



KINEMATIKA GELOMBANG

TOPIK 2

KULIAH GELOMBANG OPTIK

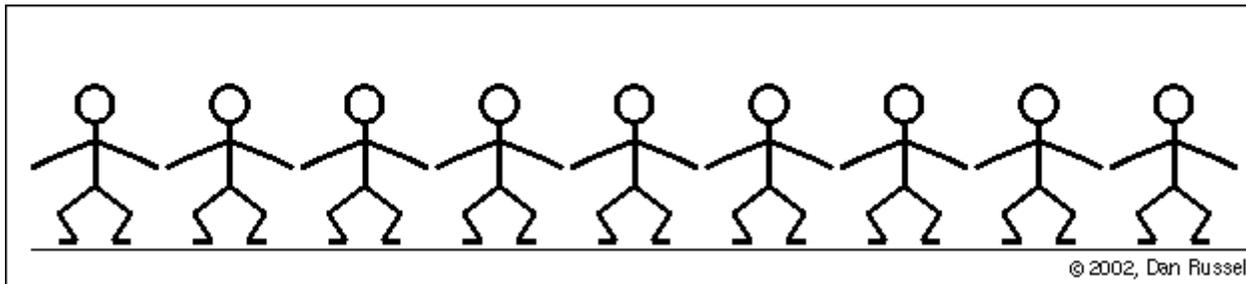
ANDHY SETIAWAN

SUB POKOK BAHASAN

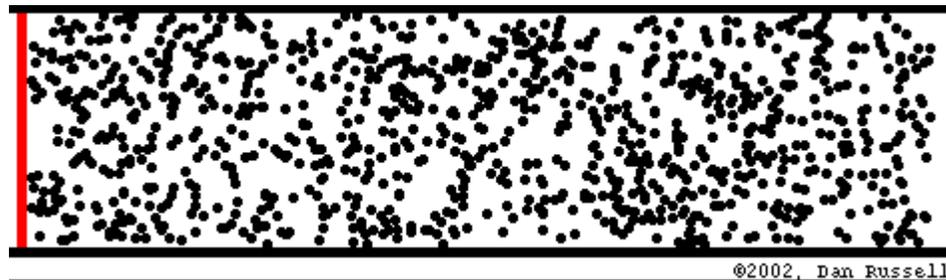
- PERSAMAAN DIFFERENSIAL GELOMBANG
- SOLUSI PERSAMAAN GELOMBANG
- SUPERPOSISI DUA GELOMBANG

PENGANTAR

ILUSTRASI PERAMBATAN PULSA

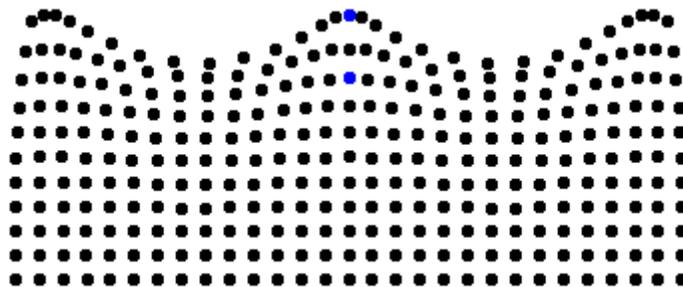
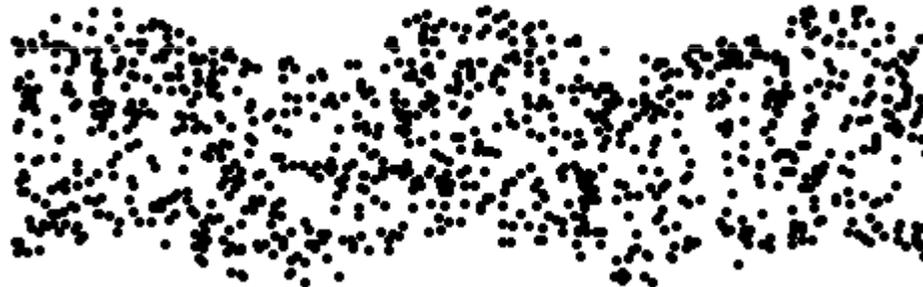
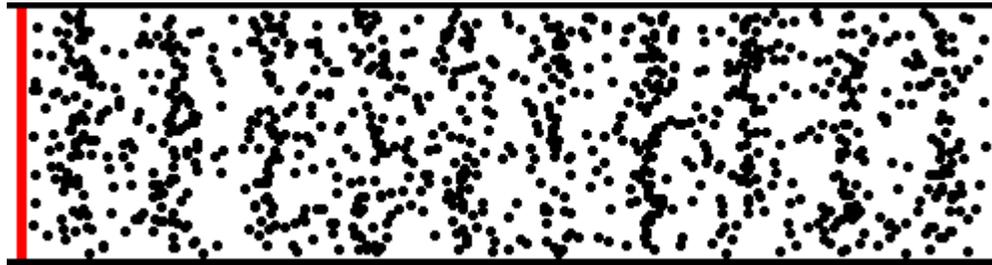


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PENGANTAR

ILUSTRASI PERAMBATAN GELOMBANG



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PERSAMAAN DIFFERENSIAL GELOMBANG arah rambat dan sudut fase

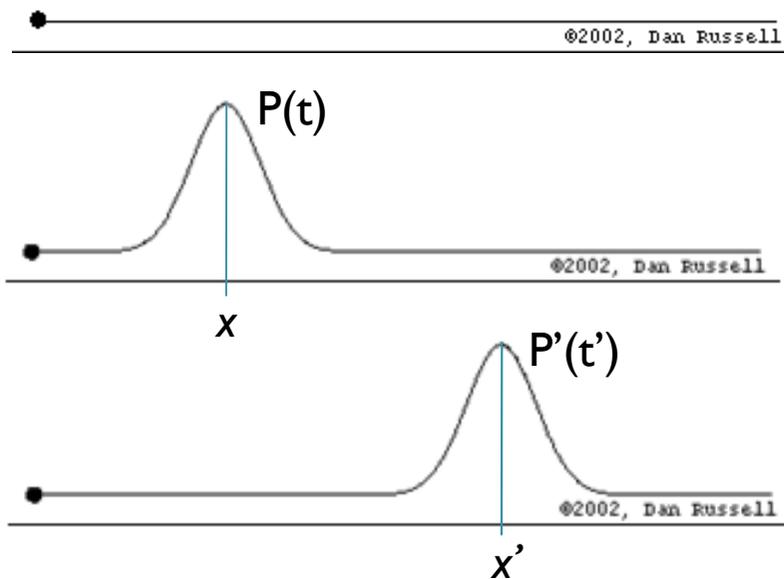
- Sistem osilasi $\psi(t)$
- fungsi gelombang $\psi(x, t)$ atau $\psi(r, t)$
- Tinjau: merambat arah x , kecepatan konstan v . $\rightarrow \psi(x, t) = f(x \pm vt)$

$$\psi(x, t) = f(\phi), \text{ dengan } \phi = x \pm vt$$

$\phi =$ sudut fase

PERSAMAAN DIFFERENSIAL GELOMBANG

arah rambat dan sudut fase



Sudut fase titik P : $\phi = x-vt$

Setelah t' : $\phi' = x'-vt'$

$$\phi = \phi'$$

$$x-vt = x'-vt'$$

$$x-vt = x+\Delta x - v(t+\Delta t)$$

$$0 = \Delta x - v \Delta t$$

$$\Delta x = v \Delta t$$

Maka $\Delta x > 0$, sehingga :

sudut fase $\phi = x-vt \rightarrow$ arah rambat ke kanan

sudut fase $\phi = x+vt \rightarrow$ arah rambat ke kiri (**coba buktikan**)

PERSAMAAN DIFFERENSIAL GELOMBANG

penurunan persamaan

- $\phi = x \pm vt$ konstan \rightarrow kedudukan setiap titik yang sama

Kecepatan fase

$$\frac{d\phi}{dt} = 0 \rightarrow \frac{d(x \pm vt)}{dt} = 0 \rightarrow \frac{dx}{dt} \pm v = 0 \rightarrow v = \mp \frac{dx}{dt}$$

- Perubahan fungsi terhadap x dan t

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial \phi} \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial \phi} \rightarrow \frac{\partial \psi}{\partial \phi} = \frac{\partial \psi}{\partial x} \rightarrow \frac{\partial \psi}{\partial x} \mp \frac{1}{v} \frac{\partial \psi}{\partial t} = 0$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial \phi} \frac{\partial \phi}{\partial t} = \pm v \frac{\partial \psi}{\partial \phi} \rightarrow \frac{\partial \psi}{\partial \phi} = \pm \frac{1}{v} \frac{\partial \psi}{\partial t}$$

PERSAMAAN DIFFERENSIAL GELOMBANG

penurunan persamaan

- Turunan kedua terhadap x dan t

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial \phi} \right) = \frac{\partial}{\partial \phi} \left(\frac{\partial \psi}{\partial x} \right) = \frac{\partial}{\partial \phi} \left(\frac{\partial \psi}{\partial \phi} \right) = \frac{\partial^2 \psi}{\partial \phi^2}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial t} \right) = \frac{\partial}{\partial t} \left(\pm v \frac{\partial \psi}{\partial \phi} \right) = (\pm v) \frac{\partial}{\partial \phi} \left(\frac{\partial \psi}{\partial t} \right) = (\pm v) \frac{\partial}{\partial \phi} \left(\pm v \frac{\partial \psi}{\partial \phi} \right) = v^2 \frac{\partial^2 \psi}{\partial \phi^2}$$

$$\frac{\partial^2 \psi}{\partial \phi^2} = \frac{\partial^2 \psi}{\partial x^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

$$\frac{\partial^2 \psi}{\partial \phi^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\nabla^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

Merupakan ungkapan gelombang datar
(Front wave berupa bidang datar)

Untuk koordinat bola

$$\nabla^2 \psi = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial \psi}{\partial r}$$

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r} \frac{\partial \psi}{\partial r} - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0 \quad \text{(Buktikan)}$$

PERSAMAAN DIFFERENSIAL GELOMBANG

prinsip superposisi

- Jika ψ_1 dan ψ_2 solusi dari pers. Gelombang, maka berlaku:

$$\frac{\partial^2 \psi_1}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \psi_1}{\partial t^2} = 0$$

$$\frac{\partial^2 \psi_2}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \psi_2}{\partial t^2} = 0$$

dijumlahkan

$$\frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 (\psi_1 + \psi_2)}{\partial t^2} = 0$$

Jadi $(\psi_1 + \psi_2)$ merupakan solusi dari pers. Gelombang juga

Prinsip superposisi

SOLUSI PERSAMAAN GELOMBANG

Solusi paling sederhana dari persamaan : $\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$ adalah

$$\psi(x,t) = \psi_0 \cos k(x-vt) \longrightarrow \psi_0 = \psi_{\text{maks}}$$

k = bilangan gelombang/vektor gelombang (menunjukkan arah rambat gelombang)

$$\psi(x,t) = \psi_0 \cos k(x-vt)$$

$$\psi(x,t) = \psi_0 \cos (kx - kvt)$$

$$\psi(x,t) = \psi_0 \cos (kx - \omega t)$$

k = frekuensi spatial

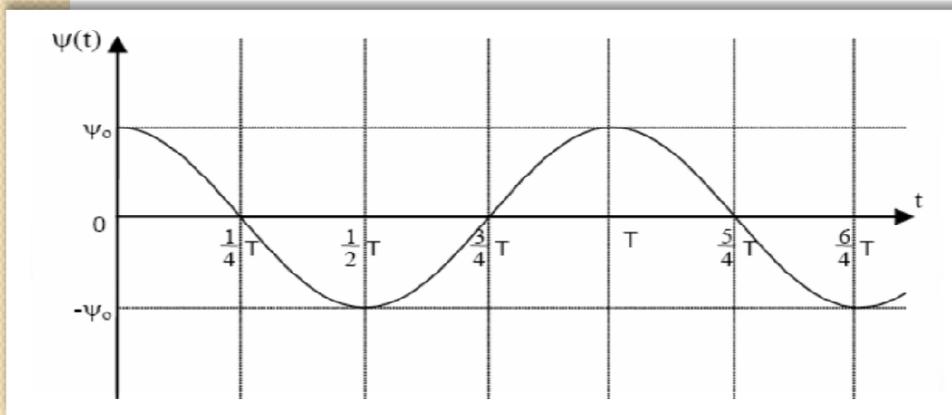
ω = frekuensi temporal

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T}$$

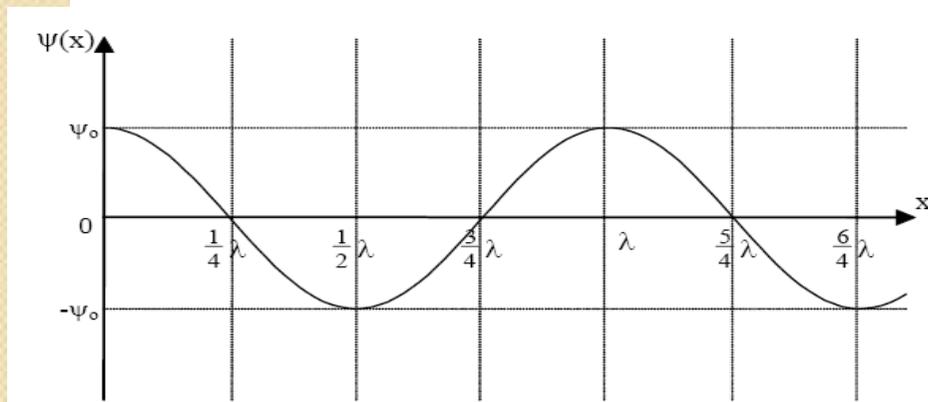
λ = perioda spatial

T = perioda temporal



Mengungkapkan
pola eksitasi
gelombang

Gelombang dalam sisi temporal



Mengungkapkan
perambatan
gelombang

Gelombang dalam sisi spatial

Sehingga solusi persamaan gelombang dapat pula diungkapkan dengan:

$$\Psi(x, t) = A \cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

SUPERPOSISI DUA GELOMBANG

Misalkan dua buah gelombang dengan arah getar pada bidang yang sama, masing-masing frekuensinya ω_1 dan ω_2 serta bilangan gelombangnya k_1 dan k_2

$$\psi_1(x,t) = A \cos(k_1x - \omega_1t) \quad \text{dan} \quad \psi_2(x,t) = A \cos(k_2x - \omega_2t)$$

Hasil superposisinya adalah:

$$\Psi(x,t) = A[\cos(k_1x - \omega_1t) + \cos(k_2x - \omega_2t)]$$

$$\begin{aligned} &\Psi(x,t) \\ &= 2A \left[\cos \left\{ \frac{(k_1 - k_2)x - (\omega_1 - \omega_2)t}{2} \right\} \cos \left\{ \frac{(k_1 + k_2)x - (\omega_1 + \omega_2)t}{2} \right\} \right] \end{aligned}$$

$$\Delta k = k_1 - k_2 \quad \Delta \omega = \omega_1 - \omega_2 \quad \text{Maka:}$$

$$\Psi(x, t) = 2A \left[\cos \left\{ \frac{\Delta k x - \Delta \omega t}{2} \right\} \cos \left\{ \frac{(2k_1 - \Delta k)x - (2\omega_1 - \Delta \omega)t}{2} \right\} \right]$$

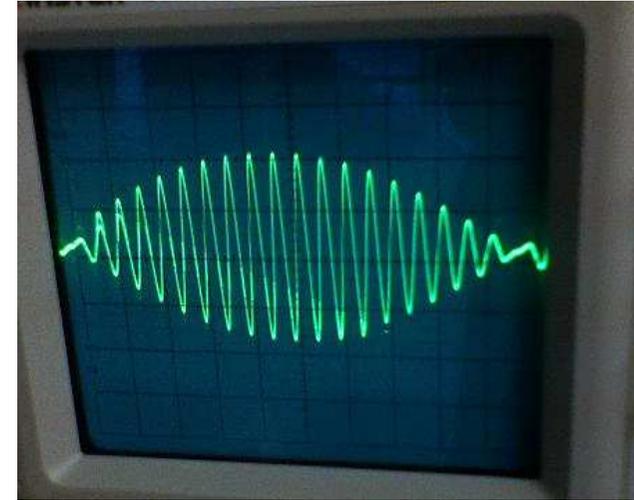
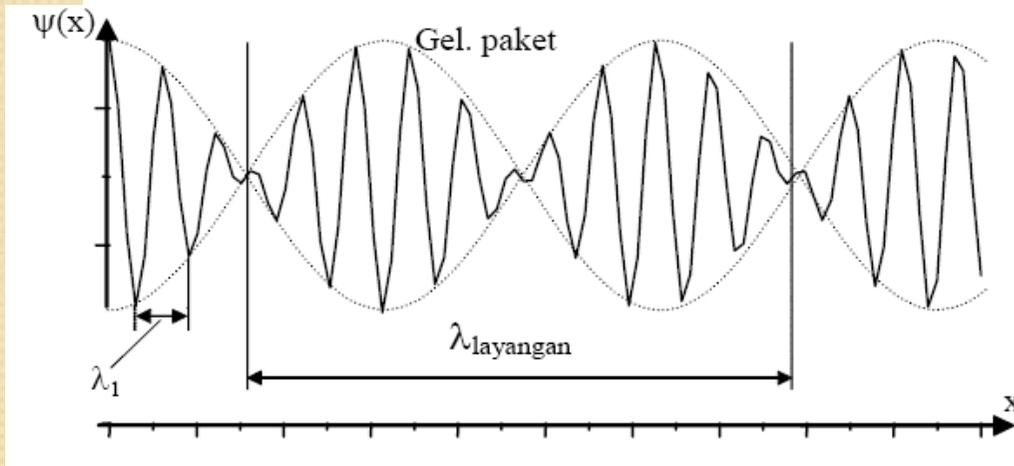
Untuk sisi spasial (dengan t konstan, misal t=0)

$$\Psi(x, t) = 2A \left[\cos \frac{\Delta k x}{2} \cos \frac{(2k_1 - \Delta k)x}{2} \right]$$

Δk sangat kecil, sehingga $2k_1 - \Delta k \approx 2k_1$

$$\Psi(x, t) = 2A \left[\cos \frac{\Delta k x}{2} \cos k_1 x \right]$$

Bila kita gambarkan hasil superposisinya, maka :

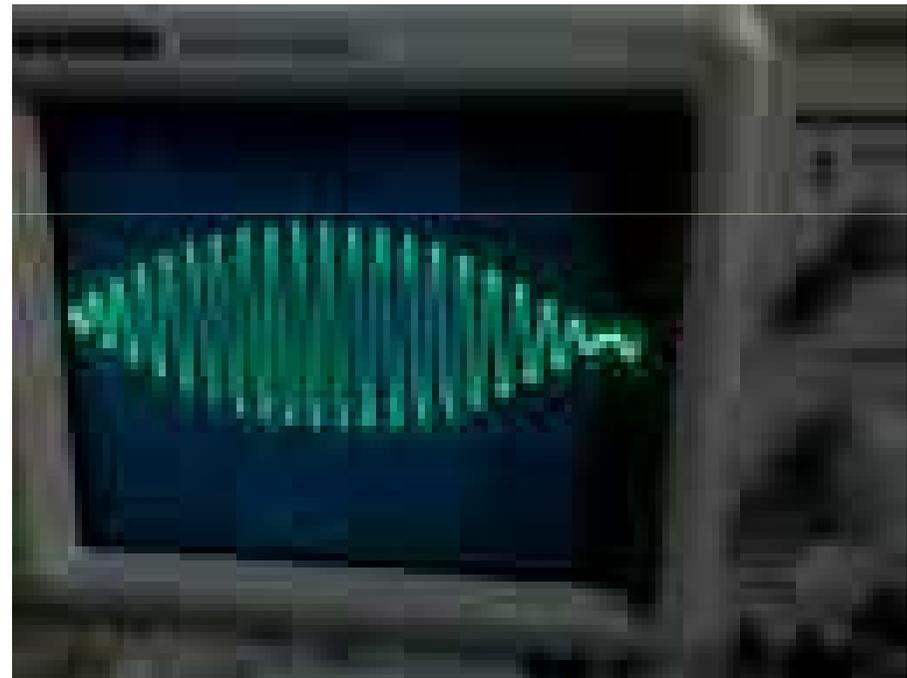
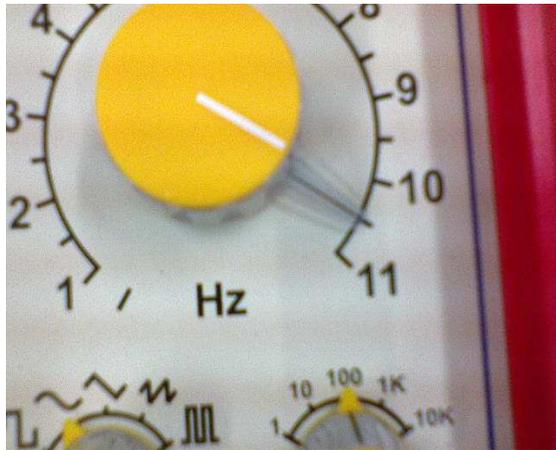


Hasil superposisi kedua gelombang dengan perbedaan frekuensi yang kecil ini disebut **layangan**, hasilnya berupa gelombang paket yang terselubung (envelope), dan kecepatan gelombang paket ini disebut dengan kecepatan group.

Kecepatan fase: $v = \frac{\omega_1}{k_1}$

Kecepatan group: $v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk} = \frac{d(kv)}{dk} = v + k \frac{dv}{dk}$

Layangan



SUPERPOISI DUA GELOMBANG arah getar saling tegak lurus

Tinjauan dua gelombang dengan frekuensi yang sama dan arah getar yang tegak lurus: Misal arah getarnya Y dan Z:

$$\psi_y(t) = A_1 \sin(\omega t + \varphi_1)$$

$$\psi_z(t) = A_2 \sin(\omega t + \varphi_2)$$

Superposisi keduanya menghasilkan:

$$\begin{aligned} & -\frac{\Psi_y}{A_1} \sin \varphi_2 + \frac{\Psi_z}{A_2} \sin \varphi_1 \\ & = \sin(\omega t) \{ \cos(\varphi_2) \sin(\varphi_1) \\ & \quad - \cos(\varphi_1) \sin(\varphi_2) \} \end{aligned}$$

$$\begin{aligned} & \frac{\Psi_y}{A_1} \cos \varphi_2 - \frac{\Psi_z}{A_2} \cos \varphi_1 \\ & = \cos(\omega t) \{ \cos(\varphi_2) \sin(\varphi_1) \\ & \quad - \cos(\varphi_1) \sin(\varphi_2) \} \end{aligned}$$

Kuadratkan kedua persamaan, kemudian dijumlahkan, menghasilkan:

$$\sin^2(\delta) = \left(\frac{\Psi_y}{A_1}\right)^2 + \left(\frac{\Psi_z}{A_2}\right)^2 - \frac{2\Psi_y\Psi_z}{A_1A_2}\cos(\delta)$$

Dengan beda sudut fase: $\delta = \varphi_1 - \varphi_2$

Persamaan ini merupakan persamaan umum elips, karena itu superposisinya disebut terpolarisasi elips.

Untuk beberapa kasus khusus, yaitu: $\delta = \pi/2, 3\pi/2, 5\pi/2, \dots$, persamaanya jadi:

$$\left(\frac{\Psi_y}{A_1}\right)^2 + \left(\frac{\Psi_z}{A_2}\right)^2 = 1$$

Terjadi polarisasi elips putar kanan, dan bila amplitudo kedua gelombang sama ($A_1=A_2$), maka superposisinya terpolarisasi lingkaran putar kanan.

Bila: $\delta = 0, 2\pi, 4\pi, \dots$ Persamaan menjadi: $\Psi_y = \frac{A_1}{A_2}\Psi_z$

Terjadi polarisasi linear