

BAGIAN 2
TOPIK 5

MODULASI GELOMBANG

Isi Materi

- ① Modulasi Amplitudo (AM)
- ① Modulasi Frekuensi (FM)

MODULASI AMPLITUDO DAN MODULASI ANGULAR (SUDUT)

- Modulasi → proses perubahan karakteristik atau besaran gelombang pembawa, menurut pola gelombang modulasinya.
- Secara umum persamaan gelombang pembawa:

$$\psi_p(t) = A(t)\cos(\omega_p t + \phi(t))$$

Dan sinyal informasi/data:

$$\psi_m(t)$$

- Apabila besaran yang dirubah dari gelombang pembawa tersebut adalah
 - amplitudo, → modulasi amplitudo (AM) → $A(t) \propto \psi_m(t)$
 - sudut fase → modulasi angular (modulasi sudut)
 - modulasi fase → $\phi(t) \propto \psi_m(t)$
 - modulasi frekuensi → $\frac{d\phi(t)}{dt} \propto \psi_m(t)$

MODULASI AMPLITUDO (AM)

Modulasi amplitudo → sinyal DSB ditambah dengan komponen gelombang pembawanya.

$$\psi(t) = \psi_p \psi_m + \psi_p$$

$$\psi(t) = \psi_{po} \cos(\omega_p t) \psi_m + \psi_{po} \cos(\omega_p t)$$

$$\psi(t) = \psi_{po} [\psi_m + 1] \cos(\omega_p t)$$

$$\psi(t) = A(t) \cos(\omega_p t)$$



$$\psi(t) = A(t)\cos(\omega_p t)$$

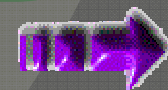
$A(t) \rightarrow$ faktor modulasi, yang mengungkapkan perubahan amplitudo (envelope) dari gelombang AM.

Dalam domain frekuensi persamaan menjadi :

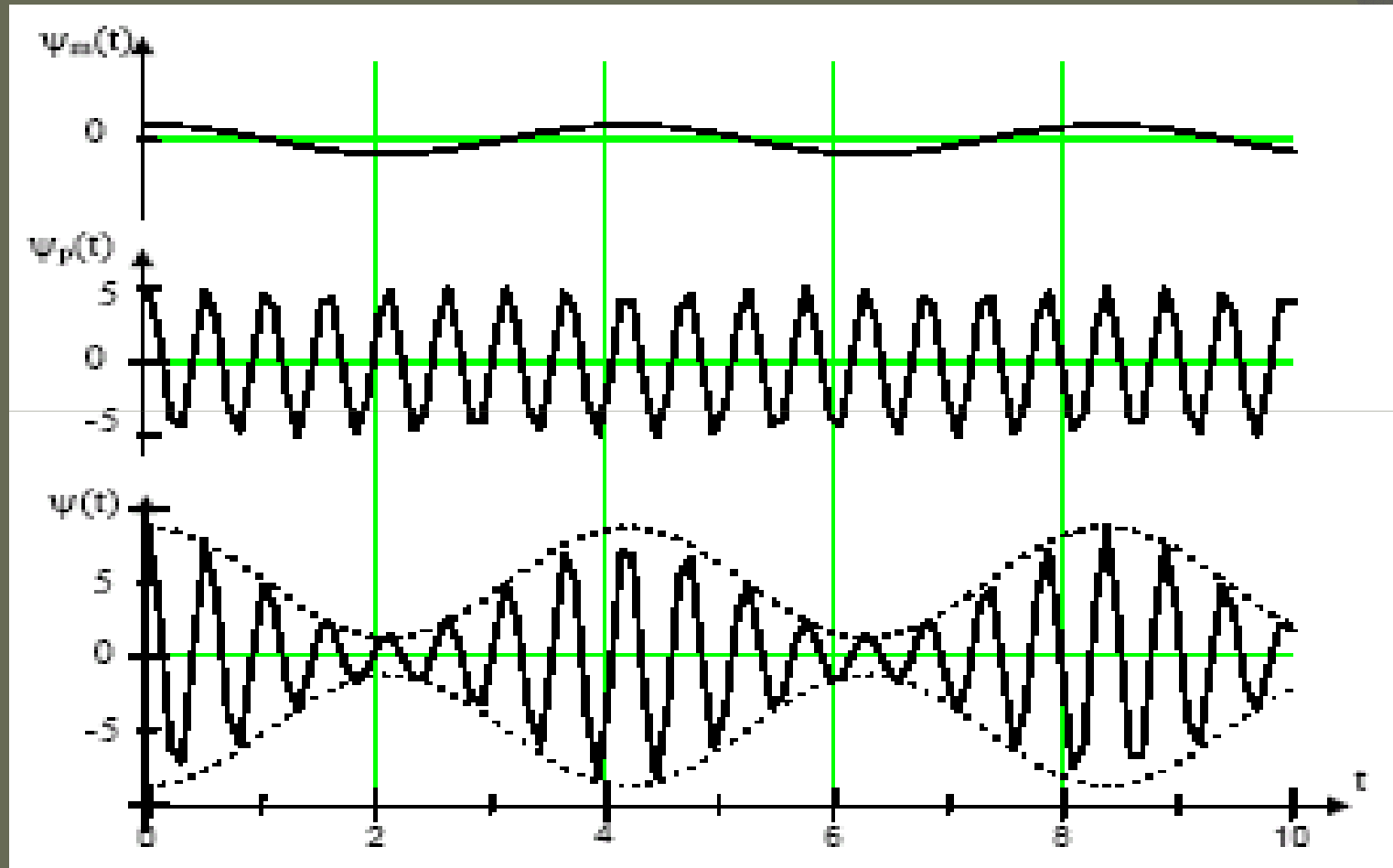
$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt$$

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_{po} [\psi_m + 1] \cos(\omega_p t) e^{-i\omega t} dt$$

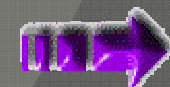
$$g(\omega) = \frac{1}{2\pi} \psi_{po} \int_{-\infty}^{\infty} [\psi_{mo} \cos(\omega_m t) \cos(\omega_p t) + \cos(\omega_p t)] e^{-i\omega t} dt$$



Sebagai contoh, untuk $\psi_m(t) = 0.75 \cos(1.5t)$ dan $\psi_p(t) = 5 \cos(12t)$, gelombang hasil modulasinya ditunjukkan seperti pada gambar 5.7.



Gelombang modulasi, gelombang pembawa dan hasil modulasi AM



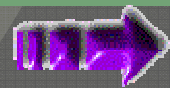
Daya rata-rata:

$$\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi(t)]^2 dt$$

$$\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \psi_{po}^2 [\psi_m(t) + 1]^2 \cos^2(\omega_p t) dt$$

$$\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \psi_{po}^2 [\psi_m(t) + 1]^2 \left[\frac{1 + \cos(2\omega_p t)}{2} \right] dt$$

$$\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{T} \frac{\psi_{po}^2}{2} \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} \{\psi_m^2 + 2\psi_m + 1\} dt + \int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t) + 1]^2 \cos(2\omega_p t) dt \right]$$



$$\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{T} \frac{\psi_{po}^2}{2} \left[\underbrace{\int_{-\frac{T}{2}}^{\frac{T}{2}} \{\psi_m^2 + 2\psi_m + 1\} dt}_{\text{Bagian 1}} + \underbrace{\int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t) + 1]^2 \cos(2\omega_p t) dt}_{\text{Bagian 2}} \right]$$

Bagian 1

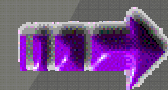
Bagian 2

Untuk $\omega_p \gg \omega_m$ suku ke dua ruas kanan persamaan ini sama dengan nol dan

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \psi_m(t) dt = 0$$

Maka daya rata-rata menjadi:

$$\bar{P} = \bar{P}_p \bar{P}_m + \bar{P}_p$$



Bagian 1

$$\int_{-T/2}^{T/2} [\{\Psi_m(t)\}^2 + 2\Psi_m(t) + 1] dt = \int_{-T/2}^{T/2} \{\Psi_m(t)\}^2 dt + \int_{-T/2}^{T/2} 2\Psi_m(t) dt + \int_{-T/2}^{T/2} dt$$

$$= \int_{-T/2}^{T/2} \{\Psi_m(t)\}^2 dt + \int_{-T/2}^{T/2} 2\Psi_{mo} \cos(\omega_m t) dt + T \Big|_{-T/2}^{T/2}$$

$$= \int_{-T/2}^{T/2} \{\Psi_m(t)\}^2 dt + 2\Psi_{mo} \frac{1}{\omega_m} \sin(\omega_m t) \Big|_{-T/2}^{T/2} + \left[\frac{T}{2} - \left(-\frac{T}{2} \right) \right]$$



$$= \int_{-T/2}^{T/2} \{\Psi_m(t)\}^2 dt + 2 \frac{\Psi_{mo}}{\omega_m} \left[\sin \frac{2\pi T}{T} \frac{T}{2} - \sin \frac{2\pi}{T} \left(-\frac{T}{2} \right) \right] + T$$

$$= \int_{-T/2}^{T/2} \{\Psi_m(t)\}^2 dt + 0 + T$$

$$\int_{-T/2}^{T/2} [\{\Psi_m(t)\}^2 + 2\Psi_m(t) + 1] dt = \int_{-T/2}^{T/2} \{\Psi_m(t)\}^2 dt + T$$



Bagian 2

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t) + 1]^2 \cos(2\omega_p t) dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\psi_{mo}^2 \cos^2(\omega_m t) \cos(2\omega_p t) + 2\psi_{mo} \cos(\omega_m t) \cos(2\omega_p t) + \cos(2\omega_p t) \right] dt$$

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t) + 1]^2 \cos(2\omega_p t) dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{\psi_{mo}^2}{2} \left[(1 + \cos(2\omega_m t)) \cos(2\omega_p t) + \frac{2\psi_{mo}}{2} (\cos(\omega_m - 2\omega_p)t + \cos(\omega_m + 2\omega_p)t) + \cos(2\omega_p t) \right] dt$$

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t) + 1]^2 \cos(2\omega_p t) dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\frac{\psi_{mo}^2}{2} \cos(2\omega_p t) + \frac{\psi_{mo}^2}{2} \cos(2\omega_m t) \cos(2\omega_p t) + \psi_{mo} \cos(\omega_m - 2\omega_p)t + \psi_{mo} \cos(\omega_m + 2\omega_p)t + \cos(2\omega_p t) \right] dt$$



$$\int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t) + 1]^2 \cos(2\omega_p t) dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\frac{\psi_{mo}^2}{2} \cos(2\omega_p t) + \frac{\psi_{mo}^2}{2} \frac{1}{2} (\cos 2(\omega_m - \omega_p)t + \cos 2(\omega_m + \omega_p)t) + \psi_{mo} \cos(\omega_m - 2\omega_p)t + \psi_{mo} \cos(\omega_m + 2\omega_p)t + \cos(2\omega_p t) \right] dt$$

Jika $\omega_p \gg \omega_m$ maka persamaan diatas menjadi :

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} [\psi_m(t) + 1]^2 \cos(2\omega_p t) dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\frac{\psi_{mo}^2}{2} \cos(2\omega_p t) + \frac{\psi_{mo}^2}{4} (\cos 2\omega_p t + \cos 2\omega_p t) + \psi_{mo} \cos 2\omega_p t + \psi_{mo} \cos 2\omega_p t + \cos 2\omega_p t \right] dt = 0$$



$$\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{T} \frac{\Psi_{p0}}{2} \int_{-T/2}^{T/2} [\{\Psi_m(t)\}^2 + 2\Psi_m(t) + 1] dt + 0$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \frac{\Psi_{p0}}{2} \left[\int_{-T/2}^{T/2} \{\Psi_m(t)\}^2 dt + \frac{\Psi_{m0}}{\omega_m} + T \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \frac{\Psi_{p0}}{2} \int_{-T/2}^{T/2} \{\Psi_m(t)\}^2 dt + \lim_{T \rightarrow \infty} \frac{1}{T} \frac{\Psi_{p0}}{2} T$$

$$= \frac{\Psi_{p0}}{2} \frac{\Psi_{m0}}{2} + \frac{\Psi_{p0}}{2}$$

$$\bar{P} = \bar{P}_p \bar{P}_m + \bar{P}_p$$

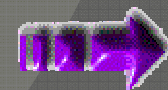
Efisiensi daya transmisi $\varepsilon \rightarrow$ perbandingan daya gelombang DSB terhadap daya gelombang hasil modulasinya :

$$\varepsilon = \frac{P_p P_m}{P_p + P_p P_m} = \frac{P_m}{1 + P_m}$$

Demodulasi AM

Cara yang biasa digunakan untuk demodulasi sinyal AM, yaitu dengan detektor hukum kuadrat terkecil (square law). Tahap pertama dilakukan deteksi dengan detektor yang memiliki hubungan antara masukan $\psi_i(t)$ dan keluaran $\psi_o(t)$ sebagai berikut :

$$\Psi_o(t) = a_1 \Psi_i(t) + a_2 \{\Psi_i(t)\}^2$$



$$\Psi_o(t) = a_1 \Psi_{po} [\Psi_m(t) + 1] \cos(\omega_p t) + a_2 \Psi_{po}^2 [\Psi_m(t) + 1]^2 \cos^2(\omega_p t)$$

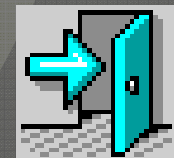
$$\Psi_o(t) = a_1 \Psi_{po} [\Psi_m(t) + 1] \cos(\omega_p t) + \frac{1}{2} a_2 \Psi_{po}^2 \left[\{\Psi_m(t)\}^2 + 2\Psi_m(t) + 1 \right] [1 + \cos(2\omega_p t)]$$

Sinyal yang akan diperoleh kembali adalah suku:

$$a_2 \Psi_{po}^2 \Psi_m(t)$$

Tahap berikutnya memisahkan suku ini dengan filter sederhana asal dipenuhi:

$$|\Psi_m(t)| < 1$$



MODULASI FREKUENSI (FM)

Pada modulasi ini sudut fase dari gelombang pembawa berubah menurut pola perubahan gelombang modulasi. Karena itu modulasi ini tidak bersifat linier, dan tidak dapat diuraikan dengan prinsip superposisi.

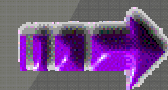
Misalkan gelombang pembawa dinyatakan dengan :

$$\psi_p(t) = \psi_{po} \cos(\omega_p t + \varphi)$$

Maka hasil modulasinya dinyatakan dengan :

$$\psi(t) = \psi_{po} \cos(\omega_p t + \varphi(t))$$

$$\psi(t) = \psi_{po} \cos [\theta(t)]$$



Kemudian dari definisi frekuensi sudut, dapat kita nyatakan :

$$\omega(t) = \frac{d\theta(t)}{dt} = \frac{d[\omega_p t + \varphi(t)]}{dt}$$

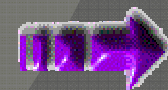
$$\omega(t) = \omega_p + \frac{d\varphi(t)}{dt}$$

$$\omega(t) = \omega_p + \omega'(t)$$

Dengan: $\omega'(t) = \frac{d\varphi(t)}{dt}$

Definisikan $\omega'(t) = K \Psi_m(t)$

K disebut konstanta deviasi frekuensi.



Dari persamaan $\omega'(t) = \frac{d\varphi(t)}{dt}$ dan $\omega'(t) = K \Psi_m(t)$

Kita peroleh: $\omega'(t) = \frac{d\varphi(t)}{dt}$

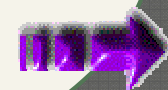
$$K \Psi_m(t) = \frac{d\varphi(t)}{dt}$$

$$\int K \Psi_m(t) dt = \int d\varphi(t)$$

$$K \int \Psi_m(t) dt = \varphi(t)$$

$$K \int \Psi_{mo} \cos(\omega_m t) dt = \varphi(t)$$

$$\frac{K}{\omega_m} \Psi_{mo} \sin(\omega_m t) = \varphi(t)$$



$$\frac{K}{\omega_m} \Psi_{mo} \sin(\omega_m t) = \varphi(t)$$

$$\beta \sin(\omega_m t) = \varphi(t)$$

dengan: $\beta = \frac{K}{\omega_m} \Psi_{mo}$ disebut indeks modulasi FM.

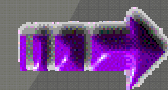
Jadi hasil modulasinya menjadi :

$$\psi(t) = \psi_{po} \cos(\omega_p t + \varphi(t))$$

$$\psi(t) = \psi_{po} \cos(\omega_p t + \beta \sin(\omega_m t))$$

atau dalam bentuk kompleks:

$$\psi(t) = \psi_{po} \operatorname{Re} \left(e^{i\{\omega_p t + \beta \sin(\omega_m t)\}} \right)$$



sedangkan

$$e^{i\beta \sin(\omega_m t)} = \sum_{n=-\infty}^{\infty} c_n e^{in\omega_m t}$$

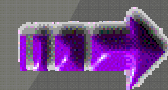
dengan:

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} e^{i\beta \sin(\omega_m t)} e^{-in\omega_m t} dt$$

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i\beta \sin(\omega_m t) - in\omega_m t} dt$$

$$c_n = J_n(\beta)$$

dimana $J_n(\beta)$ ini merupakan
fungsi Bessel jenis satu orde n .



Sehingga kita peroleh:

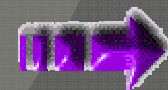
$$e^{i\beta\sin(\omega_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{in\omega_m t}$$

maka

$$\psi(t) = \psi_{po} \operatorname{Re} \left[e^{i\{\omega_p t + \beta \sin(\omega_m t)\}} \right]$$

$$\psi(t) = \psi_{po} \operatorname{Re} \left[\sum_{n=-\infty}^{\infty} J_n(\beta) e^{i\omega_p t} e^{in\omega_m t} \right]$$

$$\psi(t) = \psi_{po} \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[\omega_p + n\omega_m] t$$



Dalam domain frekuensi :

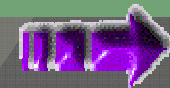
$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi(t) e^{-i\omega t} dt$$

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi_{po} \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[\omega_p + n\omega_m] t e^{-i\omega t} dt$$

$$g(\omega) = \Psi_{po} \sum_{n=-\infty}^{\infty} J_n(\beta) \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{e^{i(\omega_p + n\omega_m)t} + e^{-i(\omega_p + n\omega_m)t}}{2} \right) e^{-i\omega t} dt$$

$$g(\omega) = \frac{\Psi_{po}}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(e^{-i(\omega - \omega_p - n\omega_m)t} + e^{-i(\omega + \omega_p + n\omega_m)t} \right) dt$$

$$g(\omega) = \frac{\Psi_{po}}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(\omega - \omega_p - n\omega_m) + \delta(\omega + \omega_p + n\omega_m)]$$



Dari persamaan $\psi(t) = \psi_{po} \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[\omega_p + n\omega_m]t$ dan

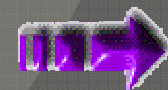
$$g(\omega) = \frac{\psi_{po}}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(\omega - (\omega_p + n\omega_m)) + \delta(\omega + (\omega_p + n\omega_m))]$$

tampak bahwa :

- Hasil frekuensi modulasi dengan sinyal nada tunggal mengandung komponen pembawa dan frekuensi side band yang tak berhingga banyaknya.

$$\omega = \omega_p + n\omega_m, \text{ dengan } n = 1, 2, 3, \dots$$

- Amplitudo masing-masing komponen bergantung pada β . Atau bergantung pada karakteristik informasi $\psi_m(t)$.



- Untuk pita sempit (narrow band), $\beta \ll 1$ rad, maka :

$$J_0(\beta) \approx 1$$

$$J_1(\beta) \approx \beta/2$$

$$J_n(\beta) \approx 0, \text{ untuk } n > 1$$

Jadi pada kasus ini, spektrum frekuensi hanya mengandung komponen ω_p dan $\pm (\omega_p + \omega_m)$, seperti pada hasil modulasi AM.

Grafik fungsi gelombang dalam domain frekuensi FM:

