Lanjutan teori kesalahan
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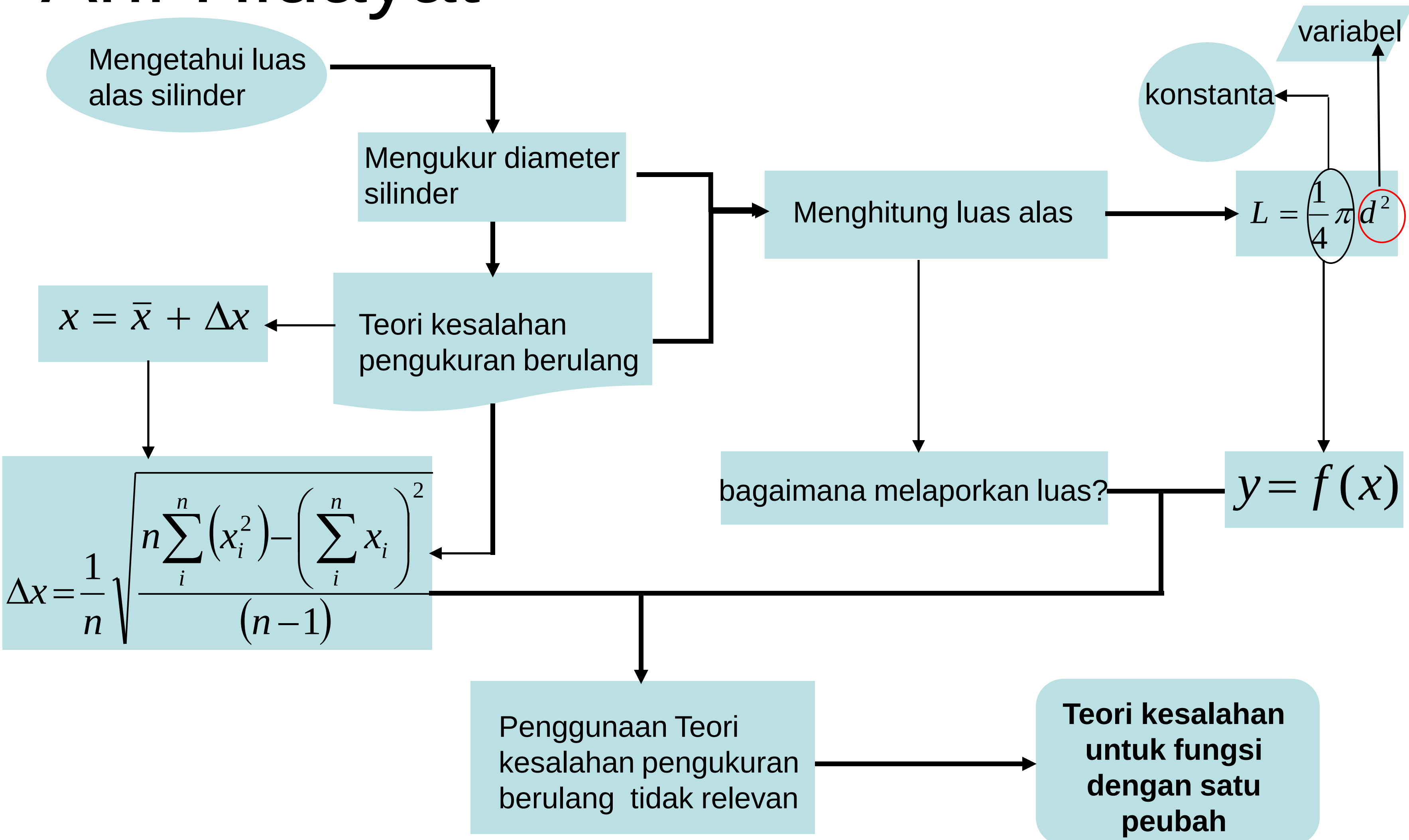


Teori kesalahan untuk fungsi dengan satu peubah

$y = f(x)$

$x = \bar{x} + \Delta x$

$y = f(\bar{x} + \Delta x)$ → Deret Taylor → $x \approx \bar{x}$ → Matfis 2

$$y \cong f(\bar{x}) \pm \left(\frac{\partial f}{\partial x}\right)_{\bar{x}} \Delta x + \frac{1}{2}\left(\frac{\partial^2 f}{\partial x^2}\right)_{\bar{x}} (\Delta x)^2 \pm \ldots$$

jika simpangan data cukup kecil, numerik suku ke-2 dst jauh lebih kecil dari suku pertama, sehingga dapat diabaikan

$$y = f(\bar{x}) \pm \left(\frac{\partial f}{\partial x}\right)_{\bar{x}} \Delta x$$

$\bar{y}$

$$\left|\left(\frac{\partial f}{\partial x}\right)_{\bar{x}}\right| |\Delta x| \rightarrow = |y - \bar{y}| \rightarrow = \Delta y$$

Kita hanya mencari nilai positipnya saja, Mengapa?

$y = f(x)$

$x = \bar{x} + \Delta x$ ↔ $y = \bar{y} \pm \Delta y$

Hanya ada satu peubah

→ Simpangan baku

→ ½ Nilai skala terkecil

Contoh 1: pengukuran tunggal

- Jika diameter penampang sebuah kawat penghantar d = (2,62 ± 0,01) mm, tentukan ketidakpastian luas penampang kawat itu ?

(2,62 ± 0,01) → Alat ukur ? → Jangka sorong

$$d = (2{,}62 \pm 0{,}01)\,\text{mm} \longrightarrow \text{½ nilai skala terkecil}$$

$$\bar{d} = 2{,}62\,\text{mm} \longrightarrow \Delta d = 0{,}01\,\text{mm}$$

$$\bar{A} = \frac{\pi}{4}\bar{d}^2\,\text{mm}^2 \longrightarrow \bar{A} = \frac{3{,}14}{4}(2{,}62)^2\,\text{mm}^2 \longrightarrow \bar{A} = 5{,}39\,\text{mm}^2$$

$$\Delta A = \left|\left(\frac{\partial}{\partial d}A\right)_{\bar{d}}\right| . |\Delta d|\,\text{mm}^2 \longrightarrow \Delta A = 2\left(\frac{\pi}{4}d\right).\Delta d\,\text{mm}^2 \longrightarrow \Delta A = \frac{\pi d}{2}.\Delta d\,\text{mm}^2 \quad : A$$

$$\frac{\Delta A}{A} = \frac{\frac{\pi . d}{2}\Delta d\,\text{mm}^2}{\frac{\pi}{4}d^2\,\text{mm}^2} \longrightarrow \frac{\Delta A}{A} = 2\frac{\Delta d}{d} \longrightarrow \frac{\Delta A}{A} = 2\frac{0{,}01}{2{,}62} \longrightarrow \frac{\Delta A}{A} = 0{,}00763$$

$$\Delta A = 0{,}00763 \,.\, 5{,}39 = 0{,}04 \longrightarrow A = (5{,}39 + 0{,}04)\,\text{mm}^2 \longrightarrow A = (\bar{A} + \Delta A)\,\text{mm}^2$$

Contoh 2: pengukuran berulang

- Jika diameter penampang sebuah kawat penghantar berdasarkan percobaan pengukuran berulang 10 kali diperoleh hasil seperti di bawah ini, tentukan ketidakpastian luas penampang kawat itu.

No	d_i (mm)
1	2,63
2	2,62
3	2,61
4	2,63
5	2,61
6	2,61
7	2,63
8	2,60
9	2,60
10	2,61

No	d_i (mm)	$\left|d_i - \bar{d}\right|$ (mm)	(mm²) . 10⁻⁴ $\left|d_i - \bar{d}\right|^2$
1	2,63	0,01	1
2	2,62	0,00	0
3	2,61	0,01	1
4	2,63	0,01	1
5	2,61	0,01	1
6	2,61	0,01	1
7	2,63	0,01	1
8	2,60	0,02	4
9	2,60	0,02	4
10	2,61	0,01	1
∑	26,25	0,11	15

$$\bar{d} = 2{,}62\,\text{mm}$$

↓

$$\Delta d = \sqrt{\frac{\sum_{1}^{10}\left(d_i - \bar{d}\right)^2}{n(n-1)}}$$

↓

$$\Delta d = \sqrt{\frac{\left(15.10^{-4}\right)^2}{(90)}}$$

↓

$$\Delta d = 0{,}000041\,\text{mm}$$

↓

Karena aturan angka signifikan dan penyesuaian dengan ketelitian alat

↓

$$\bar{A} = \frac{\pi}{4}\bar{d}^2\ \text{mm}^2$$

↓

$$\bar{A} = \frac{3{,}14}{4}2{,}62^2\ \text{mm}^2$$

↓

$$\bar{A} = 5{,}38\,\text{mm}^2$$

$$y = \bar{y} \pm \Delta y$$

Bagaimana menentukan Δy untuk pengukuran berulang?

$$\Delta y_i = \left(\frac{\partial y}{\partial x}\right)_{\bar{y}} \Delta x_i$$

Δy

$$\Delta y = \frac{\sqrt{\sum_{1}^{n}\left[(y_i - \bar{y})^2\right]}}{n(n-1)} = S_Y$$

$$\Delta y = S_Y = \frac{\sqrt{\sum_{1}^{n}\left[\left(\frac{\partial y}{\partial x}\right)_{\bar{X}}^{2} \Delta x^2\right]}}{n(n-1)}$$

$$\Delta y^2 = S_Y^{\ 2} = \left(\frac{\partial y}{\partial x}\right)_{\bar{X}}^{2} \frac{\sum_{1}^{n}\left(\Delta x^2\right)}{n(n-1)}$$

$$\Delta x^2 = S_X^{\ 2} = \frac{\sum_{1}^{n}\left(\Delta x^2\right)}{n(n-1)}$$

$$\Delta x^2 = S_X^{\ 2} = \frac{\sum_{1}^{n}\left(\Delta x^2\right)}{n(n-1)}$$

$$\Delta y^2 = \left(\frac{\partial y}{\partial x}\right)_{\bar{X}}^{2} S_X$$

$$\Delta y = \left(\frac{\partial}{\partial x}\right)_{\bar{X}} \sqrt{S_X}$$

$$\Delta y = \left(\frac{\partial y}{\partial x}\right)_{\bar{X}} . \sqrt{\Delta x}$$

Nilai Δx dari pengukuran berulang (simpangan)

$$y = \bar{y} \pm \Delta y$$

$$y = f(\bar{x}) \pm \left(\frac{\partial f}{\partial x}\right)_{\bar{x}} \Delta x$$ **Pengukuran tunggal**

$$y = f(\bar{x}) \pm \left(\frac{\partial f}{\partial x}\right)_{\bar{x}} \sqrt{\Delta x}$$ **Pengukuran berulang**

Mari Lanjutkan hitung Luas
untuk pengukuran berulang:

↓

$$\frac{\Delta A}{A} = \frac{\frac{\pi . d^2}{2}\sqrt{\Delta d}\ \text{mm}^2}{\frac{\pi}{4} d^2\ \text{mm}^2}$$

↓

$$\frac{\Delta A}{A} = \frac{2\sqrt{\Delta d}\ \text{mm}^2}{d\ \text{mm}^2}$$

↓

$$\Delta A = \frac{2(5{,}39\ \text{mm}^2)\ \sqrt{0{,}000041}\ \text{mm}}{2{,}62\ \text{mm}}$$

↓

$$\Delta A = \left|\left(\frac{\partial}{\partial d} A\right)_{\bar{d}}\right| . \left|\sqrt{\Delta d}\right|\ \text{mm}^2$$

↓

$$\Delta A = 0{,}02\ \text{mm}^2$$

↓

$$A = \left(\bar{A} \pm \Delta A\right) = \left(5{,}39 \pm 0{,}02\right)\text{mm}^2$$

Pengukuran tunggal

↑

$$A = \left(5{,}39 + 0{,}04\right)\ \text{mm}^2$$

$$y = \left(\bar{y} \pm \Delta y\right)\ \text{mm}^2$$

$$A = \left(5{,}38 \pm 0{,}02\right)\text{mm}^2$$

↓

Pengukuran berulang

Mengapa di peroleh Δy yang lebih kecil ?

↓

Tujuan pengukuran berulang berupaya memperkecil sumber-sumber kesalahan dalam pengukuran

Kasus yang lain

Menghitung percepatan gravitasi bumi

Percobaan bandul sederhana

Asumsi-asumsi fisis

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Menghitung *g*

$$g = 4\pi^2 \frac{l}{T^2}$$

konstanta

Variabel ke-1

Variabel ke-2

Mengukur periode ayunan T

Mengukur panjang tali *l*

$$x = \bar{x} + \Delta x$$

$$\Delta x = \frac{1}{n}\sqrt{\frac{n\sum_{i}^{n}\left(x_i^2\right) - \left(\sum_{i}^{n} x_i\right)^2}{(n-1)}}$$

Teori kesalahan pengukuran berulang

$$z = f(x, y)$$

Bagaimana melaporkan percepatan gravitasi?

Penggunaan Teori kesalahan untuk fungsi dengan satu peubah tidak relevan

Teori kesalahan untuk fungsi dengan dua peubah

Δ x dan Δy pengukuran tunggal

Δ x dan Δy pengukuran berulang

Δ x / Δy salah satu pengukuran berulang atau tunggal

$$z = f(x, y) \quad \begin{cases} x = x_0 \pm \Delta x \\ y = y_0 \pm \Delta y \end{cases} \quad z = f(x_0 \pm \Delta x \,,\, y_0 \pm \Delta y)$$

Deret Taylor di $x=x_0$ dan $y=y_0$

Suku ke-2 dst di abaikan

$$z = z_0(x, y) + \left\{ \left| \left(\frac{\partial z}{\partial x}\right)_{X_0, Y_0} \right| |\Delta x| + \left| \left(\frac{\partial z}{\partial y}\right)_{X_0, Y_0} \right| |\Delta y| \right\}$$

Δx : pengukuran tunggal

Δy : pengukuran tunggal

Δx : pengukuran tunggal

Δy : pengukuran berulang

Δx : pengukuran berulang

Δy : pengukuran berulang

Menentukan percepatan gravitasi dng percobaan Bandul sederhana

Mengukur periode ayunan 1 kali ← → Mengukur panjang tali1 kali

Δx : pengukuran tunggal

Δy : pengukuran berulang

T = (2,00 ± 0,05) s

ℓ = (1,0000 ± 0,0005).10^2 cm

$$g = 4\pi \frac{l}{T^2} \longrightarrow \bar{g} = 4.3{,}14 \frac{100{,}00\ \text{cm}}{(2{,}00\,\text{s})^2} \longrightarrow \bar{g} = 985\ \frac{\text{cm}}{\text{s}^2}$$

$$\Delta g = \left(\frac{\partial g}{\partial l}\right)_T \Delta l + \left(\frac{\partial g}{\partial T}\right)_l \Delta T \longrightarrow \Delta g = \left|\frac{4\pi^2}{T^2}\right| |\Delta l| + \left|2\frac{4\pi^2 l}{T^3}\right| |\Delta T|$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\,\frac{\Delta T}{T} \longrightarrow \frac{\Delta g}{g} = \left(\frac{0{,}05}{100{,}00}\right) + 2\left(\frac{\Delta 0{,}05}{2{,}00}\right) \longrightarrow \Delta g = 0{,}05\ .\ 985\ \text{cm}^2$$

$$\Delta g = 5\ \text{cm}^2$$

$$g = (\bar{g} \pm \Delta g) = (9{,}85 \pm 0{,}5)\ \text{cm}^2$$

Menentukan percepatan gravitasi dng Bandul sederhana

Mengukur periode ayunan 10 kali → **Menentukan percepatan gravitasi dng Bandul sederhana** ← Mengukur panjang tali10 kali

Δx : pengukuran berulang

Δy : pengukuran berulang

Bagaimana melaporkannya?

$$Z = f\left(x, y\right) \longrightarrow \Delta Z_i = \left(\frac{\partial Z}{\partial x}\right)\Delta x_i + \left(\frac{\partial Z}{\partial y}\right)\Delta y_i \longrightarrow \Delta Z = S_Z = \sqrt{\frac{\sum_1^n \left(Z_i - \overline{Z}\right)^2}{n\left(n-1\right)}}$$

$$\left(S_Z\right)^2 = \frac{\sum_1^n \left[\left(\frac{\partial Z}{\partial x}\right)\Delta x_i + \left(\frac{\partial Z}{\partial y}\right)\Delta y_i\right]^2}{n\left(n-1\right)}$$

$$\left(S_Z\right)^2 = \frac{\left(\frac{\partial Z}{\partial x}\right)^2 \sum_1^n \left(\Delta x_i\right)^2 + \left(\frac{\partial Z}{\partial y}\right) \sum_1^n \left(\Delta y_i\right)^2 + 2\left(\frac{\partial Z}{\partial x}\right)\left(\frac{\partial Z}{\partial y}\right)\sum_1^n \left(\Delta y_i\right)^2\left(\Delta x_i\right)^2}{n\left(n-1\right)}$$

$$\left(S_Z\right)^2 = \left(\frac{\partial Z}{\partial x}\right)^2 \left(S_X\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2 \left(S_X\right)^2 \longrightarrow S_Z = \sqrt{\left(\frac{\partial Z}{\partial x}\right)^2 \left(S_X\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2 \left(S_X\right)^2}$$

DATA

Menentukan percepatan gravitasi dng Bandul sederhana

Mengukur periode ayunan 10 kali

Mengukur panjang tali10 kali

Data Periode (T)

Data Pj Tali(T)

No	T (s)
1	2,03
2	2,02
3	2,01
4	2,03
5	2,01
6	2,01
7	2,01
8	2,00
9	2,00
10	2,01

No	T^2 (s^2)	$\lvert T^2_i - \bar{T}^2 \rvert$ (s^2)	$\lvert T^2_i - \bar{T}^2 \rvert^2$ (s^4)
1	2,03		
2	2,02		
3	2,01		
4	2,03		
5	2,01		
6	2,01		
7	2,01		
8	2,00		
9	2,00		
10	2,01		
∑	20,13	9,5	11.01

No	l (m)
1	1,04
2	1,02
3	1,06
4	1,06
5	1,02
6	1,02
7	1,04
8	1,00
9	1,06
10	1,04

No	l_i (mm)	$\lvert l_i - \bar{l} \rvert$ (m)	$\lvert l_i - \bar{l} \rvert^2$ (m^2)
1	1,04		
2	1,02		
3	1,06		
4	1,06		
5	1,02		
6	1,02		
7	1,04		
8	1,00		
9	1,06		
10	1,04		
∑	10,36	28	14.8

$$\bar{T} = 2{,}01\,\mathrm{s}^2$$

$$\bar{l} = 103{,}6\,\mathrm{cm}$$

$$\Delta T = \sqrt{\frac{\sum_{1}^{10}\left(T_i - \bar{T}^2\right)^2}{n(n-1)}}$$

$$\Delta l = \sqrt{\frac{\sum_{1}^{10}\left(l_i - \bar{l}\right)^2}{n(n-1)}}$$

$$\Delta T = \sqrt{\frac{(11.01)^2}{(90)}}$$

$$\Delta l = \sqrt{\frac{(28)^2}{(90)}}$$

$$\Delta T = 1{,}34\,\mathrm{mm}$$

$$\Delta l = 8{,}711\,\mathrm{cm}$$

$$\bar{g} = 4\pi \frac{\bar{l}}{T^2} = 10{,}11\,\frac{cm}{s^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\,\frac{\Delta T}{T}$$

$$\frac{S_Z}{\bar{g}} = \sqrt{\left(2\left(\frac{S_T}{T}\right)\right)^2 + \left(\frac{S_l}{l}\right)^2}$$

$$\frac{S_Z}{\bar{g}} = \sqrt{\left(2\left(\frac{1{,}16}{2{,}01}\right)\right)^2 + \left(\frac{0{,}000041}{103{,}6}\right)^2}$$

$$g = \bar{g} + \Delta g$$

$$g = 10{,}11 + \Delta g$$

Menentukan percepatan gravitasi dng Bandul sederhana

Mengukur periode ayunan 10 kali

Mengukur panjang tali 1 kali

Δx : pengukuran tunggal

Δy : pengukuran berulang

No	*T* (s)
1	2,03
2	2,02
3	2,01
4	2,03
5	2,01
6	2,01
7	2,01
8	2,00
9	2,00
10	2,01

tugas

Yang dilakukan di LFD

$l = (1{,}0000 \pm 0{,}0005).10^2$ cm

Ada 2 cara

$\Delta T = 3\,S_T$

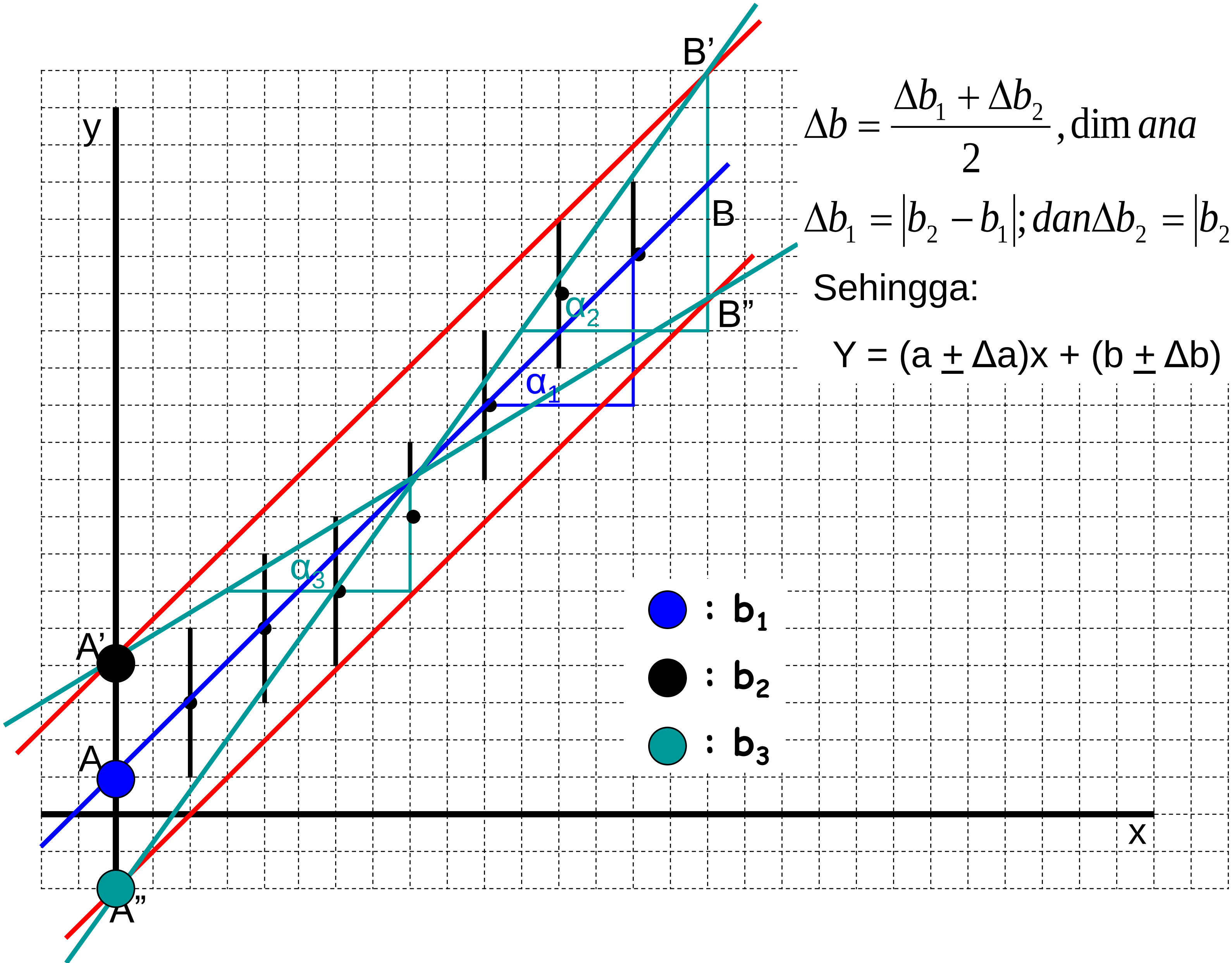
$\Delta T = S_T$

$$l = \bar{l} + \Delta l$$

$$l = \left(\bar{l} + \frac{1}{3}\Delta l\right)$$

Dimensi isotropik

Grafik Metode Garis Sejajar (Review)



$$\Delta b = \frac{\Delta b_1 + \Delta b_2}{2}, \text{dim}\,ana$$

$$\Delta b_1 = |b_2 - b_1|; dan \Delta b_2 = |b_2 - b_3|$$

Sehingga:

Y = (a ± Δa)x + (b ± Δb)

Bagaimana Jika Menggambar Grafik dengan bantuan Program Komputer?

- 6 laporan akhir jika melibatkan grafik harus menggunakan metode garis sejajar
- Grafik dan teori kesalahan menggunakan program komputer wajib digunakan hanya sebagai pembanding
- Bagaimana Program Komputernya?
- Ms. Excel ; Microcal Origin ; SPSS; MathLab