Application of Physics to Finance and Economics: Quantum Field Theory in Forward Rates and Hedging

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Malahayati University

Content :

Introduction

Financial Market Instruments

stocks

bonds

rights

warrant

option

futures

forward

1^{st} /bank

deal

2^{nd} /bank

Sent an amount of foreign exchange/goods in certain date with the deterministic value when the contracts are signed

If the exchange rate in the future < exchange rate on signed contacts, he / she makes a profit

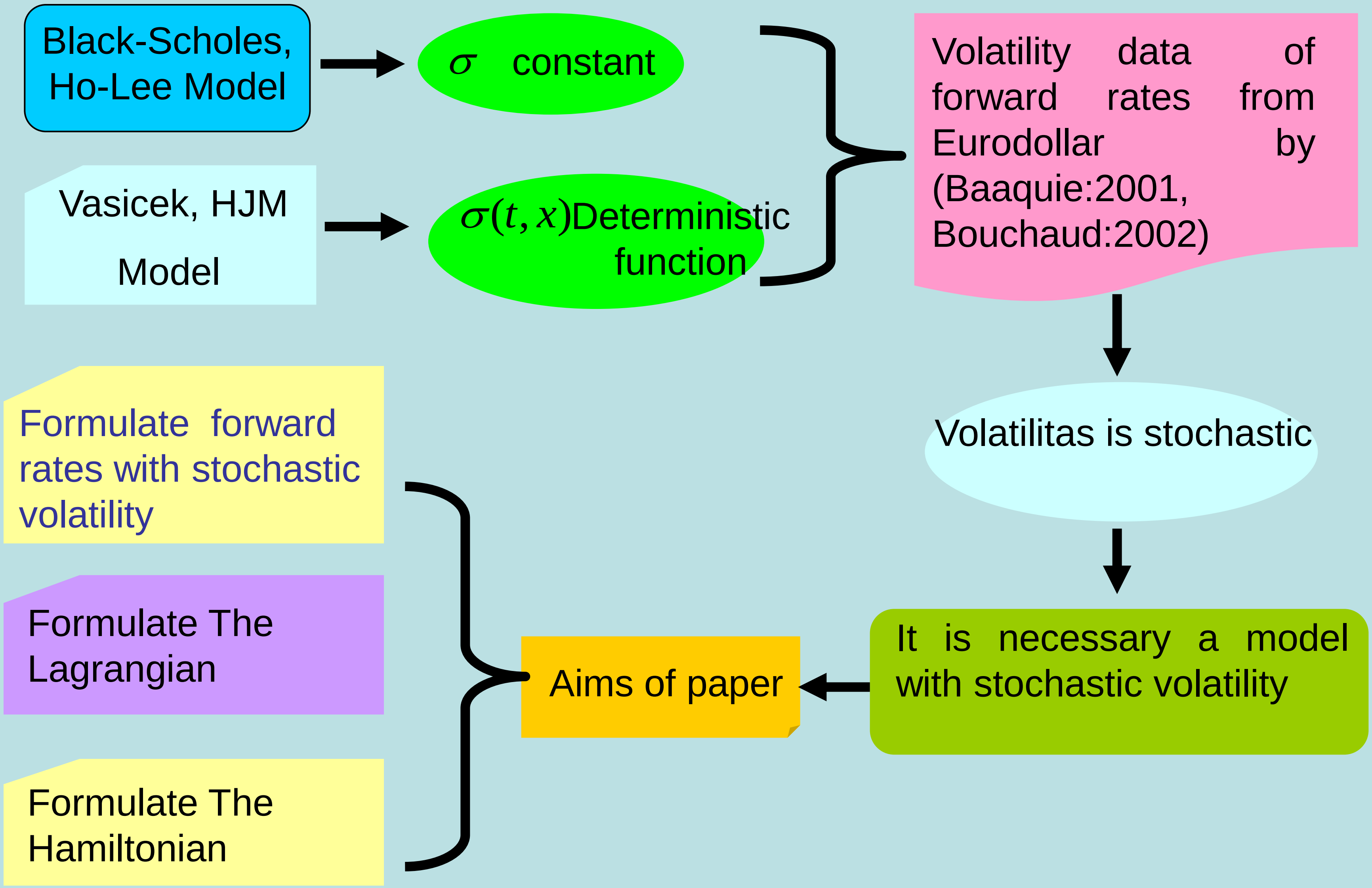
If the exchange rate > exchange rate on signed contracts, he / she makes a lose

Forward are important since the contract value is140 billion$ / year

The Fluctuation of Forward Rates reresents by volatility which indicates a risk level

Memodelkan Volatilitas Forward Rates ini σ merupakan bahasan dalam Tesis

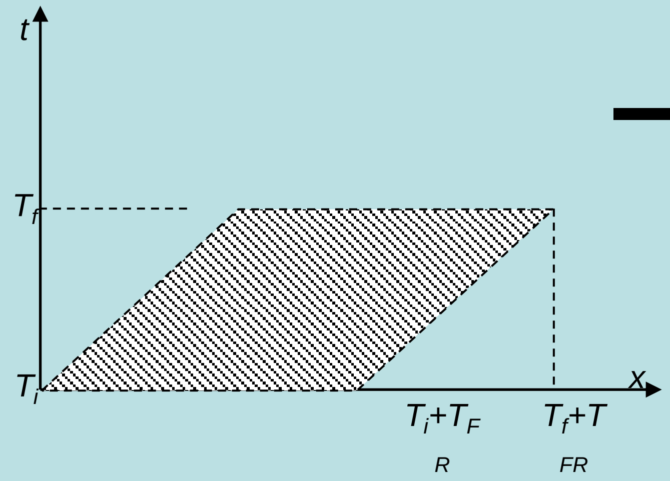
Models of Volatility of Forward Rates



DeterministicVolatility

Domain $\mathcal{P}$

Forward Rates



Lagrangian $\mathcal{L}$

Kinetic

Potential

Flux of lagrangian

$$\mathcal{L}[f] = \mathcal{L}_{kinetik}[f] + \mathcal{L}_{rigiditas}[f]$$

$$\mathcal{L} = -\frac{1}{2}\left[\left\{\frac{\frac{\partial f(t,x)}{\partial t} - \alpha(t,x)}{\sigma(t,x)}\right\}^2 + \frac{1}{\mu^2}\left\{\frac{\partial}{\partial x}\left(\frac{\frac{\partial f(t,x)}{\partial t} - \alpha(t,x)}{\sigma(t,x)}\right)\right\}^2\right]$$

Lagrangian forStochastic Volatility

Stochastic Volatility

- Volatility as a forward rates function
- Volatility as independen quantum field

Volatility as forward rates function

$$\sigma(t,x)\,,\; f(t,x) = \sigma_0(t,x)\, f^{\nu}(t,x)$$

$$\frac{\partial f(t,x)}{\partial t} \to f_0 \frac{\partial \phi(t,x)}{\partial t}$$

$$\mathcal{L} = -\frac{1}{2}\left[\left\{ \frac{f_0 \frac{\partial \phi(t,x)}{\partial t} - \alpha(t,x)}{\sigma_0(t,x) e^{\nu\phi(t,x)}} \right\}^2 + \frac{1}{\mu^2} \left\{ \frac{\partial}{\partial x} \left(\frac{f_0 \frac{\partial \phi(t,x)}{\partial t} - \alpha(t,x)}{\sigma_0(t,x) e^{\nu\phi(t,x)}} \right) \right\}^2 \right]$$

Volatility as independent quantum field

Introduced another quantum field

$h(t,x)$ → $\sigma_0(t,x) = \sigma_0 \, e^{-h(t,x)}$ → **interacting** $f(t,x)$ — $h(t,x)$

asumsi

- ξ → $h(t,x)$ not deterministic
- κ → Control the fluctuation $h(t,x)$ In x direct
- ρ → Field correlation coefficient $f(t,x)$ and $h(t,x)$
- $\beta(t,x)$ → Analogue with drift term $\alpha(t,x)$

$$\mathcal{L} = -\frac{1}{2(1-\rho)^2}\left(\frac{\frac{\partial f}{\partial t}-\alpha}{\sigma} - \rho\,\frac{\frac{\partial h}{\partial t}-\beta}{\xi}\right)^2 - \frac{1}{2}\left(\frac{\frac{\partial h}{\partial t}-\beta}{\xi}\right)^2$$

$$-\frac{1}{2\mu^2}\left(\frac{\partial}{\partial x}\left(\frac{\frac{\partial f}{\partial t}-\alpha}{\sigma}\right)\right)^2 - \frac{1}{2\kappa^2}\left(\frac{\partial}{\partial x}\left(\frac{\frac{\partial h}{\partial t}-\beta}{\xi}\right)^2\right)$$

→ $<\sigma(t,x)>$

$$\sigma_0 \exp\left\{-\int_{t_0}^{t} dt' \, \beta(t',x)\right\} + O(\xi,\kappa,\rho)$$

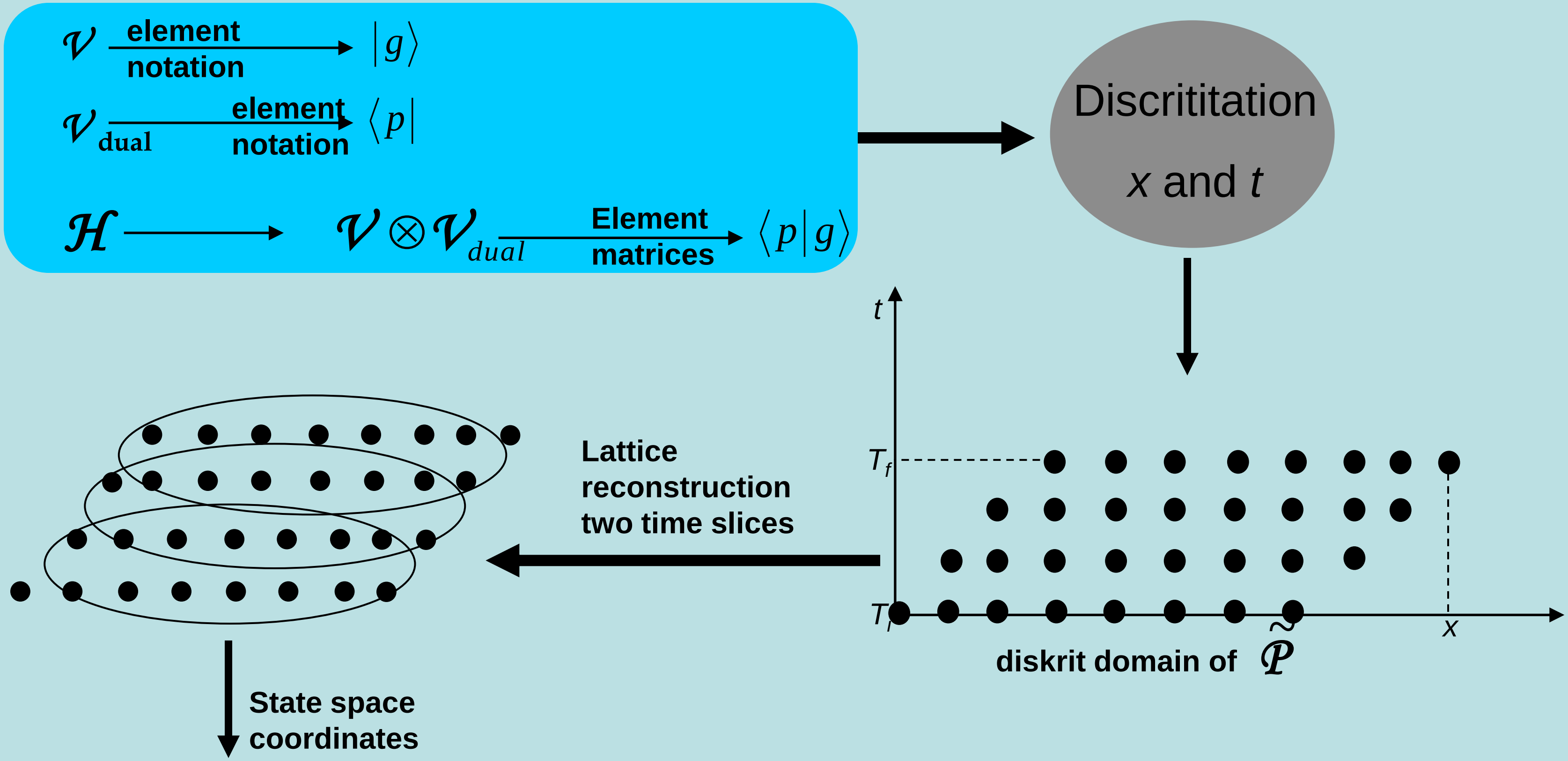
Hamiltonian and State Space

There are state spaces:

$\mathcal{V} \xrightarrow[\text{notation}]{\text{element}} |g\rangle$

$\mathcal{V}_{\text{dual}} \xrightarrow[\text{notation}]{\text{element}} \langle p|$

$\mathcal{H} \longrightarrow \mathcal{V} \otimes \mathcal{V}_{dual} \xrightarrow[\text{matrices}]{\text{Element}} \langle p|g\rangle$



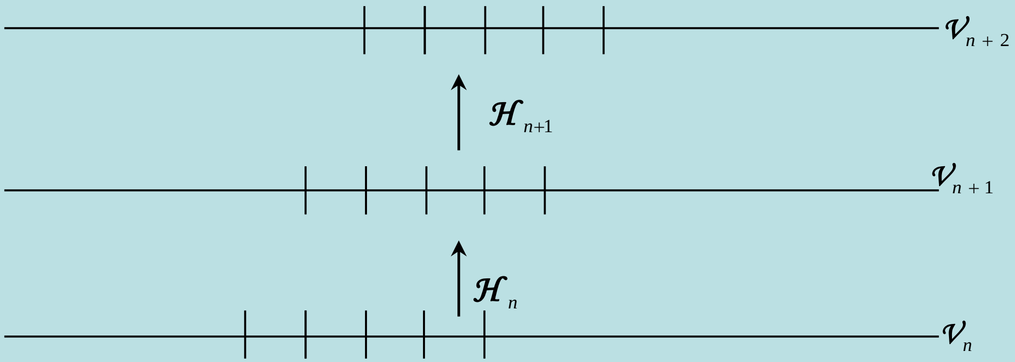
$$\langle \tilde{f}_n | = \bigotimes_{n \le l \le n+N_{FR}} \langle f_{n,l} | \equiv \langle f_{n,n} | \langle \tilde{F}_n | \rightarrow$$

State space coordinates $\mathcal{V}_n$

$$| f_{n+1} \rangle = \bigotimes_{n+1 \le l \le n+1+N_{FR}} | f_{n+1,l} \rangle \equiv | F_n \rangle | f_{n,n+1+N_{FR}} \rangle \rightarrow$$

State space coordinates $\mathcal{V}_{n+1}$

Hamiltonian Propagation



→ complete basis

$$I_n \equiv \int df_{n,n} \, . \, d\tilde{F}_n \left| f_{n,n} ; \tilde{F}_n \right\rangle \left\langle f_{n,n} ; \tilde{F}_n \right|$$

states vector

stock states vector

$$P(t,T) \equiv \left\langle f_t \middle| P(t,T) \right\rangle = e^{-\int_t^T f(x)dx}$$

bonds states vector →

$$\left| \mathcal{B}(t) \right\rangle = \sum_l c_l \left| P(t,T_l) \right\rangle + L \left| P(t,T) \right\rangle$$

Hamiltonian Forward Rates-Volatility as Forward Rates Function

Dari Lagrangian → **Didefinisikan Propagator** →

$$D(x,x';t) = \frac{\mu}{2\sinh(\mu T_{FR})} \left[\cosh\left(\mu T_{FR} - \mu \left| x - x' \right|\right) + \cosh\left(\mu T_{FR} - \mu(x + x' - 2t)\right) \right]$$

Hamiltonian Forward Rates-Volatility as Forward Rates Function

$$\mathcal{H}_{\phi}(t) = -\frac{1}{2f_0^2}\int \sigma_0 e^{\nu\phi}(x) D(x,x';t)\sigma_0 e^{\nu\phi}(x')\frac{\delta^2}{\delta\phi(x)\delta\phi(x')} - \frac{1}{f_0}\int \alpha\frac{\delta}{\delta\phi}$$

Hamiltonian Forward Rates-Volatility as a Quantum Field

Dari lagrangian notasi matriks

$$\mathcal{M}(x,x';t) = \begin{bmatrix} \frac{1}{1-\rho^2} - \frac{1}{\mu^2}\frac{\partial^2}{\partial x^2} & -\frac{\rho}{1-\rho^2} \\ -\frac{\rho}{1-\rho^2} & \frac{1}{1-\rho^2} - \frac{1}{\kappa^2}\frac{\partial^2}{\partial x^2} \end{bmatrix}\delta(x-x')$$

⬇

$$S(n) = -\frac{1}{2\in}\int\left[\sigma^{-1}A\ \xi^{-1}B\right](x)\mathcal{M}(x,x';t)\begin{bmatrix}\sigma^{-1} & A \\ \xi^{-1} & B\end{bmatrix}(x')$$

⬇

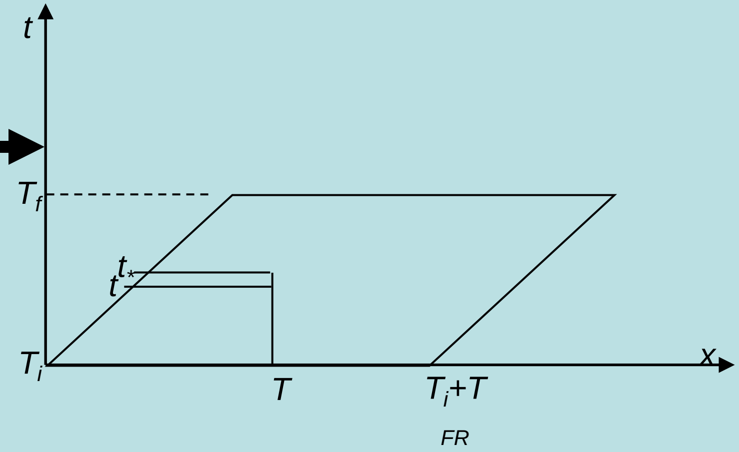
$$\mathcal{H}(t) = \frac{1}{2}\int\begin{bmatrix}\sigma\frac{\delta}{i\delta\tilde{f}} & \xi\frac{\delta}{i\delta\tilde{h}}\end{bmatrix}\mathcal{M}^{-1}\begin{bmatrix}\sigma\frac{\delta}{i\delta\tilde{f}} \\ \xi\frac{\delta}{i\delta\tilde{h}}\end{bmatrix} - \int\left\{\alpha\frac{\delta}{\delta\tilde{f}} + \beta\frac{\delta}{\delta\tilde{h}}\right\}$$

Hamiltonian Forward Rates-No Arbitrage

A security hamiltonian

$$\left(\mathcal{H}_s + r\right)\left|S\right\rangle = 0$$

Domain



If no arbitrage or for zero coupon bond

$$\mathcal{H}(t)\left|P(t+\in,T)\right\rangle = 0$$

Spot rate is transformation factor for bonds obligasi

Lagrangian for Volatility as a Forward Rates Function –No Arbitrage

$$\mathcal{L}_{umum} = \mathcal{L}[\phi] + \int U(t,x)\frac{\partial \phi}{\partial t} + \int W(t,x)$$

with U and W is local arbitrage local that is function of $f(t,x)$

no arbitrage

$$U(t,x) = W(t,x) = 0$$

Lagrangian for Volatility as quantum field –no arbitrage

$$\alpha(t,x)=\sigma(t,x)\int_t^x dx'\,G(x,x';t)\sigma(t,x')$$

$$\Downarrow$$

$$v(t,x)=\int_t^x dx' G(x,x';t)\sigma(t,x')$$

Let define a local function in volatility field

$$\Downarrow$$

$$\mathcal{L}(t,x)=\frac{-1}{2(1-\rho^2)}\left(\sigma^{-1}\frac{\partial f}{\partial t}-v-\rho\frac{\frac{\partial h}{\partial t}-\beta}{\xi}\right)^2-\frac{1}{2}\left(\frac{\frac{\partial h}{\partial t}-\beta}{\xi}\right)^2$$

$$-\frac{1}{2\mu^2}\left(\frac{\partial}{\partial x}\left(\sigma^{-1}\frac{\partial f}{\partial t}-v\right)\right)^2-\frac{1}{2K^2}\left(\frac{\partial}{\partial x}\left(\frac{\frac{\partial h}{\partial t}-\beta}{\xi}\right)\right)^2$$

function of $\beta(t,x)$ and μ, κ, ρ, ξ **parameters need to be determined from market data**

Conclusion

- Forward Rates with Stochastic Volatility is investigated with let the volatility as forward rates function; and volatility as a quantum field
- Obtained Lagrangian and Hamiltonian for forward rates with analyze state space system, with forward rates as a parralellgram domain
- Forward Rates with stochastic volatility has an amount of free parameters that need to be determined with investigate the market. These parameter are reported in the next paper in ICMNS (international conference on mathematics and natural sciences) october 28th – 30^{th,} 2008.