MODULASI GELOMBANG

TOPIK 5

Bagian 1

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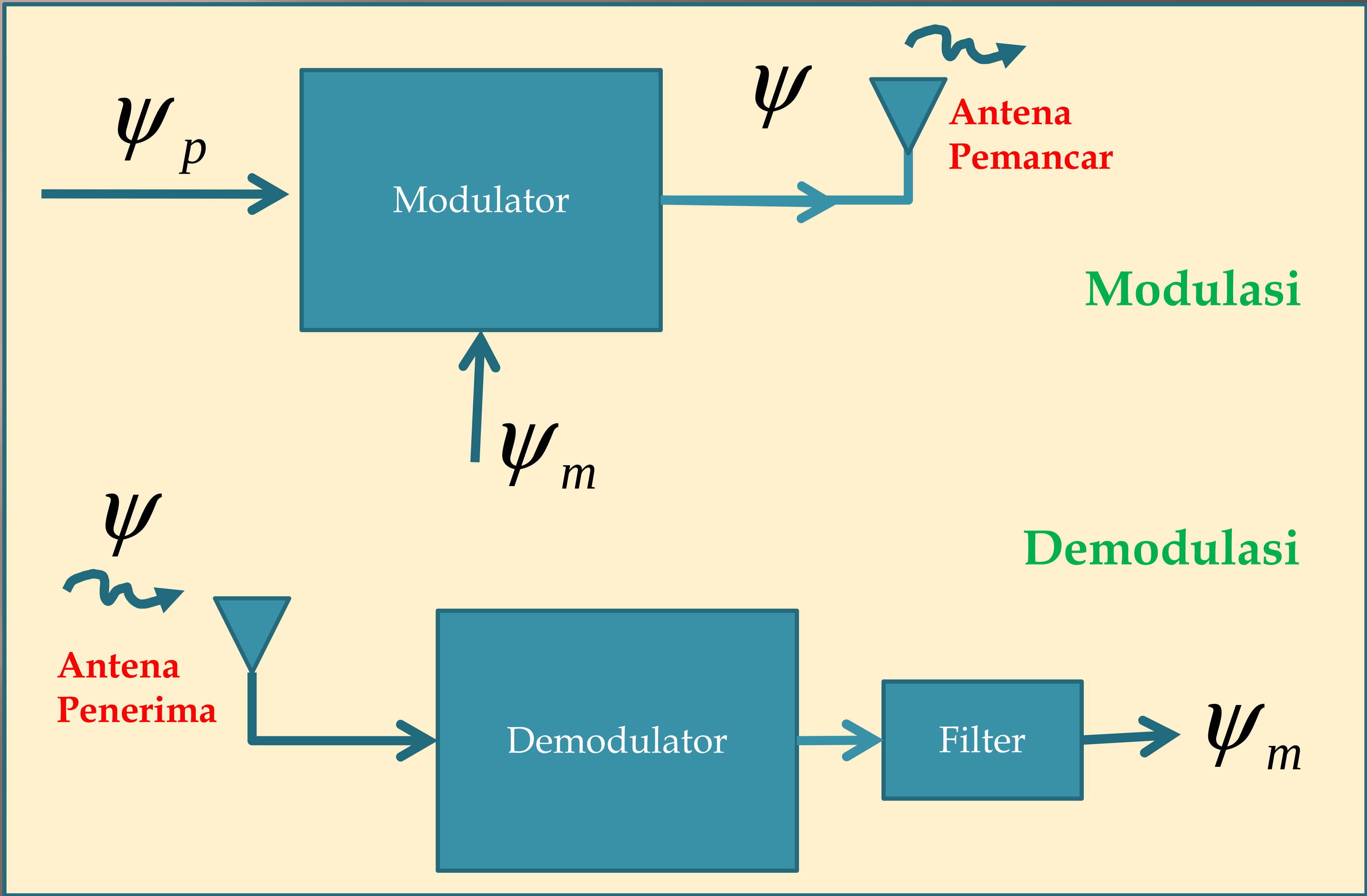
Isi Materi

- Pendahuluan
- Transformasi Fourier dan Fungsi Delta Dirac
- Modulasi Double Side Band (DSB)

A. Pendahuluan

- Modulasi → proses perubahan karakteristik suatu gelombang menurut pola gelombang lain, dengan cara menumpangkan (memboncengkan) suatu gelombang pada gelombang lainnya.
- Dalam teknik komunikasi, gelombang atau sinyal pita dasar (base band) dikirimkan dengan modulasi gelombang pembawa yang berfrekuensi tinggi.
- sinyal pita dasar → gelombang informasi atau gelombang modulasi → ψ_m
- Gelombang pembawa → ψ_p

Teknik modulasi dan demodulasi secara umum



Transformasi Fourier dan Fungsi Delta Dirac

- **Transformasi Fourier:** operasi yang menghubungkan kelakuan suatu fungsi dalam dua domain yang berkonjugasi.

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt \qquad \psi(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(\omega) e^{i\omega t} d\omega$$

$$g(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(x) e^{-ikx} dx \qquad \psi(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(k) e^{ikx} dk$$

- **Fungsi Delta Dirac**

$$\delta(\omega - \omega_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-it(\omega-\omega_0)} dt \qquad \delta(t - t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega(t-t_0)} d\omega$$

$$\delta(x - x_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ik(x-x_0)} dk \qquad \delta(k - k_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ix(k-k_0)} dx$$

$\delta(x - x_0) = 0$ untuk $\mathrm{x} \neq \mathrm{x}_0$

$\delta(x - x_0) = \infty$ untuk $\mathrm{x} = \mathrm{x}_0$

Modulasi DSB

Hasil modulasi secara umum dapat kita ungkapkan dengan :

$$\psi(t) = \psi_p(t)\psi_m(t)$$

$$\psi(t) = \psi_{po}\cos(\omega_p t)\psi_{mo}\cos(\omega_m t)$$

$$\psi(t) = \frac{1}{2}\psi_{po}\psi_{mo}\left[\cos(\omega_p - \omega_m)t + \cos(\omega_p + \omega_m)t\right]$$

Operasi di atas disebut dengan *mixing*, hasilnya berupa dua komponen gelombang (side band), masing-masing berfrekuensi $\omega_p+\omega_m$ dan $\omega_p-\omega_m$.

$\omega_p+\omega_m$: pita sisi atas (upper side band)

$\omega_p-\omega_m$: pita sisi bawah (lower side band)

Tampak secara eksplisit bahwa akibat modulasi, terjadi translasi frekuensi gelombang modulasi dari ωm menjadi $\omega_p \pm \omega_m$.

Gelombang pembawa dalam domain frekuensi :

$$g_p(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi_p(t)e^{-i\omega t}\,dt$$

$$g_p(\omega) = \frac{1}{2\pi}\psi_{po}\int_{-\infty}^{\infty}\cos(\omega_p t)e^{-i\omega t}\,dt$$

$$g_p(\omega) = \frac{1}{4\pi}\psi_{po}\int_{-\infty}^{\infty}(e^{i\omega_p t} + e^{-i\omega_p t})e^{-i\omega t}\,dt$$

$$g_p(\omega) = \frac{1}{4\pi}\psi_{po}\int_{-\infty}^{\infty}\left[e^{-i(\omega-\omega_p)t} + e^{-i(\omega+\omega_p)t}\right]dt$$

$$g_p(\omega) = \frac{1}{2}\psi_{po}\left[\delta(\omega-\omega_p) + \delta(\omega+\omega_p)\right]$$

Untuk gelombang modulasi :

$$g_m(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi_m(t) e^{-i\omega t}\, dt$$

$$g_m(\omega) = \frac{1}{2\pi}\psi_{mo}\int_{-\infty}^{\infty} \cos(\omega_m t) e^{-i\omega t}\, dt$$

$$g_m(\omega) = \frac{1}{4\pi}\psi_{mo}\int_{-\infty}^{\infty} (e^{i\omega_m t} + e^{-i\omega_m t}) e^{-i\omega t}\, dt$$

$$g_m(\omega) = \frac{1}{4\pi}\psi_{mo}\int_{-\infty}^{\infty} \left[e^{-i(\omega-\omega_m)t} + e^{-i(\omega+\omega_m)t}\right] dt$$

$$g_m(\omega) = \frac{1}{2}\psi_{mo}\left[\delta(\omega-\omega_m) + \delta(\omega+\omega_m)\right]$$

Hasil modulasinya dalam domain frekuensi :

$$g(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\psi(t)e^{-i\omega t}dt$$

$$g(\omega)=\frac{1}{2\pi}\frac{1}{2}\psi_{mo}\psi_{po}\int_{-\infty}^{\infty}(\cos(\omega_p-\omega_m)t+\cos(\omega_p+\omega_m)t)e^{-i\omega t}dt$$

$$g(\omega)=\frac{1}{4\pi}\psi_{mo}\psi_{po}\int_{-\infty}^{\infty}\left[\frac{e^{i(\omega_p-\omega_m)t}+e^{-i(\omega_p-\omega_m)t}}{2}\right.$$

$$\left.+\frac{e^{i(\omega_p+\omega_m)t}+e^{-i(\omega_p+\omega_m)t}}{2}\right]e^{-i\omega t}dt$$

$$g(\omega) = \frac{1}{4}\psi_{mo}\psi_{po}\,\frac{1}{2\pi}\int_{-\infty}^{\infty}\Big[e^{-i(\omega-(\omega_p-\omega_m))t} + e^{-i(\omega+(\omega_p-\omega_m))t} + e^{-i(\omega-(\omega_p+\omega_m))t} + e^{-i(\omega+(\omega_p+\omega_m))t}\Big]dt$$

$$g(\omega) = \frac{1}{4}\psi_{po}\psi_{mo}\Big[\delta\big(\omega-(\omega_p-\omega_m)\big)+\delta\big(\omega+(\omega_p-\omega_m)\big) + \delta\big(\omega-(\omega_p+\omega_m)\big)+\delta\big(\omega+(\omega_p+\omega_m)\big)\Big]$$

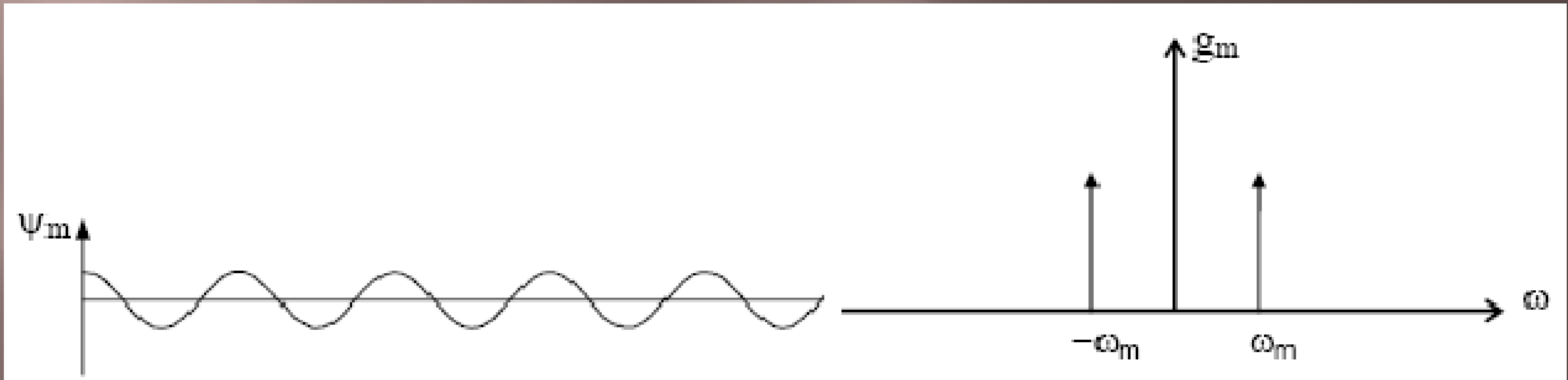
$$g(\omega) = \frac{1}{2}\psi_{po}\big[g_m(\omega-\omega_p)+g_m(\omega+\omega_p)\big]$$

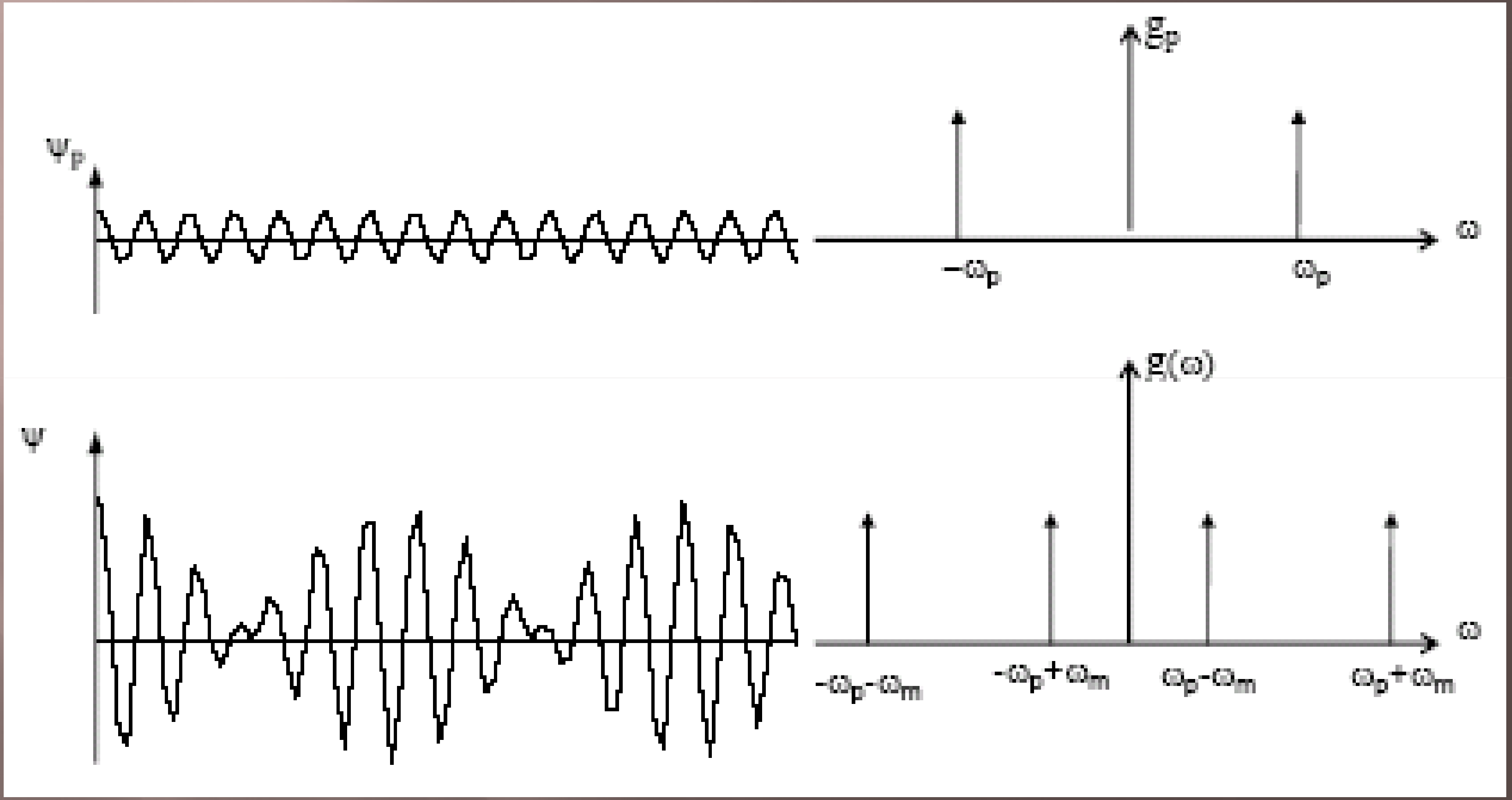
Lebar pita (bandwidth) B :

B = upper side - lower side

$= 2\ \omega m$

Grafik dalam domain waktu dan domain frekuensi untuk modulasi DSB ini diperlihatkan seperti pada gambar 5.6.





ψ_p
g_p
$-\omega_p$
ω_p
ω
ψ
$g(\omega)$
$-\omega_p-\omega_m$
$-\omega_p+\omega_m$
$\omega_p-\omega_m$
$\omega_p+\omega_m$
ω

Daya rata-rata yang diteruskan :

$$\overline{p} = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi(t)]^2 \, dt$$

$$\overline{p} = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \{\psi_m(t)\}^2 \psi_{po}{}^2 \cos^2(\omega_p t) dt$$

$$\overline{p} = \lim_{T\to\infty} \frac{1}{T} \frac{\psi_{po}{}^2}{2} \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} \{\psi_m(t)\}^2 \, dt + \int_{-\frac{T}{2}}^{\frac{T}{2}} \{\psi_m(t)\}^2 \cos(2\omega_p t) dt \right]$$

Untuk $\omega_p >> \omega_m$ suku ke dua ruas kanan persamaan ini sama dengan nol, maka daya rata-rata :

$$\overline{P} = \overline{P}_p \, \overline{P}_m$$

$$\overline{P}_p = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \psi_{po}^{\;2} \cos^2(\omega_p t)\, dt = \frac{1}{2} \psi_{po}^{\;2}$$

$$\overline{P}_m = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t)]^2 \, dt$$

Demodulasi DSB

Demodulasi diartikan sebagai operasi untuk memperoleh kembali sinyal modulasi ψm(t) dari gelombang hasil modulasi ψ(t). Demodulasi DSB dilakukan dengan dua tahap sebagai berikut :

a. Gelombang hasil modulasi dikalikan dengan osilator lokal yang sinkron dengan gelombang pembawa ψp(t). Osilator lokal: 2 cos(ωpt).

$$\psi'(t) = \psi(t) 2\cos(\omega_p t)$$

$$\psi'(t) = \psi_{po}\psi_m(t)\left[1 + \cos(2\omega_p t)\right]$$

$$\psi'(t) = \psi_{po}\psi_m(t) + \psi_{po}\psi_{mo}\cos(\omega_m t)\cos(2\omega_p t)$$

$$\psi'(t) = \psi_{po}\psi_m(t) + \frac{1}{2}\psi_{po}\psi_{mo}\left[\cos(\omega_m t - 2\omega_p t) + \cos(\omega_m t + 2\omega_p t)\right]$$

Dalam domain frekuensi :

$$g'(\omega) = \frac{1}{2\pi}\psi_{po}\psi_{mo}g_m(\omega)$$

$$g'(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi'(t)e^{-i\omega t}dt$$

$$g'(\omega) = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty}\psi_{po}\psi_m(t)e^{-i\omega t}dt + \frac{1}{2}\psi_{po}\psi_{mo}\right.$$

$$\left.\int_{-\infty}^{\infty}\left[\cos(\omega_m t - 2\omega_p t) + \cos(\omega_m t + 2\omega_p t)\right]e^{-i\omega t}dt\right]$$

$$g^{'}(\omega) = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} \psi_{po}\psi_{mo}\cos(\omega_m t)e^{-i\omega t}dt + \frac{1}{2}\psi_{po}\psi_{mo}\right.$$

$$\left.\int_{-\infty}^{\infty}\left[\cos(\omega_m t - 2\omega_p t) + \cos(\omega_m t + 2\omega_p t)\right]e^{-i\omega t}dt\right]$$

$$g^{'}(\omega) = \frac{\psi_{po}\psi_{mo}}{2\pi}\left[\int_{-\infty}^{\infty}\left[\frac{e^{-\omega_m t} + e^{\omega_m t}}{2}\right]e^{-i\omega t}dt\right] + \frac{1}{4\pi}\psi_{po}\psi_{mo}\int_{-\infty}^{\infty}\left[\frac{e^{i(\omega_m - 2_p)t} + e^{-i(\omega_m - 2_p)t}}{2}\right]e^{-i\omega t}dt$$

$$+ \frac{1}{4\pi}\psi_{po}\psi_{mo}\int_{-\infty}^{\infty}\left[\frac{e^{i(\omega_m + 2\omega_P)t} + e^{-i(\omega_m + 2\omega_P)t}}{2}\right]e^{-i\omega t}dt$$

$$g'(\omega)=\frac{\psi_{po}\psi_{mo}}{4\pi}\left[\int_{-\infty}^{\infty}\left[e^{-i(\omega-\omega_m)t}+e^{-i(\omega+\omega_m)t}\right]dt+\frac{1}{8\pi}\psi_{po}\psi_{mo}\int_{-\infty}^{\infty}\left[e^{-i(\omega-(\omega_m-2_p))t}+e^{-i(\omega+(\omega_m-2\omega_p))t}\right]dt\right.$$

$$\left.+\frac{1}{8\pi}\psi_{po}\psi_{mo}\int_{-\infty}^{\infty}\left[e^{-i(\omega-(\omega_m+2\omega_p))t}+e^{-i(\omega+(\omega_m+2\omega_p))t}\right]dt\right]$$

$$g'(\omega)=\frac{\psi_{po}\psi_{mo}}{2}\left[\delta(\omega-\omega_m)+\delta(\omega+\omega_m)\right]+\frac{\psi_{po}\psi_{mo}}{4}\left[\delta(\omega-\omega_m+2\omega_p)+\delta(\omega+\omega_m-2\omega_p)\right]$$

$$+\frac{\psi_{po}\psi_{mo}}{4}\left[\delta(\omega-\omega_m-2\omega_p)+\delta(\omega+\omega_m+2\omega_p)\right]$$

Kemudian, dapat dituliskan dalam bentuk :

$$g'(\omega)=\psi_{po}g_m(\omega)+\frac{\psi_{po}}{2}\left[g_m(\omega+2\omega_p)+g_m(\omega-2\omega_p)\right]$$

b. Karena $\omega_p >> \omega_m$, maka $\omega m << 2\omega_p - \omega_m$; berarti sinyal ψ_{po} $g_m(\omega)$ dapat dipisahkan dengan tapis lolos rendah (lowpass filter) dengan frekuensi pancung (cut off) ω_{co}.

$$\omega_m < \omega_{co} < 2\omega_p - \omega_m$$