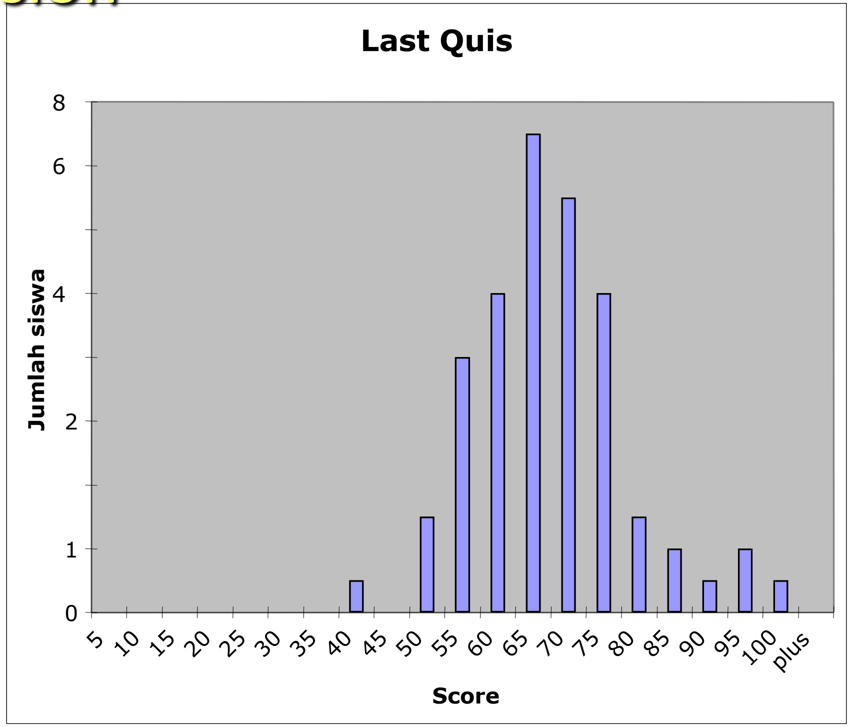
Tuesday, 3rd Nov 2009

Today's Topics:

- I really appreciate If all Of U can stand till the end of the course because of the language that U choose for U in this session
- Homework 6:
  - Sorry for delay
  (I've got sick)
- Last Quis:
  - Average: 65%
- Now:
  - Linear momentum
  - Collision



Linear Momentum

- **Definition:** For a single particle, the linear momentum $\boldsymbol{p}$ is defined as:

$$\boldsymbol{p} = m\boldsymbol{v}$$

($\boldsymbol{p}$ is a vector since $\boldsymbol{v}$ is a vector).

So $p_x = mv_x$ etc.

- Newton's 2nd Law:

$$\mathbf{F} = m\mathbf{a} = m\frac{\Delta \mathbf{v}}{\Delta t} = \frac{\Delta(m\mathbf{v})}{\Delta t}$$

$$\Rightarrow \quad \mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}$$

- Units of linear momentum are *kg m/s*.

Impulse-momentum theorem:

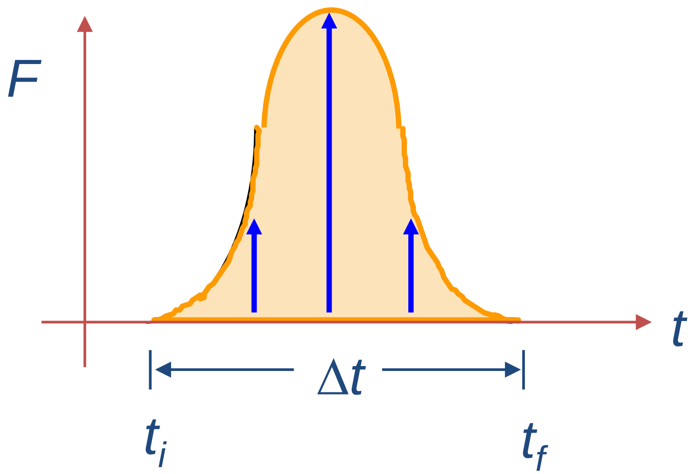
- The impulse of the force action on an object equals the change in momentum of the object:

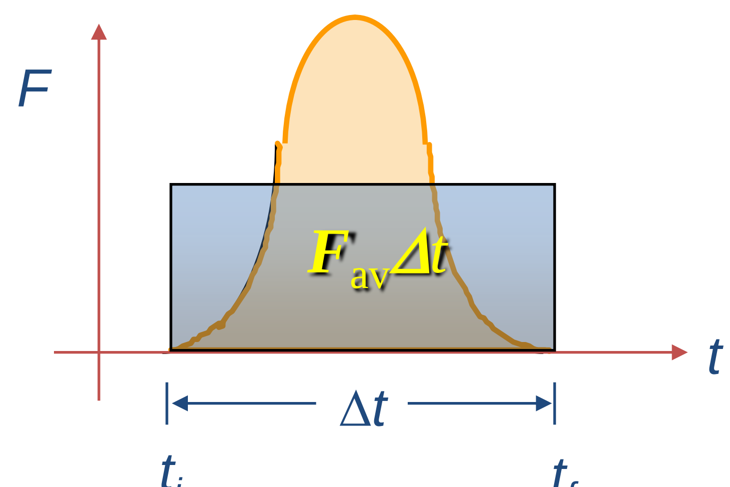
$$\mathbf{F}\Delta t = \Delta \mathbf{p} = m\mathbf{v}_f - m\mathbf{v}_i$$

Impulse has units of *Ns*.

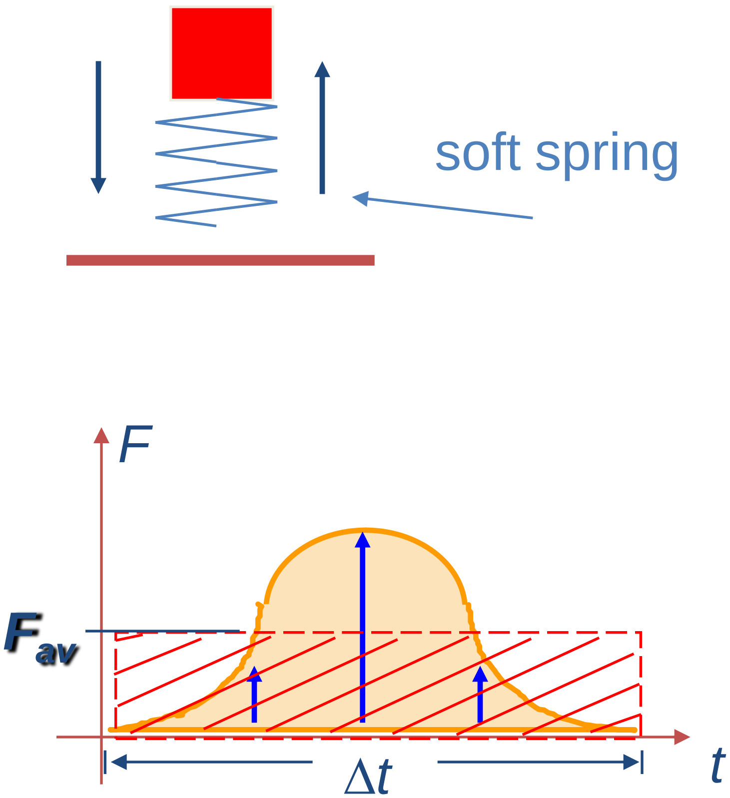
$$\text{Impulse} \equiv \mathbf{F}\Delta t = \Delta \mathbf{p}$$

- The impulse imparted by a force during the time interval Δt is equal to area under the force-time graph from beginning to the end of the time interval.

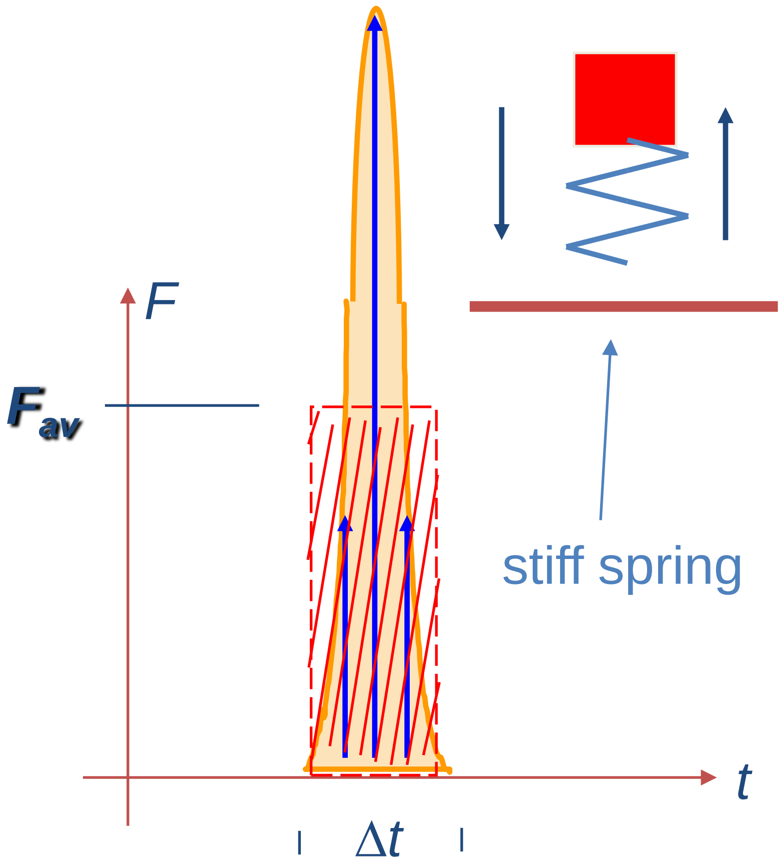




Average Force and Impulse



Δt big, F_{av} small



Δt small, F_{av} big

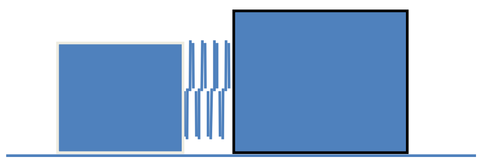
Momentum Conservation

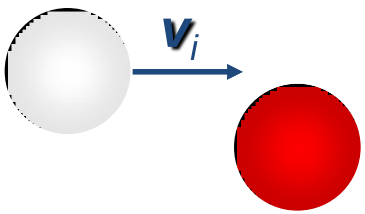
$$\boldsymbol{F}_{EXT} = \frac{\Delta p}{\Delta t} \quad \Rightarrow \quad \boxed{\frac{\Delta \boldsymbol{p}}{\Delta t} = 0} \quad \Leftarrow \quad \boldsymbol{F}_{EXT} = 0$$

- The concept of momentum conservation is one of the most fundamental principles in physics.
- This is a component (vector) equation.
  - We can apply it to any direction in which there is no external force applied.
- You will see that we often have momentum conservation even when energy is not conserved.

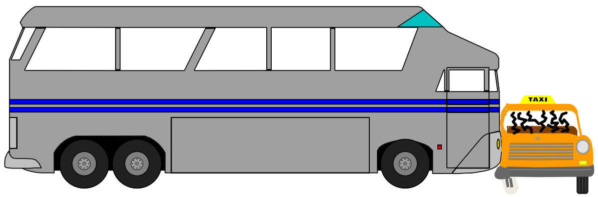
Elastic vs. Inelastic Collisions

- A collision is said to be *elastic* when energy as well as momentum is conserved before and after the collision. $K_{before} = K_{after}$
  - Carts colliding with a spring in between, billiard balls, etc.





- A collision is said to be *inelastic* when energy is not conserved before and after the collision, but momentum is conserved. $K_{before} \neq K_{after}$
  - Car crashes, collisions where objects stick together, etc.



Inelastic collision in 1-D: Example 1

- A block of mass *M* is initially at rest on a frictionless horizontal surface. A bullet of mass *m* is fired at the block with a muzzle velocity (speed) *v*. The bullet lodges in the block, and the block ends up with a speed *V*. In terms of *m, M,* and *V* :
  - What is the momentum of the bullet with speed *v* ?
  - What is the initial energy of the system ?
  - What is the final energy of the system ?
  - Is energy conserved ?

before after

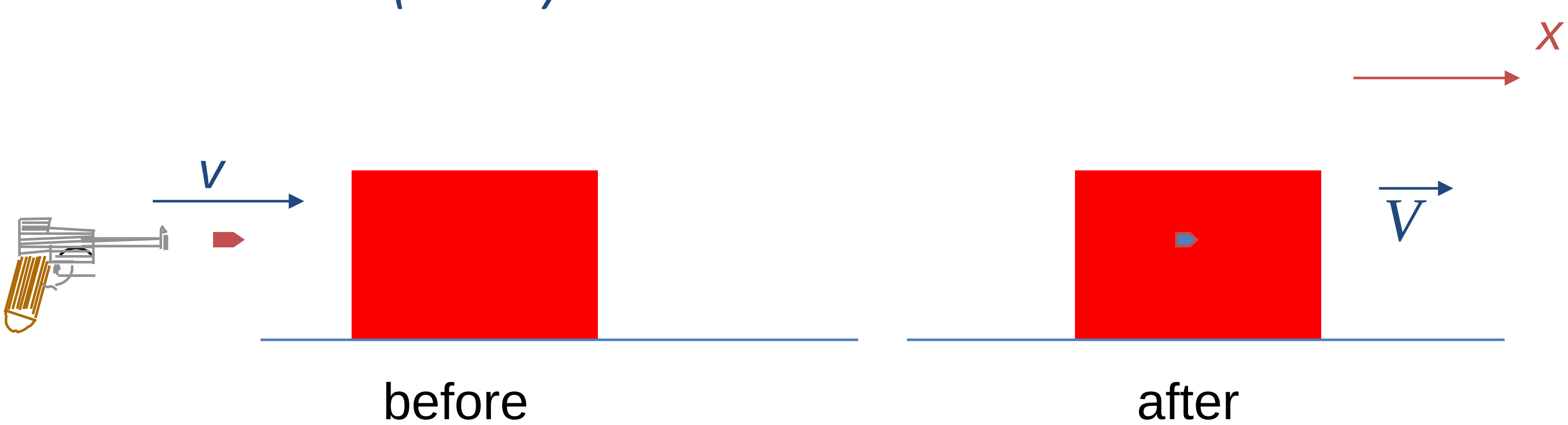
Example 1...

- Consider the bullet & block as a system. After the bullet is shot, there are no external forces acting on the system in the *x*-direction

Momentum is conserved in the x direction !

  - $P_{x,\text{before}} = P_{x,\text{after}}$
  - $mv = (M+m)\,V$

$$\Rightarrow \quad v = \left(\frac{M+m}{m}\right)V$$



$$v = \left(\frac{M+m}{m}\right)V$$

Example 1...

- Now consider the energy of the system before and after:
- Before:
- After: $E_B = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{M+m}{m}\right)^2 V^2 = \frac{1}{2}\left(\frac{M+m}{m}\right)(M+m)V^2$
- So

$$E_A = \frac{1}{2}(M+m)V^2$$

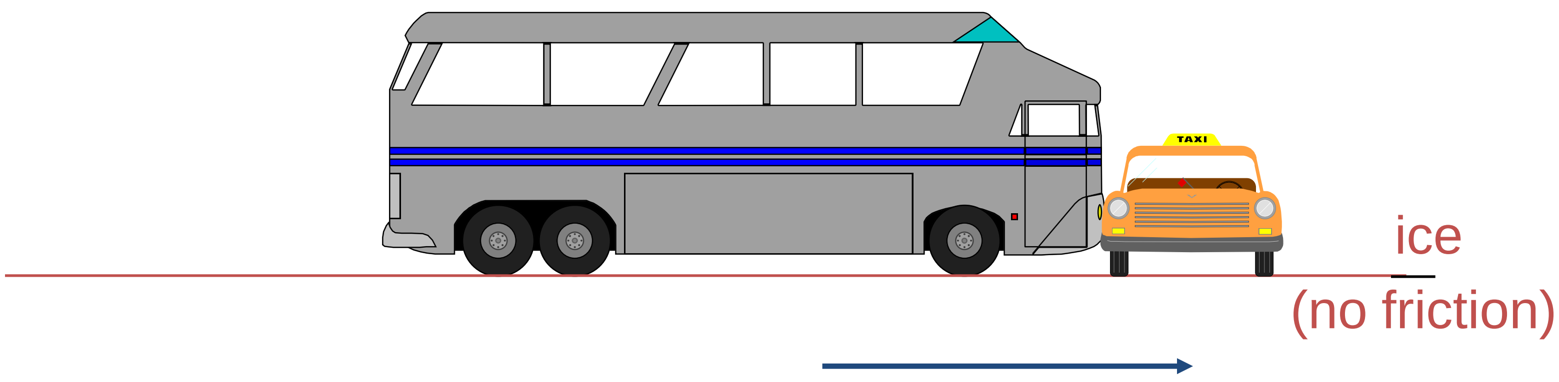
$$\boxed{E_A = \left(\frac{m}{M+m}\right)E_B}$$

Energy is NOT conserved (friction stopped the bullet)

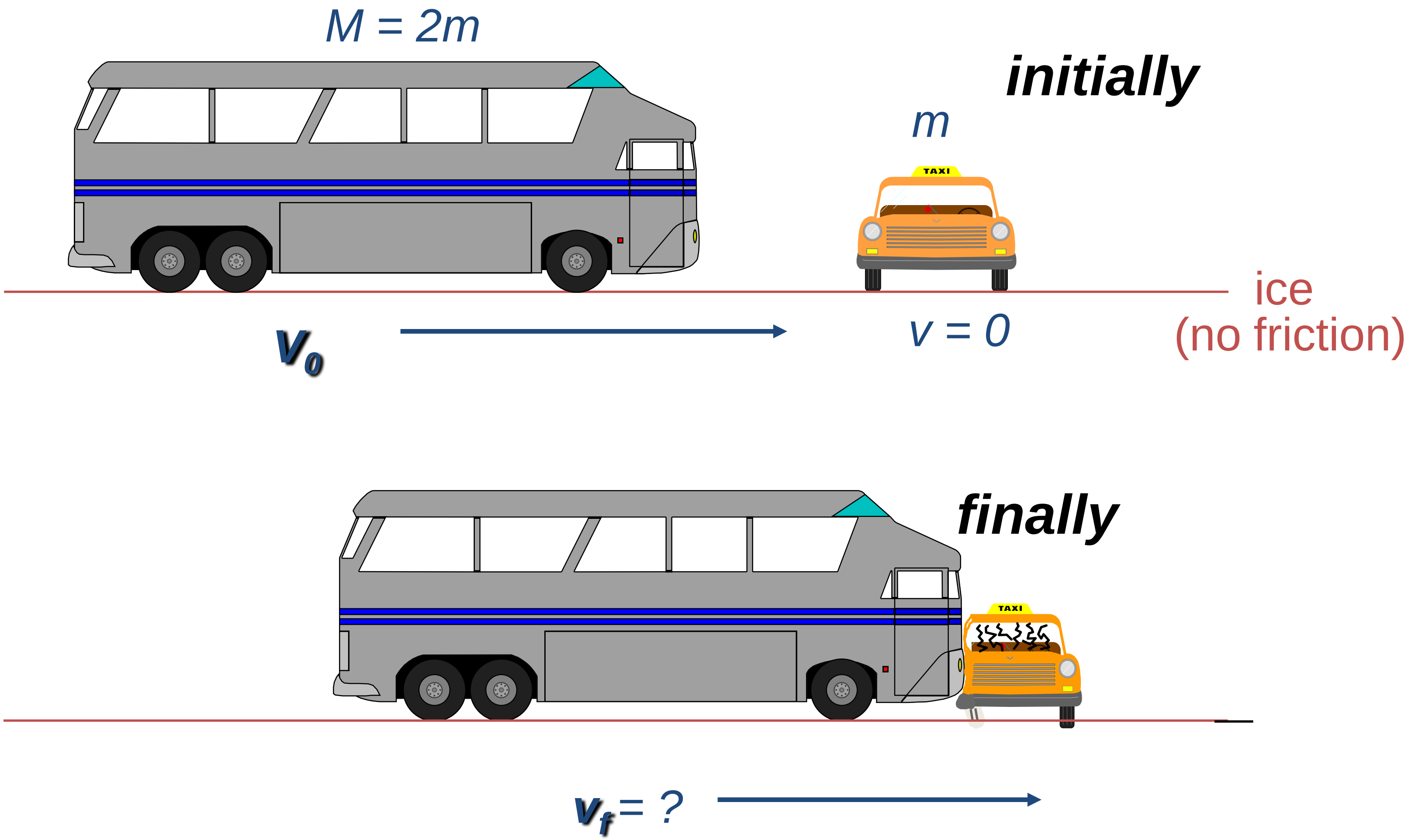
However momentum was conserved, and this was useful.

Inelastic Collision in 1-D

Winter in Storrs



Inelastic Collision in 1-D



$V_f =$ A) 0 B) $V_o/2$ C) $2V_o/3$ D) $3V_o/2$ E) $2V_o$

Inelastic Collision in 1-D

Use conservation of momentum to find ***v*** after the collision.

Before the collision:

$$\boldsymbol{P}_i = M\boldsymbol{V_o} + m(0)$$

After the collision:

$$\boldsymbol{P}_f = (M+m)\boldsymbol{V}_f$$

Conservation of momentum:

$$\Rightarrow \quad \boldsymbol{P}_i = \boldsymbol{P}_f$$

$$M\boldsymbol{V_o} = (M+m)\boldsymbol{V}_f$$

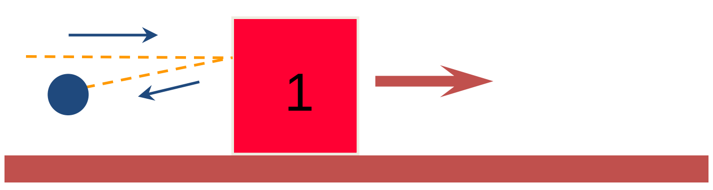
$$\Rightarrow \quad \boldsymbol{V}_f = \frac{M}{(M+m)}\boldsymbol{V_o} = \frac{2m}{(2m+m)}\boldsymbol{V_o} = \frac{2}{3}\boldsymbol{V_o}$$

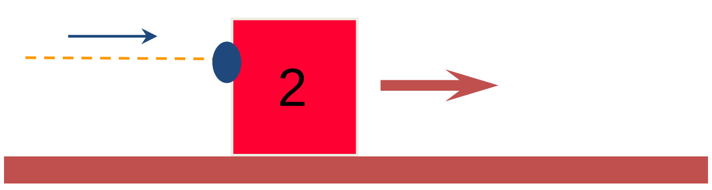
C) $2V_o/3$

Momentum Conservation

- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.
- The ball hitting box 1 bounces back, while the ball hitting box 2 gets stuck.
  - Which box ends up moving fastest ?

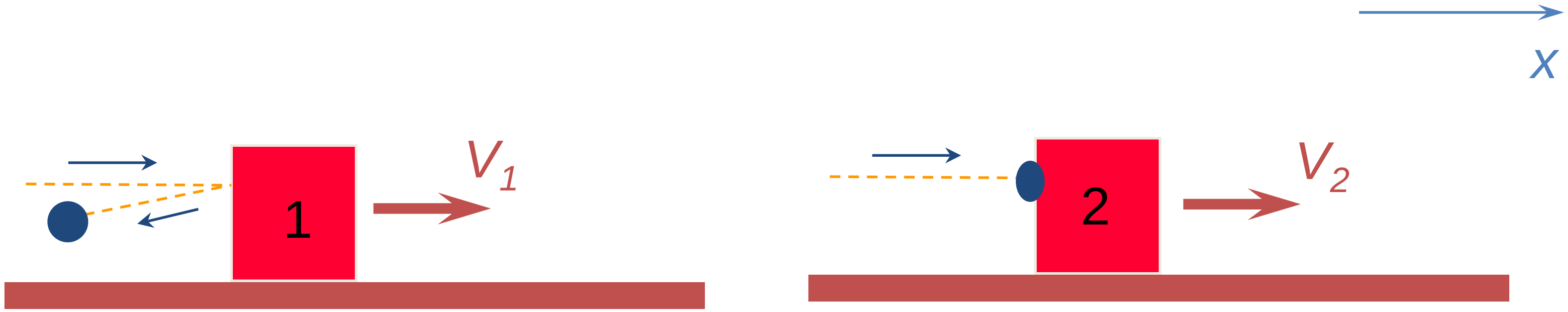
(a) Box 1 **(b) Box 2** **(c) same**





Momentum Conservation

- Since the total external force in the x-direction is zero, momentum is conserved along the x-axis.
- In both cases the initial momentum is the same (*mv* of ball).
- In case 1 the ball has negative momentum after the collision, hence the box must have more positive momentum if the total is to be conserved.
- The speed of the box in case 1 is biggest !



Momentum Conservation

$$mv = MV_1 - mv'$$

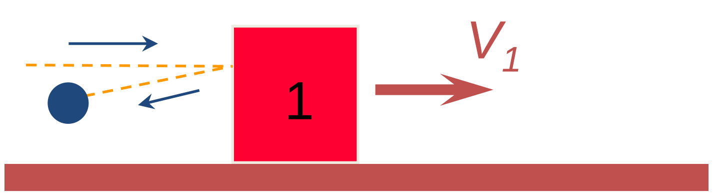
$$\Rightarrow V_1 = (mv + mv') / M$$

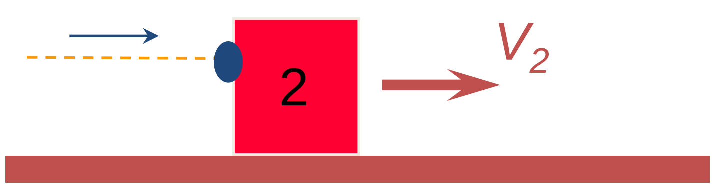
$$mv = (M+m)V_2$$

$$\Rightarrow V_2 = mv / (M+m)$$

V_1 numerator is bigger and its denominator is smaller than that of V_2.

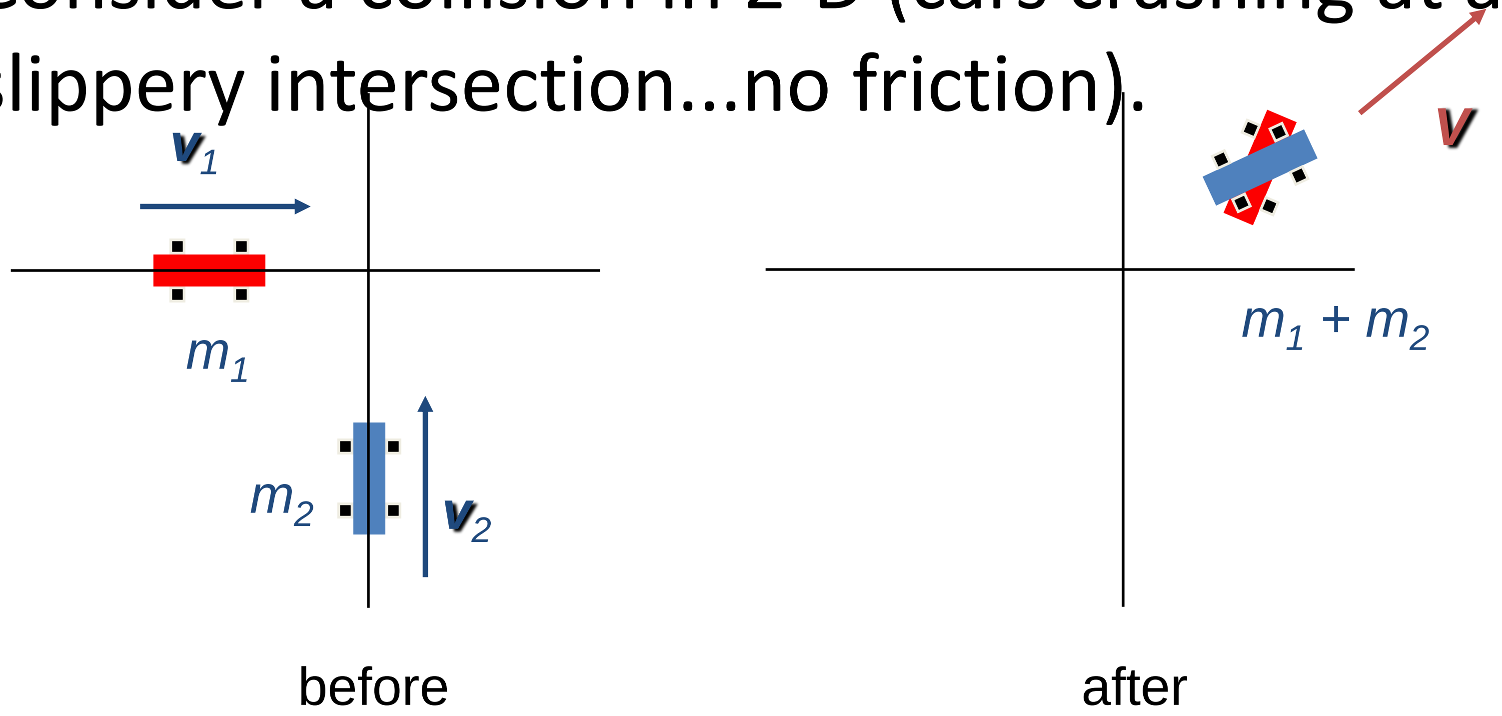
$$\Rightarrow V_1 > V_2$$





Inelastic collision in 2-D

- Consider a collision in 2-D (cars crashing at a slippery intersection...no friction).

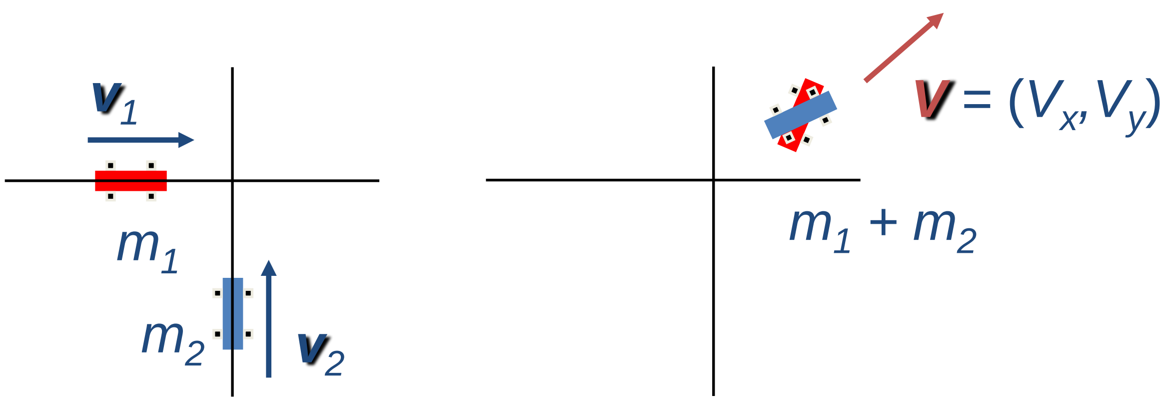


Inelastic collision in 2-D...

- There are no net external forces acting.
  - Use momentum conservation for both components.

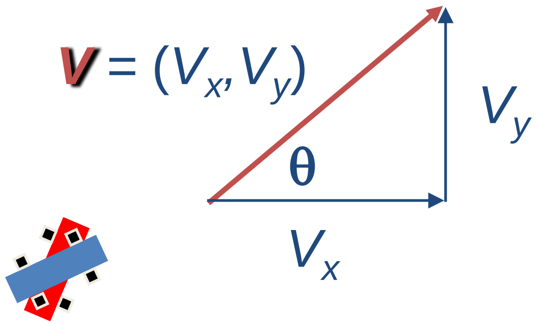
$$P_{x,a} = P_{x,b} \Rightarrow m_1 v_1 = (m_1 + m_2) V_x \Rightarrow V_x = \frac{m_1}{(m_1 + m_2)} v_1$$

$$P_{y,a} = P_{y,b} \Rightarrow m_2 v_2 = (m_1 + m_2) V_y \Rightarrow V_y = \frac{m_2}{(m_1 + m_2)} v_2$$



Inelastic collision in 2-D...

- So we know all about the motion after the collision !



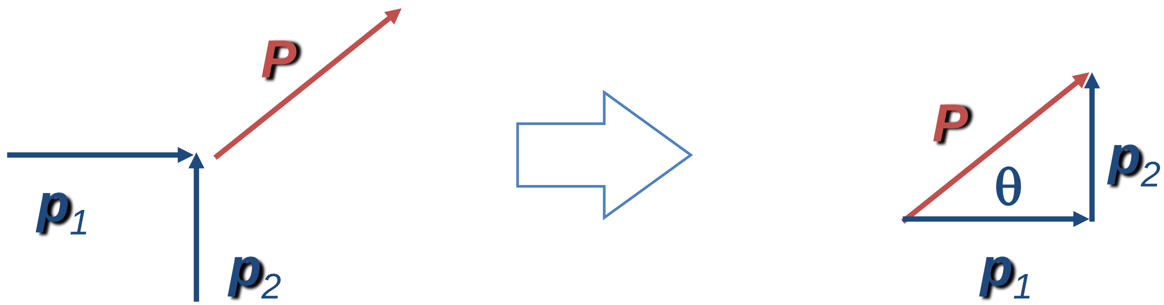
$$V_x = \frac{m_1}{(m_1 + m_2)} v_1$$

$$V_y = \frac{m_2}{(m_1 + m_2)} v_2$$

$$\tan\theta = \frac{V_y}{V_x} = \frac{m_2 v_2}{m_1 v_1} = \frac{p_2}{p_1}$$

Inelastic collision in 2-D...

- We can see the same thing using vectors:

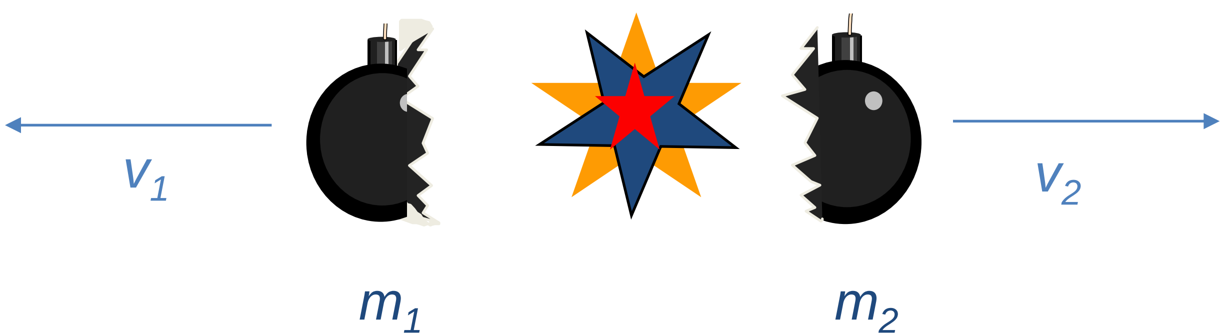


$$\tan\theta = \frac{p_2}{p_1}$$

Explosion (inelastic un-collision)

Before the explosion:

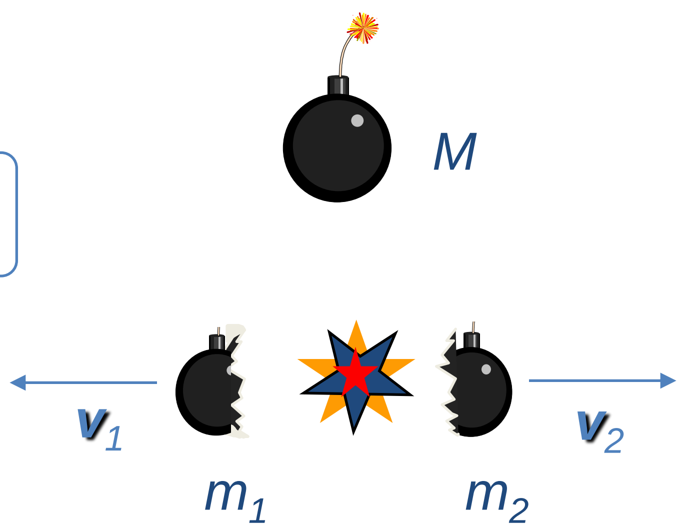
After the explosion:



Explosion...

- No external forces, so $\boldsymbol{P}$ is conserved.
- Initially: $\boldsymbol{P} = 0$
- Finally: $\boldsymbol{P} = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2 = 0$

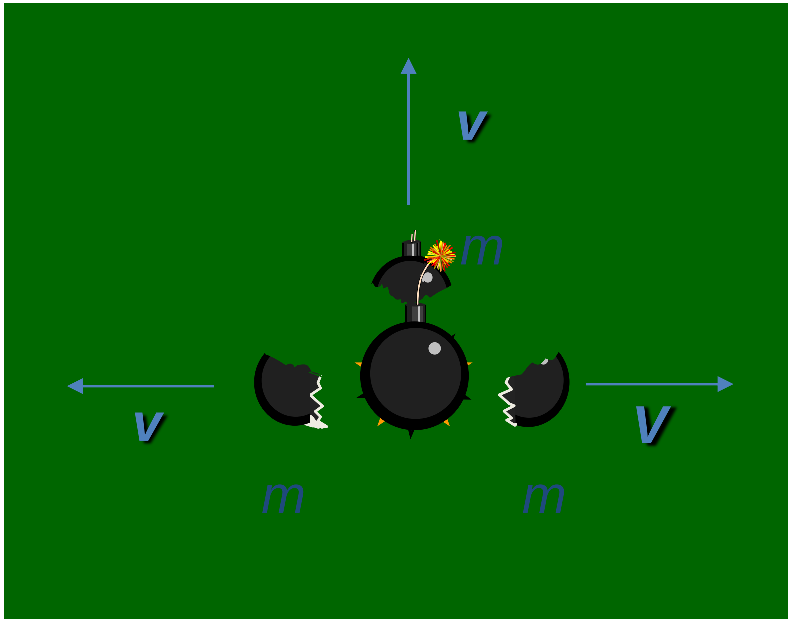
$$m_1\boldsymbol{v}_1 = - m_2\boldsymbol{v}_2$$



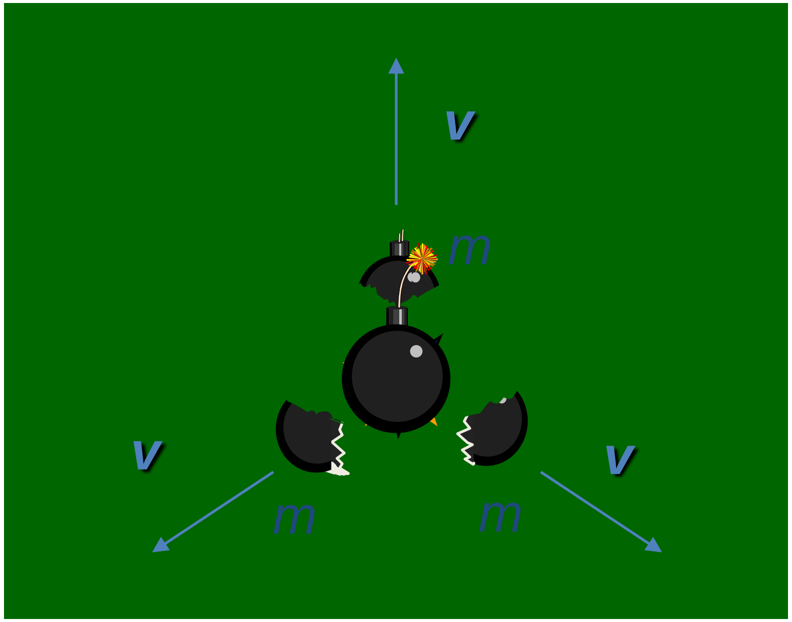
Center of Mass

- A bomb explodes into 3 identical pieces. Which of the following configurations of velocities is possible?

(a) ***1*** **(b)** ***2*** **(c)** ***both***



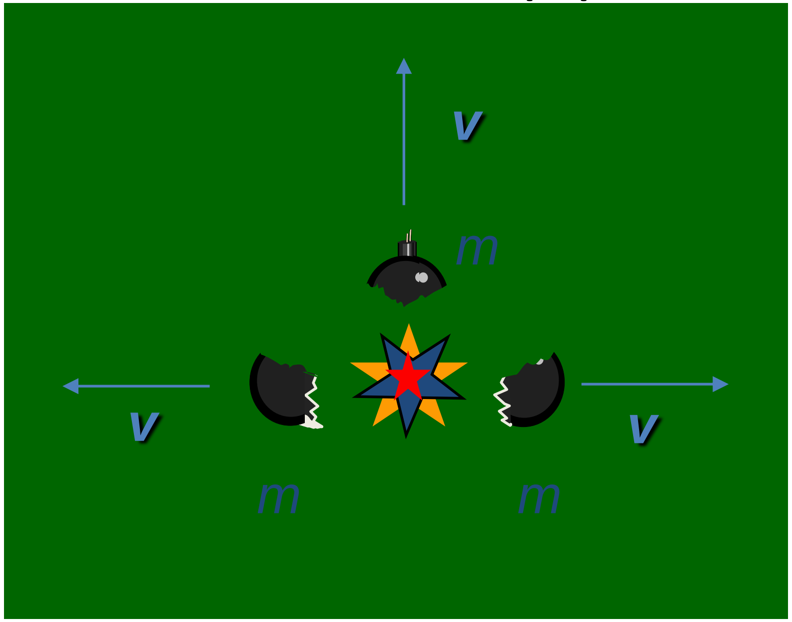
(1)



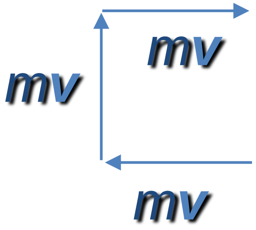
(2)

Solution

- No external forces, so ***P*** must be conserved.
- Initially: $\boldsymbol{P} = 0$
- In explosion *(1)* there is nothing to balance the upward momentum of the top piece so $\boldsymbol{P}_{final} \neq 0$.

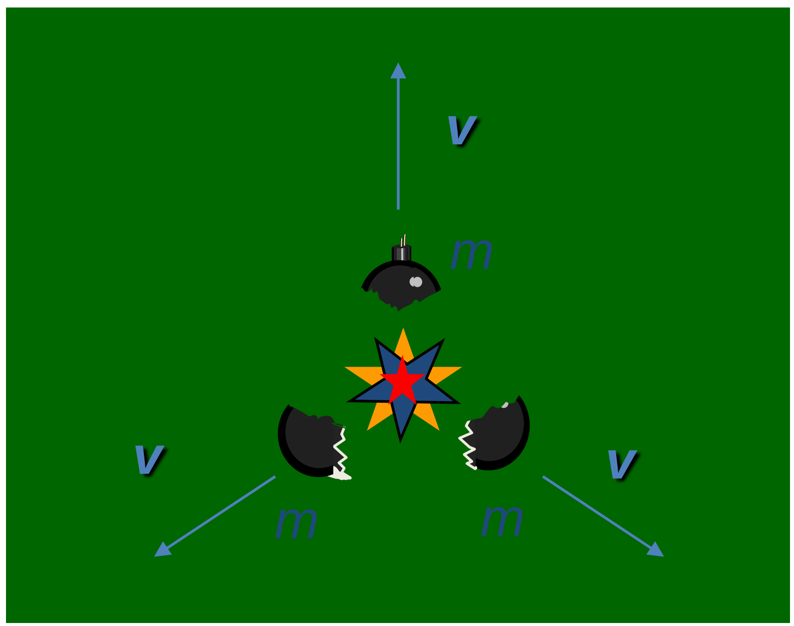


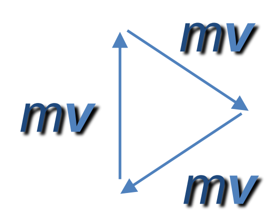
(1)



Solution

- No external forces, so ***P*** must be conserved.
- All the momenta cancel out.
- $\boldsymbol{P_{final}} = 0$.





(2)

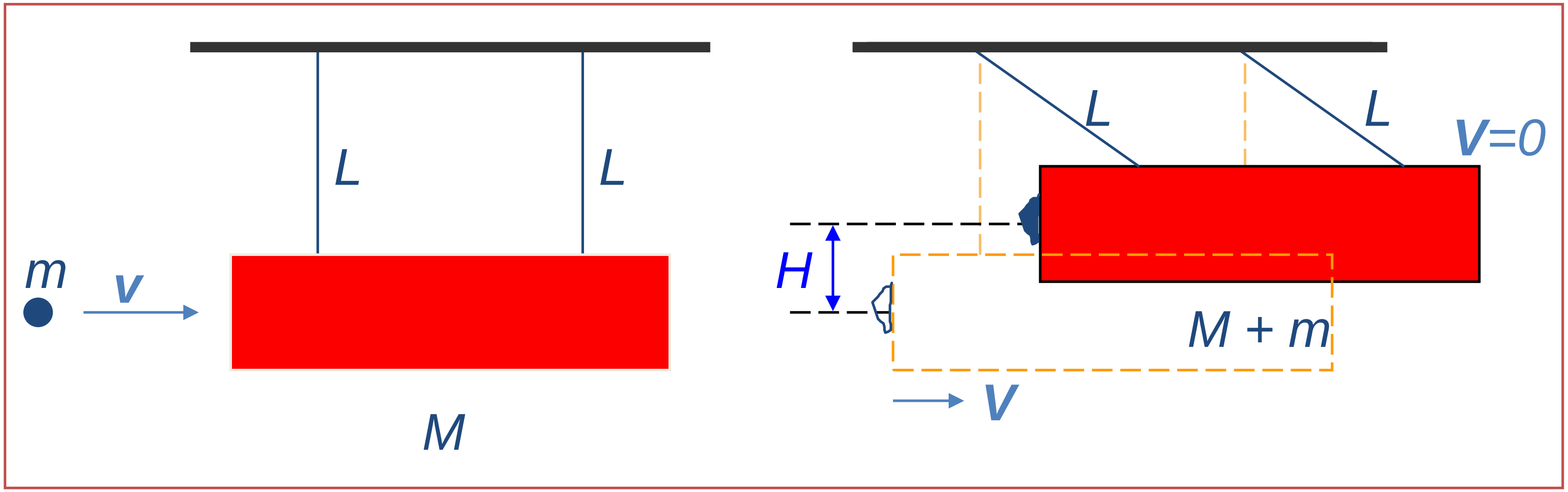
Comment on Energy Conservation

- We have seen that the total kinetic energy of a system undergoing an inelastic collision is not conserved.
  - Energy is lost:
    - Heat (bomb)
    - Bending of metal (crashing cars)

- Kinetic energy **is not** conserved since **work** is done during the collision !

- Momentum along a certain direction **is** conserved when there are no external forces acting in this direction.
  - In general, easier to satisfy than energy conservation.

Ballistic Pendulum



- A projectile of mass *m* moving horizontally with speed *v* strikes a stationary mass *M* suspended by strings of length *L*. Subsequently, *m* + *M* rise to a height of *H*.

Given *H*, what is the initial speed *v* of the projectile?

Ballistic Pendulum…

- Two stage process:

1. *m* collides with *M*, inelastically. Both *M* and *m* then move together with a velocity ***V*** (before having risen significantly).

2. *M* and *m* rise a height *H*, conserving energy *E*. (no non-conservative forces acting after collision)

Ballistic Pendulum...

- Stage 1: Momentum is conserved

in x-direction: $mv = (m + M)V$

$$V = \left(\frac{m}{m+M}\right)v$$

- Stage 2: Energy is conserved

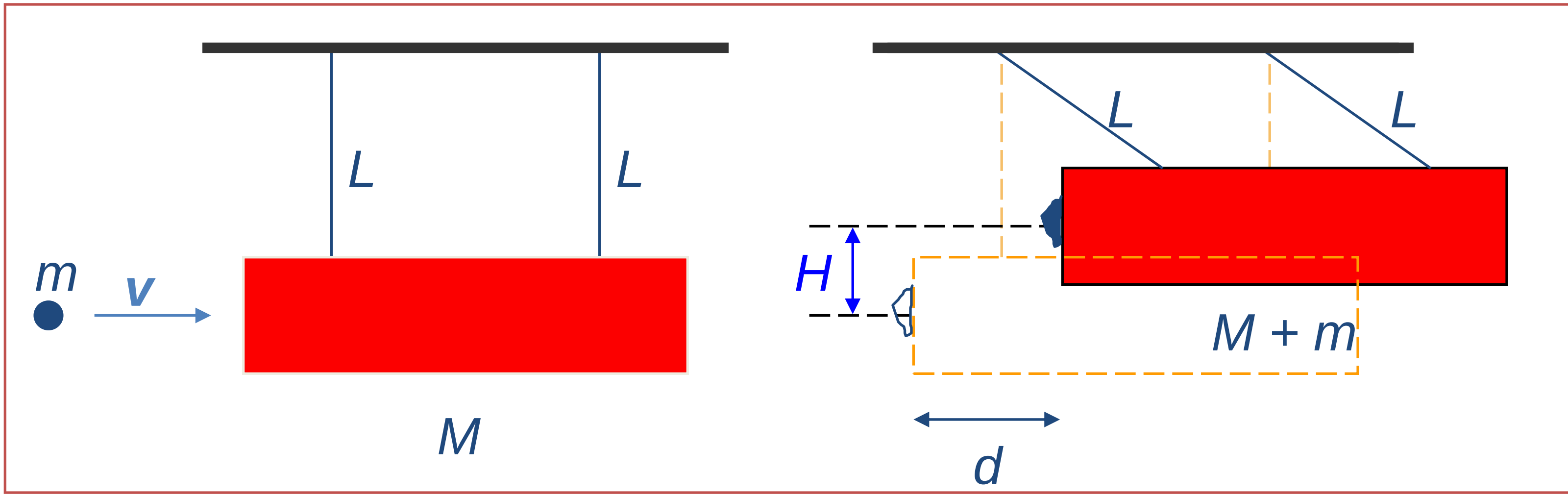
$$(E_I = E_F)$$

$$\tfrac{1}{2}(m + M)V^2 = (m + M)gH \quad \Rightarrow \quad V^2 = 2gH$$

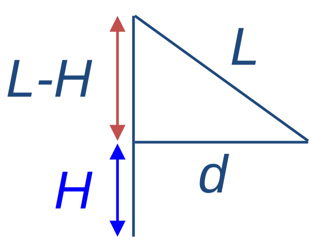
Eliminating **V** gives:

$$v = \left(1 + \frac{M}{m}\right)\sqrt{2gH}$$

Ballistic Pendulum



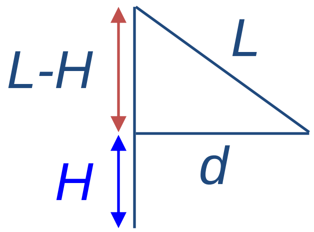
- If we measure the forward displacement *d*, not *H*:



$$L^2 = d^2 + (L - H)^2$$

$$H = L - \sqrt{L^2 - d^2}$$

Ballistic Pendulum



$$H = L - \sqrt{L^2 - d^2}$$

$$= L - L\sqrt{1 - \frac{d^2}{L^2}} \approx L - L\left(1 - \frac{d^2}{2L}\right) \approx \frac{d^2}{2L} \quad \text{for} \quad \frac{d}{L} << 1$$

$$v = \left(1 + \frac{M}{m}\right)\sqrt{2gH} \quad \Rightarrow \quad \boxed{v = \left(1 + \frac{M}{m}\right) \cdot d \cdot \sqrt{\frac{g}{L}}}$$

for $d << L$

- See Cutnell :

  p.

  Number :

  please submit by your attach files as usual before early wed next week (11th Nov, 2009)
- All of assignment is not an easy homework, so please do early