

# Dinamika Gelombang

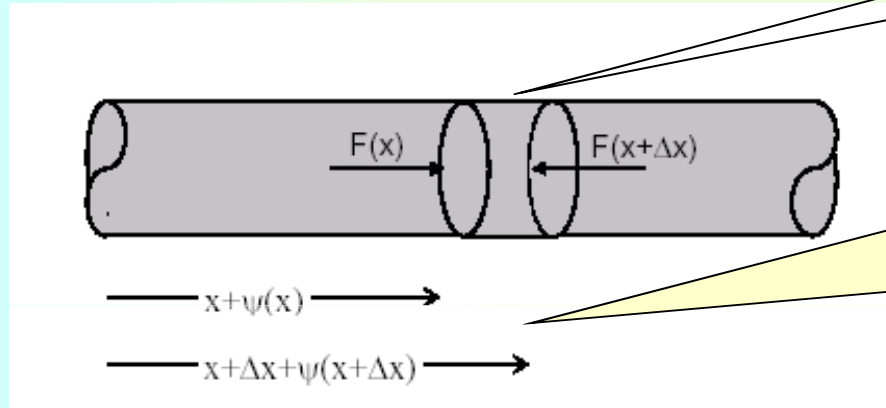
## Bagian 2

# Sub Pokok Bahasan

- Gelombang pada zat cair
- Gelombang di udara (gelombang bunyi)
- Gelombang permukaan air

# B.4 Gelombang Pada Zat Cair

Elemen Zat cair setebal  $\Delta x$  dengan luas penampang A



Elemen mengalami deformasi. Perpindahan sisi kiri dan kanan elemen tsb dinyatakan dengan  $\Psi(x)$  dan  $\Psi(x + \Delta x)$

Persamaan gerak elemen Volume zat Cair

$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = F(x) - F(x + \Delta x)$$

Hubungan antara tegangan dan regangan :

Modulus Bulk

$$\frac{F}{A} = -M \frac{\Delta V}{V}$$

Persamaan gerak elemen Volume zat Cair

$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = F(x) - F(x + \Delta x)$$

Ekspansi ke Deret Taylor

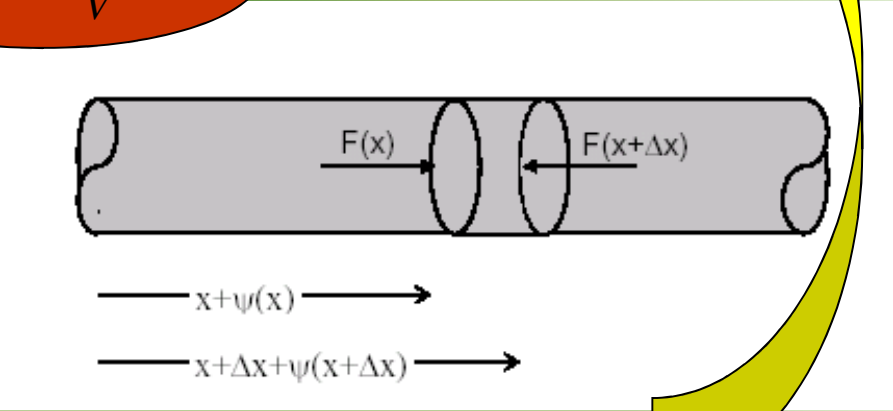
$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = \left[ F(x) - F(x) - \frac{\partial F}{\partial x} \Delta x \right]$$

$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = -\Delta x \frac{\partial F}{\partial x}$$

Hubungan antara tegangan dan regangan :

$$\frac{F}{A} = -M \frac{\Delta V}{V}$$

$$\frac{F}{A} = -M \frac{A[\Delta x + \Psi(x + \Delta x) - \Psi(x)] - A\Delta x}{A\Delta x}$$



$$\frac{F}{A} = -M \frac{A \frac{\partial \Psi}{\partial x} \Delta x}{A\Delta x} = -M \frac{\partial \Psi}{\partial x}$$

andhysetiawan

$$F = -AM \frac{\partial \Psi}{\partial x}$$

$$\frac{\partial F}{\partial x} = -AM \frac{\partial^2 \Psi}{\partial x^2}$$

Substitusi

$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = -\Delta x \frac{\partial F}{\partial x}$$



$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = AM \Delta x \frac{\partial^2 \Psi}{\partial x^2}$$

$$\frac{\partial F}{\partial x} = -AM \frac{\partial^2 \Psi}{\partial x^2}$$



$$\frac{\partial^2 \Psi}{\partial t^2} = \frac{M}{\rho} \frac{\partial^2 \Psi}{\partial x^2}$$



$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{M}{\rho} \frac{\partial^2 \Psi}{\partial x^2} = 0$$

Bandingkan dengan  
Persamaan Umum  
gelombang

$$\frac{\partial^2 \Psi}{\partial t^2} - v^2 \frac{\partial^2 \Psi}{\partial x^2} = 0$$



Cepat Rambat Gelombang :

$$v = \sqrt{\frac{M}{\rho}}$$

## C. Gelombang di Udara (Gelombang Bunyi)

UDARA

Tidak mengalami perubahan bentuk

Mempunyai respon terhadap perubahan tekanan

$$\rho = mV^{-1} \rightarrow \frac{d\rho}{dV} = -mV^{-2}$$

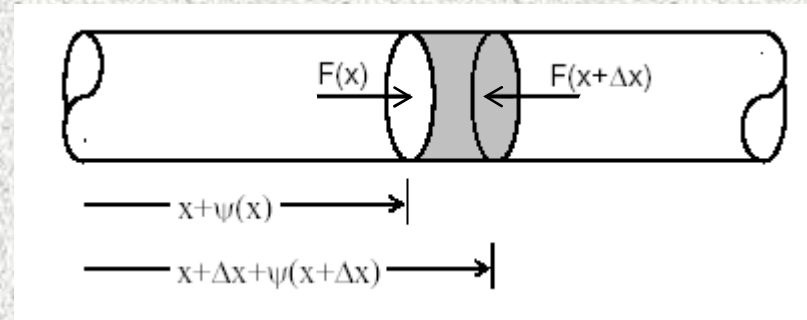
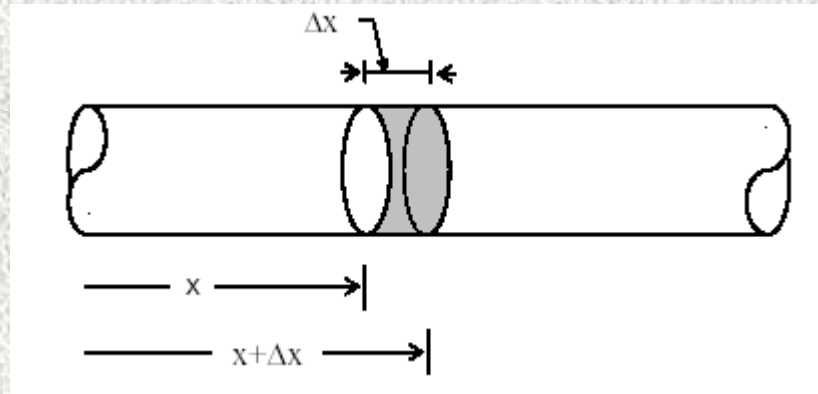
$$\frac{\rho}{d\rho} = -\frac{V}{dV}$$

Modulus Bulk

$$B = \rho \frac{dp}{d\rho}$$

$$B = -V \frac{dp}{dV}$$

## C.1 Cepat Rambat Gelombang Bunyi



Hukum II Newton

$$ma = F$$

$$\rho A \Delta x \frac{\partial^2 \Psi}{\partial t^2} = A [p(x) - p(x + \Delta x)]$$

$$\rho \frac{\partial^2 \Psi}{\partial t^2} = - \frac{\partial p}{\partial x}$$

Ekspansi ke Deret Taylor

Dalam perambatannya berlaku hukum kekekalan massa

$$\rho A [\Delta x + \Psi(x + \Delta x) - \Psi(x)] = \rho_0 A \Delta x = c$$

Ekspansi ke Deret Taylor

$$\rho A \Delta x \left\{ 1 + \frac{\partial \Psi}{\partial x} \right\} = c \quad \Rightarrow \quad \rho \left\{ 1 + \frac{\partial \Psi}{\partial x} \right\} = C$$

$\frac{\partial \Psi}{\partial x} \ll 1$

Cepat rambat Gelombang bunyi di udara

$$\left( \frac{\partial \rho}{\partial x} + \rho \frac{\partial^2 \Psi}{\partial x^2} \right) = 0 \quad \Rightarrow \quad \frac{\partial \rho}{\partial x} + \rho \frac{\partial^2 \Psi}{\partial x^2} = 0 \quad \Rightarrow \quad \frac{\partial \rho}{\partial x} = -\rho \frac{\partial^2 \Psi}{\partial x^2}$$

$$\rho \frac{\partial^2 \Psi}{\partial t^2} = -\frac{\partial p}{\partial x} = -\frac{\partial p}{\partial \rho} \frac{\partial \rho}{\partial x}$$

$$\rho \frac{\partial^2 \Psi}{\partial t^2} = -\frac{\partial p}{\partial \rho} \frac{\partial \rho}{\partial x}$$

$$B = \rho \frac{\partial p}{\partial \rho} \quad \Rightarrow \quad \frac{\partial p}{\partial \rho} = \frac{B}{\rho}$$

$$\rho \frac{\partial^2 \Psi}{\partial t^2} = B \frac{\partial^2 \Psi}{\partial x^2}$$

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{B}{\rho} \frac{\partial^2 \Psi}{\partial x^2} = 0$$

$$\frac{\partial^2 \Psi}{\partial t^2} - v^2 \frac{\partial^2 \Psi}{\partial x^2} = 0$$



Gelombang dalam gas bersifat **adiabatik**

$$pV^\gamma = c$$



$$p\rho^{-\gamma} = c$$



$$\frac{dp}{d\rho} = \frac{\gamma p}{\rho}$$



$$dp\rho^{-\gamma} - p\gamma\rho^{-\gamma-1}d\rho = 0$$



$$B = \rho \frac{dp}{d\rho}$$



$$B = \gamma p$$

$$B = \gamma p$$

substitusi

$$v = \sqrt{\frac{B}{\rho}}$$

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

$$p = \rho \frac{RT}{M}$$

*Hk. Gay – Lussac*

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\beta = \sqrt{\frac{\gamma R}{M}}$$

$$v = \beta \sqrt{T}$$

## C.2 Intensitas Gelombang Bunyi

Dari  $B = -\frac{p}{d\psi/dx}$  Diperoleh hubungan antara gelombang tekanan dan gelombang pergeseran

$$p = -B \frac{d\Psi}{dx}$$

Daya atau arus energi gelombang bunyi:

$$P = p \cdot A \frac{\partial \Psi}{\partial t}$$

$$P = -B \frac{\partial \Psi}{\partial x} A \frac{\partial \Psi}{\partial t}$$

$$P = B \cdot A \cdot v \left( \frac{\partial \Psi}{\partial x} \right)^2$$

Rapat arus energi atau Intensitas gelombang bunyi  $P/A$

$$I = B \cdot v \cdot \left( \frac{\partial \Psi}{\partial x} \right)^2$$

$$p = -B \cdot \frac{\partial \Psi}{\partial x} = \frac{F}{A}$$

$$F = -Z \cdot \frac{\partial \Psi}{\partial t}$$

Impedansi karakteristik  
Impedansi jenis  
Rapat Impedansi

Impedansi

$$\frac{Z}{A} = \frac{B \cdot \frac{\partial \Psi}{\partial x}}{\frac{\partial \Psi}{\partial t}}$$

$$z = \frac{B}{v}$$

$$I = B \cdot v \cdot \left( \frac{\partial \Psi}{\partial x} \right)^2$$

$$I = B \cdot v \cdot \left( \frac{p}{B} \right)^2$$

$$I = \frac{1}{z} \cdot p^2$$

$$p = -B \frac{d\Psi}{dx}$$

Intensitas gelombang bunyi sering dinyatakan sebagai taraf intensitas  $\beta$  dalam satuan decibel (dB), yang menyatakan tingkat relatif dan didefinisikan sebagai berikut::

$$\beta = \log \frac{I}{I_0} \text{ Bel}$$



$$\beta = 10 \cdot \log \frac{I}{I_0} \text{ dB}$$

Dengan:

$$I_0 = 10^{-12} \text{ W} / \text{m}^2 = \text{Intensitas acuan}$$



## **Gelombang Permukaan Air**



**Anggap Air Memiliki sifat – sifat sebagai berikut**

- a. Non viskos, Viskositas yang disebabkan oleh gesekan internal, diabaikan.**
- b. Amplitudo gelombang relatif lebih kecil dibanding panjang gelombangnya.**
- c. Gaya-gaya yang bekerja hanyalah gaya gravitasi dan tegangan permukaan.**
- d. Inkompresibel, Volume tidak berubah karena perubahan tekanan, jadi rapat massanya konstan.**



Selain itu air dipandang sebagai air ideal, dengan sifat sifat :



a. Berlaku hukum kekekalan massa :

$$\frac{\partial \rho}{\partial t} = 0$$

Inkompresibel



$$\nabla \cdot (\rho v) = - \frac{\partial \rho}{\partial t}$$



$$\nabla \cdot (\rho v) = 0$$

$$v = \frac{\partial \Psi}{\partial t}$$



$$\nabla \cdot \left( \rho \frac{\partial \Psi}{\partial t} \right) = 0$$



$$\nabla \cdot \Psi = \text{Konstan}$$

b. Tidak ada gelembung.

$$\oint \Psi \cdot \hat{n} dA = 0$$

Teorema Divergensi

$$\int \nabla \cdot \Psi dV = 0$$

$$\nabla \cdot \Psi = 0 \implies \frac{\partial \Psi_x}{\partial x} + \frac{\partial \Psi_y}{\partial y} = 0$$

c. Tidak ada pusaran.

$$\oint \vec{v} \cdot d\vec{\ell} = 0$$

Teorema Stokes (Rotasi)

$$\int \nabla \times \vec{v} \cdot \hat{n} dA = 0$$

$$\nabla \times \frac{\partial \Psi}{\partial t} = 0 \implies \frac{\partial}{\partial t} (\nabla \times \Psi) = 0 \implies (\nabla \times \Psi) = 0$$
$$\hat{k} \left( \frac{\partial \Psi_y}{\partial x} - \frac{\partial \Psi_x}{\partial y} \right) = 0$$



## D.1. Penerapan Syarat Batas

Syarat batas di  $x = 0$  :

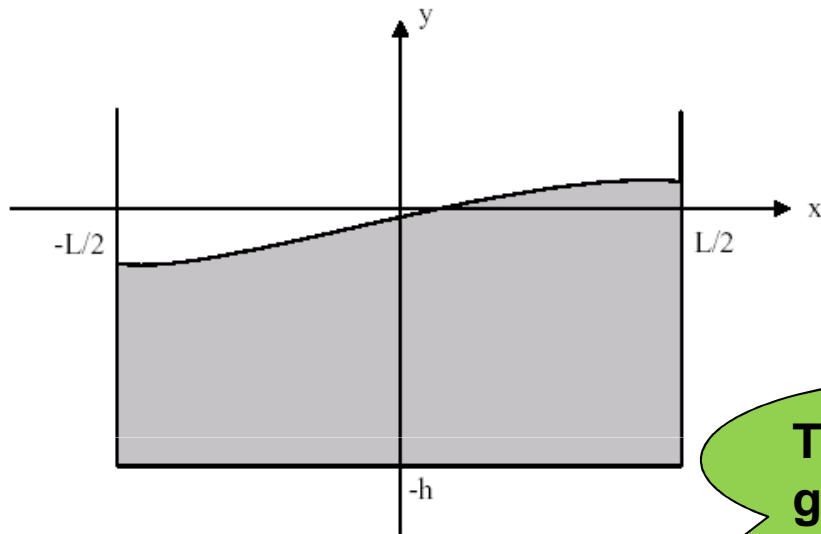
$\psi_y = 0$ , maka  $\psi_y$  mengandung faktor  $\sin(kx)$ .

$$\psi_y = \cos(\omega t) \sin(kx) f(y) \quad \dots(1)$$

Syarat batas di  $x = \pm \frac{L}{2}$  :

$\psi_x = 0$ , maka  $y_x$  mengandung faktor  $\cos(kx)$ .

$$\psi_x = \cos(\omega t) \cos(kx) g(y) \quad \dots(2)$$



Tidak ada gelembung

Pers. 1

$$\frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y} = 0$$

Pers. 2

$$\left( \frac{\partial \psi_y}{\partial x} - \frac{\partial \psi_x}{\partial y} \right) = 0$$

Tidak ada pusaran

$$-kg(y) + \frac{df(y)}{dy} = 0 \dots\dots(3)$$

$$kf(y) - \frac{dg(y)}{dy} = 0 \dots\dots(4)$$

Persamaan 3

Diferensiasikan terhadap y

$$-k \frac{dg(y)}{dy} + \frac{d^2 f(y)}{dy^2} = 0$$

$$\frac{dg(y)}{dy} = \frac{1}{k} \frac{d^2 f(y)}{dy^2} \dots\dots(5)$$

Substitusi ke persamaan 4

$$k f(y) - \frac{1}{k} \frac{d^2 f(y)}{dy^2} = 0$$

$$\frac{d^2 f(y)}{dy^2} - k^2 f(y) = 0$$

Solusi Persamaan

$$f(y) = Ae^{ky} + Be^{-ky}$$

Persamaan 4

Diferensiasikan terhadap y

$$k \frac{df(y)}{dy} - \frac{d^2 g(y)}{dy^2} = 0$$

$$\frac{df(y)}{dy} = \frac{1}{k} \frac{d^2 g(y)}{dy^2} \dots\dots(6)$$

Substitusi ke persamaan 3

$$-k g(y) + \frac{1}{k} \frac{d^2 g(y)}{dy^2} = 0$$

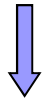
$$\frac{d^2 g(y)}{dy^2} - k^2 g(y) = 0$$

Solusi Persamaan

$$g(y) = Ce^{ky} + De^{-ky}$$

Syarat Batas :  $y = -h: \Psi_y = 0$  Maka  $f(-h) = 0$

$$f(-h) = Ae^{-kh} + Be^{kh} = 0 \quad f(y) = Ae^{ky} + (-Ae^{-2kh})e^{-ky}$$



$$f(y) = A(e^{ky} - e^{-k(2h+y)})$$



$B = -Ae^{-2kh}$  dari pers (3):  $-kg(y) + \frac{df(y)}{dy} = 0 \rightarrow g(y) = A(e^{ky} + e^{-k(2h+y)})$   
 Persamaan Gelombang arah x dan y pada persamaan (1) dan (2)

$$\Psi_y = A \cos(\omega t) \sin(kx) \{e^{ky} - e^{-k(2h+y)}\} \dots (7)$$

$$\Psi_x = A \cos(\omega t) \cos(kx) \{e^{ky} + e^{-k(2h+y)}\} \dots (8)$$

Ekspansi ke deret pangkat

Kasus khusus

a. Bila  $h \gg \lambda$ , maka

$$\Psi_y = Ae^{ky} \cos(\omega t) \sin(kx) \dots (9)$$

$$\Psi_x = Ae^{ky} \cos(\omega t) \cos(kx) \dots (10)$$

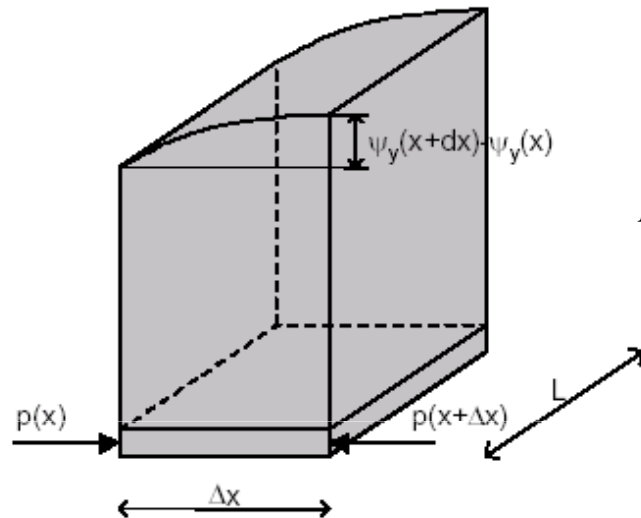
b. Bila  $h \ll \lambda$ , maka :

$$\Psi_y = 2Ak(y+h) \cos(\omega t) \sin(kx) \dots (11)$$

$$\Psi_x = 2A \cos(\omega t) \cos(kx) \dots (12)$$

## D.2. Hubungan Dispersi Gelombang Permukaan Air

Persamaan Gerak



$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = L \Delta y [p(x) - p(x + \Delta x)]$$

$$p(x) = \rho g \Psi_y(x)$$

Hukum hidrostatika

$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = L \Delta y \rho g [\Psi_y(x) - \Psi_y(x + \Delta x)]$$


Deret Taylor

$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = -L \Delta y \Delta x \rho g \frac{\partial \Psi_y}{\partial x}$$

$$\Delta m = \Delta V \rho$$

$$\Delta m = L \Delta y \Delta x \rho$$

$$\frac{\partial^2 \Psi_x}{\partial t^2} = -g \frac{\partial \Psi_y}{\partial x}$$


$$\frac{\partial^2 \Psi_x}{\partial t^2} = -g \frac{\partial \Psi_y}{\partial x}$$

$$\Psi_y = A \cos(\omega t) \sin(kx) \{e^{ky} - e^{-k(2h+y)}\}$$

$$\Psi_x = A \cos(\omega t) \cos(kx) \{e^{ky} + e^{-k(2h+y)}\}$$

$$\omega^2 \{e^{ky} + e^{-k(2h+y)}\} = gk \{e^{ky} - e^{-k(2h+y)}\}$$

Syarat batas di  $y = 0$

$$\omega^2 \{1 + e^{-k2h}\} = gk \{1 - e^{-k2h}\}$$

Persamaan  
Dispersi

$$\omega^2 = gk \frac{1 - e^{-2kh}}{1 + e^{-2kh}}$$

### D.3. Gelombang Gravitasi dan Gelombang Riak

$$\omega^2 = gk \frac{1 - e^{-2kh}}{1 + e^{-2kh}}$$

Persamaan Dispersi

Kasus Khusus

a. Bila  $h \gg \lambda \Rightarrow e^{-2kh} \cong 0$


Persamaan dispersi menjadi :

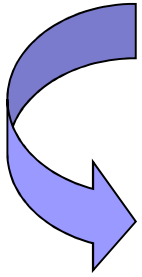
$$\omega^2 = gk \Rightarrow \omega = \sqrt{gk} \iff k = \frac{2\pi}{\lambda}$$

$$v_f = \frac{\omega}{k}$$

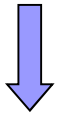
$$v_f = \sqrt{\frac{g\lambda}{2\pi}}$$

Kecepatan fase


$$v_g = \frac{d\omega}{dk}$$



$$v_g = \frac{d\sqrt{gk}}{dk}$$



$$v_g = \frac{1}{2} \sqrt{\frac{g}{k}}$$



$$v_g = \frac{1}{2} \sqrt{\frac{g\lambda}{2\pi}}$$

Gelombang ini disebut Gelombang Gravitasi

Gelombang ini bersifat dispersif

$$v_f \neq v_g$$

Kecepatan Grup

b. Bila  $h \ll \lambda$ , Maka  $e^{-2kh}$  dalam deret pangkat

$$e^{-2kh} = 1 + (-2kh) + \frac{(-2kh)^2}{2!} + \dots$$

$$e^{-2kh} = 1 - (-2kh)$$

$$\omega^2 = gk \frac{1 - e^{-2kh}}{1 + e^{-2kh}}$$

$$\omega^2 = gk \frac{1 - (1 - 2kh)}{1 + (1 - 2kh)} \implies \omega^2 = gk^2 h \implies \omega = k \sqrt{gh}$$

$$v_f = \frac{\omega}{k} \implies v_f = \frac{k}{k} \sqrt{gh} \implies v_f = \sqrt{gh}$$

$$v_g = \frac{d\omega}{dk} \implies v_g = \frac{dk \sqrt{gh}}{dk} \implies v_g = \sqrt{gh} \implies v_g = v_f$$

Gelombang Riak bersifat non Dispersif



$$p(x) = (\rho g + \gamma k^2) \Psi_y(x)$$

$$p(x + \Delta x) = (\rho g + \gamma k^2) \Psi_y(x + \Delta x)$$

$$\gamma \longrightarrow \gamma k^2 \Psi$$

Efek tegangan permukaan diperhitungkan

Tekanan pada elemen massa bertambah

$$\rho \frac{\partial^2 \Psi_x}{\partial t^2} = -(\rho g + \gamma k^2) \frac{\partial \Psi_y}{\partial x}$$

$$\frac{\partial^2 \Psi_x}{\partial t^2} = -\left(g + \frac{\gamma k^2}{\rho}\right) \frac{\partial \Psi_y}{\partial x}$$

$$\omega^2 = \left(gk + \frac{\gamma k^3}{\rho}\right) \frac{1 - e^{-2kh}}{1 + e^{-2kh}}$$

Untuk kasus  $h \gg \lambda$ , tegangan permukaan tidak diabaikan

$$e^{-2kh} \cong 0$$

$$\omega^2 = \left(gk + \frac{\gamma k^3}{\rho}\right)$$

$$\longrightarrow v = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi\gamma}{\lambda\rho}}$$

andhysetiawan

$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = L\Delta y [p(x) - p(x + \Delta x)]$$

$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = L\Delta y (\rho g + \gamma k^2) [\Psi_y(x) - \Psi_y(x + \Delta x)]$$

Deret Taylor

$$\Delta m \frac{\partial^2 \Psi_x}{\partial t^2} = L\Delta y (\rho g + \gamma k^2) \left(-\Delta x \frac{\partial \Psi_y(x)}{\partial x}\right)$$

$$\Psi_y = A \cos(\omega t) \sin(kx) \{e^{ky} - e^{-k(2h+y)}\}$$

$$\Psi_x = A \cos(\omega t) \cos(kx) \{e^{ky} + e^{-k(2h+y)}\}$$

Untuk kasus  $h \ll \lambda$  tegangan permukaan tidak diabaikan, Bagaimana dispersivitasnya?