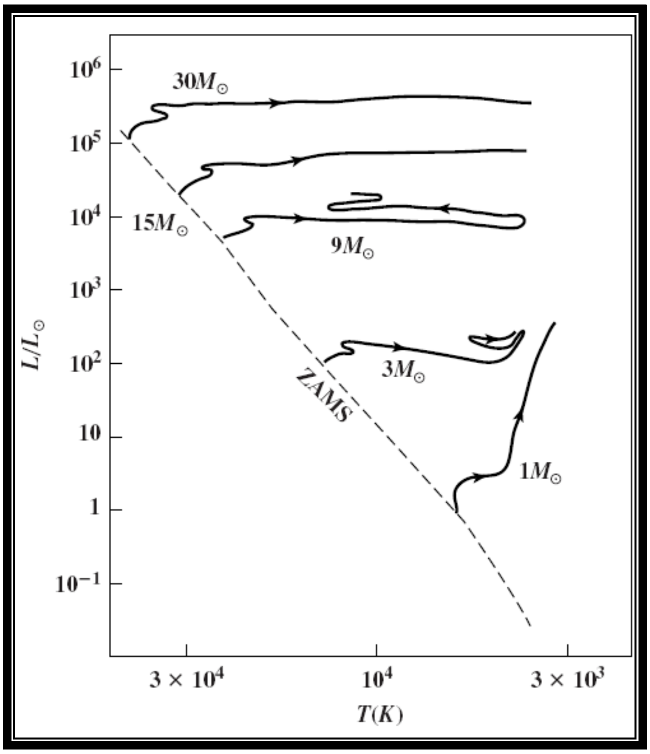

Figure 5.8 Evolutionary tracks away from the main sequence on an HR diagram. Each track is marked by the mass for the model.The dashed line is the zero-age main sequence (ZAMS).

There is a mechanism that keeps the surface temperature from becoming too low. The rate of photon diffusion increases as the absolute value of gradient temperature increases. Remember, gradient temperature is negative, so we are saying that the greater the temperature difference between some point on the inside and the surface, the greater the energy flow between those two points. If the surface temperature of the star falls too much, the photon diffusion is faster, delivering more energy to the surface, raising the surface temperature. Therefore, as the radius continues to increase, the surface temperatureremains approximately constant. The luminosity therefore increases, and the evolutionary track moves vertically. The star is then a *red giant*.

We now look at the evolution of the core while the star is becoming a red giant. The temperature of the core climbs to 10^8 K. This is hot enough for the triple-alpha process to take place, fusing the helium into carbon. The density is so high that the material no longer behaves like an ideal gas. This is called a *degenerate gas*. In an ideal gas, when the triple-alpha process starts, the extra energy generated causes an increase in pressure, which causes the gas to expand, slowing the reaction rate. This keeps the reactions going slowly. In a degenerate gas the pressure doesn't depend on temperature and no such safety valve exists. The conversion of helium to carbon takes place very quickly. We call this sudden release of energy the *helium flash*. The energy released causes a brief increase in stellar luminosity.

Following the helium flash the energy production decreases. The core is no longer degenerate, and steady fusion of helium to carbon takes place. This region is surrounded by a shell in which hydrogen is still being converted into helium. At this point the star reaches the horizontal branch on the HR diagram. The outer layers of the star are weakly held to the star, since they are so far from the center. The star begins to undergo mass loss. The subsequent evolution depends on the amount of mass that is lost.

Eventually all the helium in the core is converted into carbon and oxygen. The temperature is not high enough for further fusion, and the core again begins to contract. A helium-burning shell develops, and the rate of energy production again increases. The envelope of the star again expands. On the HR diagram the evolutionary track ascends the giant branch again, reaching what is called the *asymptotic giant branch*. Stars on the asymptotic giant branch are more luminous than red giants. The star can briefly become large enough to become a red supergiant at this stage. The star can also undergo oscillations in the rate of nuclear energy generation.

We have already said that the outer layers of a red giant are held together very weakly. Remember, the gravitational force on a mass m in the outer layer is GmM/R^2, where M is the mass of the star and R is its radius. As the star expands, M stays constant, so the pull on the outer layer falls off as $1/R^2$. Since the outer layer is weakly held, it is subject to being driven away. The actual mechanism for driving material away is still not fully understood. It may involve pressure waves moving radially outward. It may also involve radiation pressure. Photons carry energy and momentum (Remember, the momentum of a photon of energy E is E/c.). When photons from inside the star strike the gas in the outer layers, and are absorbed, their momentum is also absorbed. By conservation of momentum, the shell must move slightly outward. We do observe shells that are ejected. They are fuzzy in appearance in small telescopes, just like planets; when originally observed, they were called *planetary nebulae.*

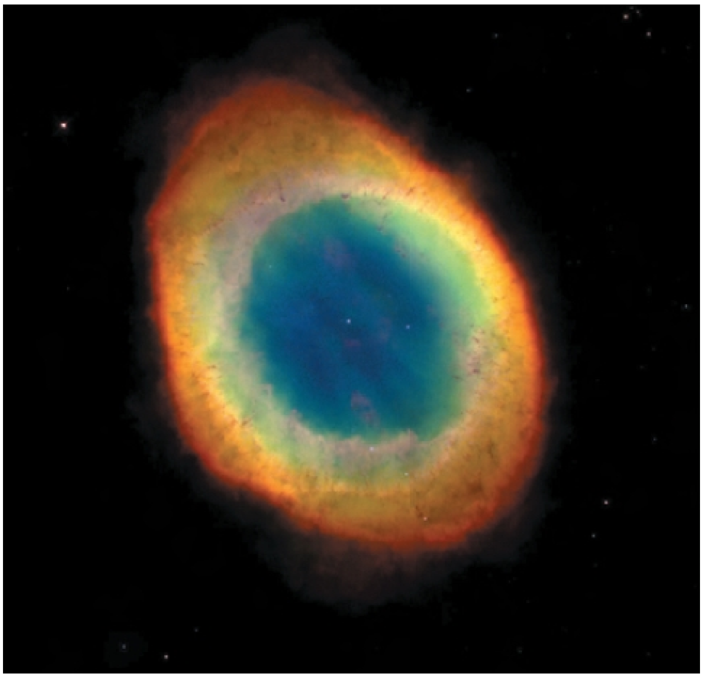

Figure 5.9 HST image of planetary nebula, the Ring Nebula (M57), in the constellation Lyra. It is at a distance of 1 kpc, and is about 0.3 pc across. In its center there is a white dwarf.

The material left behind after the planetary nebula is ejected is the remnant of the core of the star. It is mostly carbon or oxygen, and its temperature is not high enough for further nuclear fusion to take place. The gas pressure is not high enough to support the star against gravitational collapse. This collapse would continue forever if not for an additional source of pressure when a high enough density is reached. This pressure arises from *electron degeneracy*. A star supported by electron degeneracy pressure will be quite small, since it must collapse to a high density before the degeneracy pressure is high enough to stop the collapse. These objects are quite hot, being the remnant of the core of a star. These objects are the stars that appear on the HR diagram as *white dwarfs*.

More massive stars live a shorter lifetime on the main sequence than do lower mass stars. As with the lower mass stars, the main sequence lifetime for higher mass stars ends when the hydrogen in the core is used up. The core then begins to contract, and the temperature for helium fusion to heavier elements is quickly reached. The helium fusion takes place before the core can become degenerate. Therefore, in contrast with the helium flash in lower mass stars, the helium burning in more massive stars takes place steadily. At this point, the star has a helium-burning core with a hydrogen-burning shell around it (**Figure 5.10**).

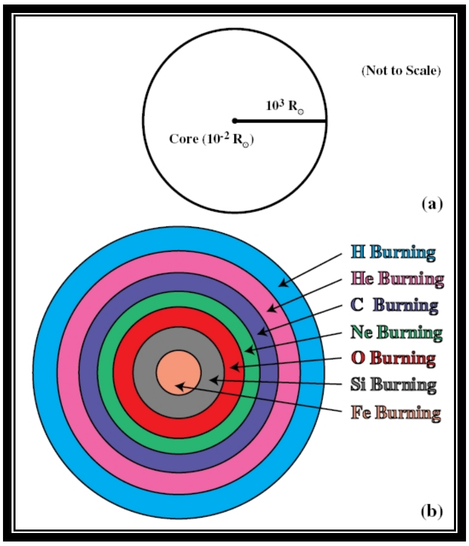

Figure 5.10 Shells in the core of a high mass star as it evolves away from the main sequence. (a) The core is only a small fraction of the total radius. (b) In the core, there is a succession of shells of different composition. Each shell has exhausted the fuels that are still burning in shells farther out.

When the helium in the core is exhausted, the temperature is high enough for the carbon and oxygen to fuse into even heavier elements. At this time, we have a carbon- and oxygen-burning core, surrounded by a helium-burning shell, which in turn is surrounded by a hydrogen-burning shell. As heavier elements are built up, the core develops more layers.

As the luminosity of the core increases, the outer layers of the star expand. The atmosphere cools with the expansion, but the size increases sufficiently for the luminosity to increase. At this point the envelope is convective, and the temperature gradient is limited by the adiabatic lapse rate. So the envelope must grow to a large size to accommodate the large temperature difference between the core and the surface. Eventually, the radius of the star reaches about $10^3\ R_\odot$. At this point the star is called a *red supergiant*.

In the core of a high mass star the buildup of heavier elements continues. The isotope of iron ^{56}Fe has the highest binding energy per nucleon. This makes it the most stable nucleus. This means that any reaction involving ^{56}Fe, be it fission or fusion, requires an input of energy. When all of the mass of the core of the star is converted to ^{56}Fe (and other stable elements, such as nickel), nuclear reactions in the core will stop.

At this stage, the core will start to cool and the thermal pressure will not be sufficient to support the core. As long as the mass of iron in the core is less than the *Chandrasekhar limit* ($< 1.44\ M_\odot$), the core can be supported by electron degeneracy pressure. However, once the core goes beyond that limit, there is nothing to support it, and it collapses. In the collapse, some energy, previously in the form of gravitational potential energy, is liberated. As the iron is destroyed, protons are liberated from nuclei. The electrons in the star can combine with these protons to form neutrons and neutrinos.

The core is driven to a very dense state in a short time, about one second. What happens next is not completely understood, but the collapse results in an explosion in which most of the mass of the star is blown away. The neutrons created probably play a role in this. They also obey the exclusion principle, and exert a degeneracy pressure. This pressure can stop the collapse and cause the material to bounce back. In addition, so many neutrinos are created, and the material is so dense, that a sufficient number of neutrinos interact with the matter forcing the material outward. Such an exploding star is called a *supernova*. This type of supernova is actually called a *type II supernova*. Another type of supernova, type I, seems to be associated with older objects in our galaxy (the mechanism for type I supernovae probably involves white dwarfs in close binary systems). During the explosion, nuclear reactions take place very rapidly, and elements much heavier than iron are created. This material is then spread out into interstellar space, along with the results of the normal nucleosynthesis during the main sequence life of the star. This enriched material is then incorporated into the next generation of stars.

The core is compressed so that normal gas pressure cannot support it. If the mass is more than 1.44 $M_\odot$ electron degeneracy pressure cannot support it. The collapse of the core continues beyond even the high densities associated with a white dwarf. As the density increases, electrons and protons are forced together to make neutrons. The resulting object is called a *neutron star*. The material thrown out in a supernova explosion is called a *supernova remnant*. It contains most of the material that was once the star. In young supernova remnants we can actually see the expansion of the ejected material. These remnants are important because they spread the products of nucleosynthesis in stars throughout the interstellar medium. There, this material enriched in "metals" will be incorporated into the next generation of stars. This explains why stars that formed relatively recently in the history of our galaxy have a higher metal abundance than the older stars. In the later stages of a supernova remnant's expansion, we still see a glowing shell, like those in **Figure 5.11**.

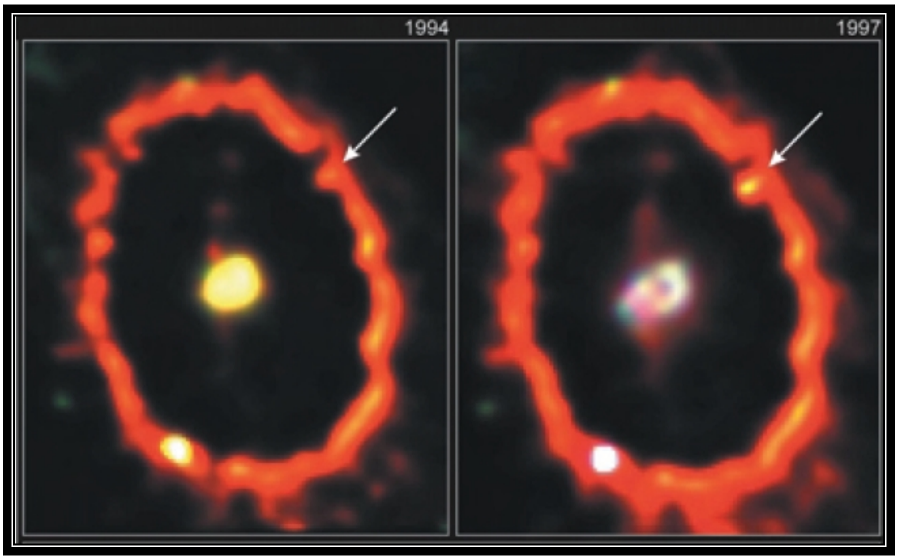

Figure 5.11 HST image of the region of SN1987A in the galaxy Large Magellanic Cloud. The small bright ring shows the interaction of the expanding supernova remnant with the surrounding medium.

Example:

Problem *Estimate the density of a neutron star and compare it with that of a neutron. Take the mass and radius of the star to be 1.4 M⊙ and 15 km respectively.*

Answer *The density of the star is the mass divided by the volume:*

$$\rho = \frac{(1.4)\left(2\times10^{30}\,\mathrm{kg}\right)}{\left(4\pi/3\right)\left(1.5\times10^{4}\,\mathrm{m}\right)^{3}} = 2\times10^{17}\,\mathrm{kgm^{-3}}$$

The density of a neutron is

$$\rho_{\mathrm{neutron}} = \frac{\left(1.7\times10^{-27}\,\mathrm{kg}\right)}{\left(4\pi/3\right)\left(1.0\times10^{-15}\,\mathrm{m}\right)^{3}} = 4\times10^{17}\,\mathrm{kgm^{-3}}$$

We see that the density of a neutron star is very close to that of a neutron. This means that the neutrons in a neutron star must be packed very close together, with very little empty space.

Astronomy Laboratory

The Hertzsprung – Russell Diagram

Procedure:

1. Use the table below to plot a selection of stars from 47 Tucanae on the color magnitude diagram. The stars have been broken into three separate lists of stars. In graph, plotting color index (B – V) along the bottom (x) axis and absolute magnitude along the vertical (y) axis. *Do not forget to format the vertical axis value in reverse order!*

2. Plot your HR diagram of the stars in Orion. In graph, plotting spectral type (or temperature) along the bottom (x) axis and luminosity (***not luminosity class!***) along the vertical (y) axis. You can use the sun's luminosity value as a unit of luminosity if you wish.

3. If you need other informations related to physical parameter of the stars (e.g. temperature etc) refer to this website: ***http://simbad.u-strasbg.fr/simbad/*** *or* ***http://www.wikipedia.org***

4. What is your conclusion about the two graphs you have plotted according to their trend of data? How are they similar? How are they different?

Lab Skills and Objectives

- **Construct the Hertzsprung – Russell diagram**
- **Compare and contrast the Hertzsprung – Russell diagram of globular cluster with stars in Orion**

47 Tucanae Star Data Table					
Star List #1		Star List #2		Star List #3	
Absolute Magnitude (M_V)	Color Index (B-V)	Absolute Magnitude (M_V)	Color Index (B-V)	Absolute Magnitude (M_V)	Color Index (B-V)
+1.1	+0.87	−1.5	+1.30	+1.7	+0.06
+1.3	+0.96	−1.4	+1.85	+1.8	+0.31
+1.3	+0.78	−1.3	+1.38	+2.2	+0.28
+1.5	+0.86	−1.3	+1.70	+2.4	+0.43
+1.7	+0.82	−1.2	+1.18	+2.5	+0.34
+2.0	+0.86	−1.1	+1.13	+2.5	+0.47
+2.1	+0.76	−1.0	+1.05	+2.7	+0.38
+2.3	+0.70	−1.0	+1.26	+2.7	+0.13
+2.4	+0.83	−0.9	+1.16	+2.7	+0.41
+2.7	+0.82	−0.8	+1.08	+2.8	+0.33
+2.8	+0.75	−0.7	+1.12	+2.9	+0.41
+2.9	+0.69	−0.6	+1.32	+3.0	+0.45
+3.0	+0.84	−0.4	+1.08	+3.1	+0.11
+3.1	+0.73	−0.1	+1.09	+3.2	+0.41
+3.2	+0.77	+0.2	+1.05	+3.2	+0.47
+3.2	+0.58	+0.6	+1.01	+3.2	+0.38

Source: Guhathakurta, P. *et al*, *The Astronomical Journal*, **104**, p. 1790

Name	Designation	m (apparent)	M (absolute)	Spectral Type	Luminosity Class
Betelgeuse	α Ori	.80	-5.6	M2	Iab
Rigel	β Ori	.14	-7.1	B8	Ia
Bellatrix	γ Ori	1.64	-3.6	B2	III
Mintaka	δ Ori	2.20	-6.0	O9	II
Alnilam	ε Ori	1.70	-6.2	B0	Ia
Alnitak	ζ Ori	2.05	-6.1	O9	Ib
	η Ori	3.35	-4.4	B0	V
Trapezium A	θ^1 Ori A	6.77	-4.4	B1	III?
Trapezium B	θ^1 Ori B	8.1	-2.9	B3	III?
Trapezium C	θ^1 Ori C	5.16	-5.7	O6	V
Trapezium D	θ^1 Ori D	6.72	-4.4	B1	III?
	θ^2 Ori	5.07	-4.8	O9	V
Hatysa	ι Ori	2.77	-5.7	O9	III
Saiph	κ Ori	2.04	-6.6	B1	Ic
Heka	λ Ori	3.66	-5.3	O8	V
	μ Ori	4.12	+0.5	A0	V
	ν Ori	4.42	-1.7	B3	V
	ξ Ori	4.38	-1.7	B3	V
	π^1 Ori	4.66	+1.0	A0	V
	π^2 Ori	4.32	+1.0	A0	V
	π^3 Ori	3.19	+3.5	F6	V
	π^4 Ori	3.69	-3.6	B2	III
	π^5 Ori	3.72	-3.6	B2	III
	π^6 Ori	4.46	-2.3	K2	II
	σ Ori	3.75	-4.8	O9	V
	τ Ori	3.59	-2.2	B5	III
	ϕ^1 Ori	4.41	-4.8	B0	IV
	ϕ^2 Ori	4.09	+0.4	G8	III
	χ^1Ori	4.41	+4.4	G0	V
	χ^2Ori	4.63	-6.8	B2	Ia