General Physics (PHY 2130)

Lecture XI

- Vibrations and waves
  - Hooke's law, spring-mass system
  - Elastic potential energy
  - Period and frequency
  - Wave motion

WAYNE STATE UNIVERSITY

Lightning Review

Last lecture:

1. Laws of Thermodynamics
   - ✓ $\Delta U = U_f - U_i = Q + W$
   - ✓ Heat engines, Carnot's cycle

Review Problem: Your friend has constructed a heat engine that (as he claims) operates with 40% efficiency. The engine receives heat from the hot reservoir (T_H=400 K) and expels heat to the cold reservoir (T_C=300 K). Your friend is

1. telling the truth
2. not completely honest with you
3. saying something that you cannot possibly verify with the provided data .

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Note: Maximally possible efficiency of a heat engine that operates in these conditions is that of a Carnot's engine, i.e.

$$e_{max} = 1 - \frac{|T_c|}{|T_h|} = 1 - \frac{300K}{400K} = \underline{0.25} \;\; or \;\; \underline{25\%}$$

Thus, your friend should check his measurements.

Vibrations and Waves

Hooke's Law

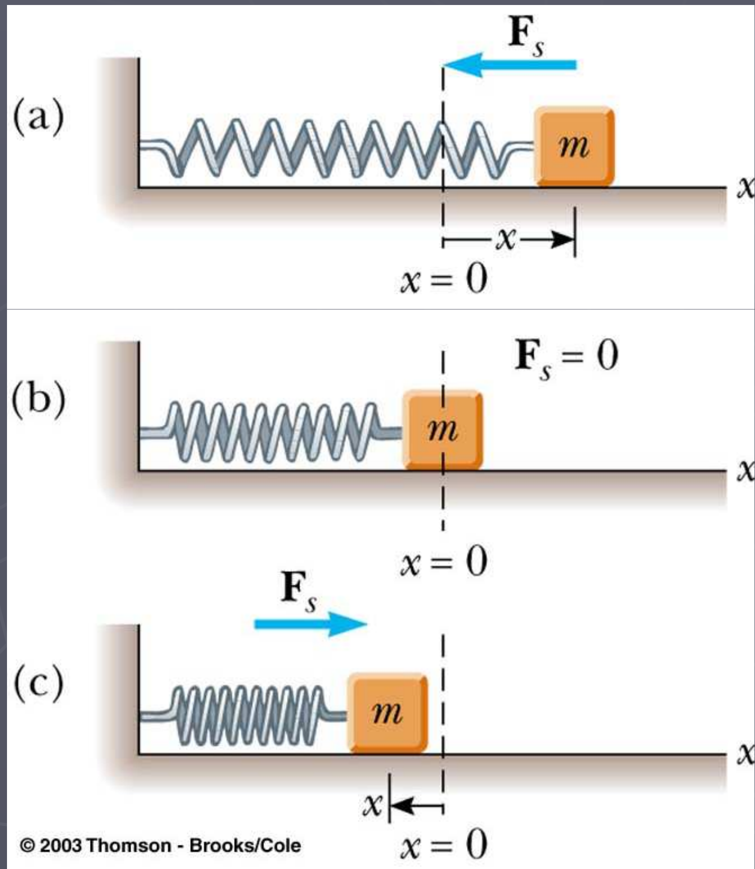
- $F_s = - k x$
  - F_s is the spring force
  - k is the spring constant
    - It is a measure of the stiffness of the spring
      - A large k indicates a stiff spring and a small k indicates a soft spring
  - x is the displacement of the object from its equilibrium position
  - The negative sign indicates that the force is always directed opposite to the displacement

Hooke's Law Force

- The force always acts toward the equilibrium position
  - It is called the *restoring force*
- The direction of the restoring force is such that the object is being either pushed or pulled toward the equilibrium position

Hooke's Law Applied to a Spring – Mass System

- When x is positive (to the right), F is negative (to the left)
- When x = 0 (at equilibrium), F is 0
- When x is negative (to the left), F is positive (to the right)



Motion of the Spring-Mass System

- Assume the object is initially pulled to $x = A$ and released from rest
- As the object moves toward the equilibrium position, F and a decrease, but v increases
- At $x = 0$, F and a are zero, but v is a maximum
- The object's momentum causes it to overshoot the equilibrium position
- The force and acceleration start to increase in the opposite direction and velocity decreases
- The motion continues indefinitely

Simple Harmonic Motion

- Motion that occurs when the net force along the direction of motion is a Hooke's Law type of force
  - The force is proportional to the displacement and in the opposite direction
- The motion of a spring mass system is an example of Simple Harmonic Motion

Simple Harmonic Motion, cont.

- Not all periodic motion over the same path can be considered Simple Harmonic motion
- To be Simple Harmonic motion, the force needs to obey Hooke's Law

Amplitude

- Amplitude, A
  - The amplitude is the maximum position of the object relative to the equilibrium position
  - In the absence of friction, an object in simple harmonic motion will oscillate between $\pm A$ on each side of the equilibrium position

Period and Frequency

- The period, T, is the time that it takes for the object to complete one complete cycle of motion
  - From x = A to x = - A and back to x = A
- The frequency, f, is the number of complete cycles or vibrations per unit time

Acceleration of an Object in Simple Harmonic Motion

- Newton’s second law will relate force and acceleration
- The force is given by Hooke’s Law
- $F = - k x = m a$
  - $a = -kx / m$
- The acceleration is a function of position
  - Acceleration is *not* constant and therefore the uniformly accelerated motion equation cannot be applied

Acceleration Defining Simple Harmonic Motion

- Acceleration can be used to define simple harmonic motion
- An object moves in simple harmonic motion if its acceleration is directly proportional to the displacement and is in the opposite direction

Elastic Potential Energy

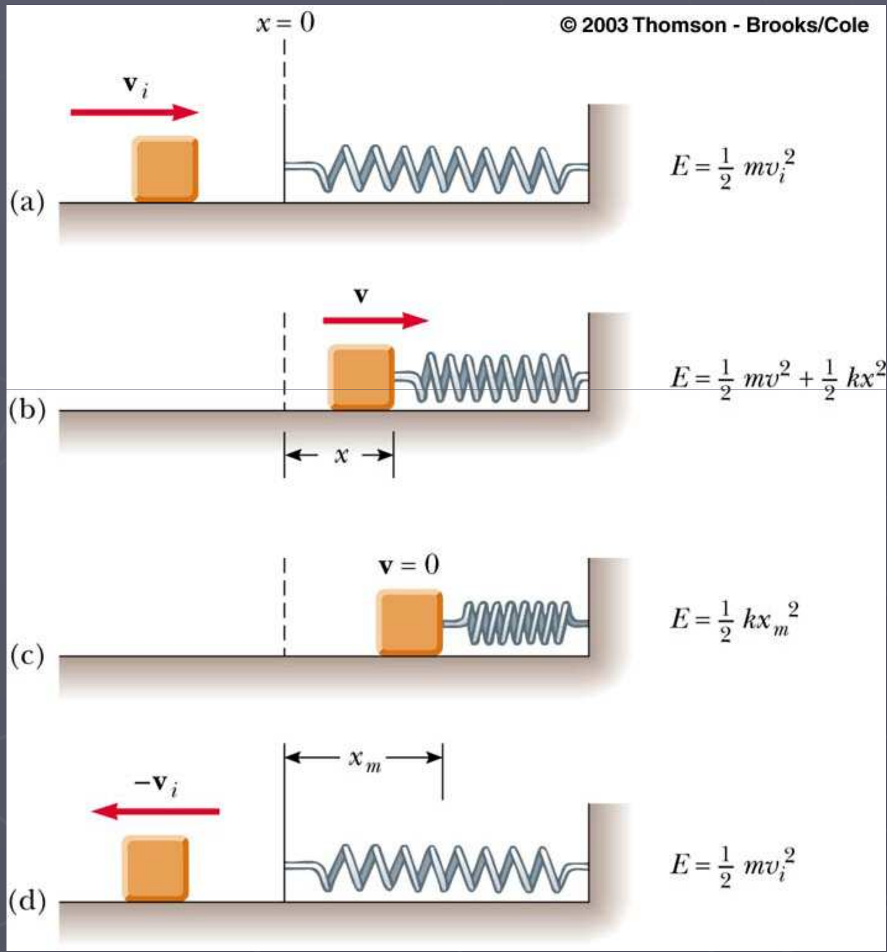
- A compressed spring has potential energy
  - The compressed spring, when allowed to expand, can apply a force to an object
  - The potential energy of the spring can be transformed into kinetic energy of the object

Elastic Potential Energy, cont

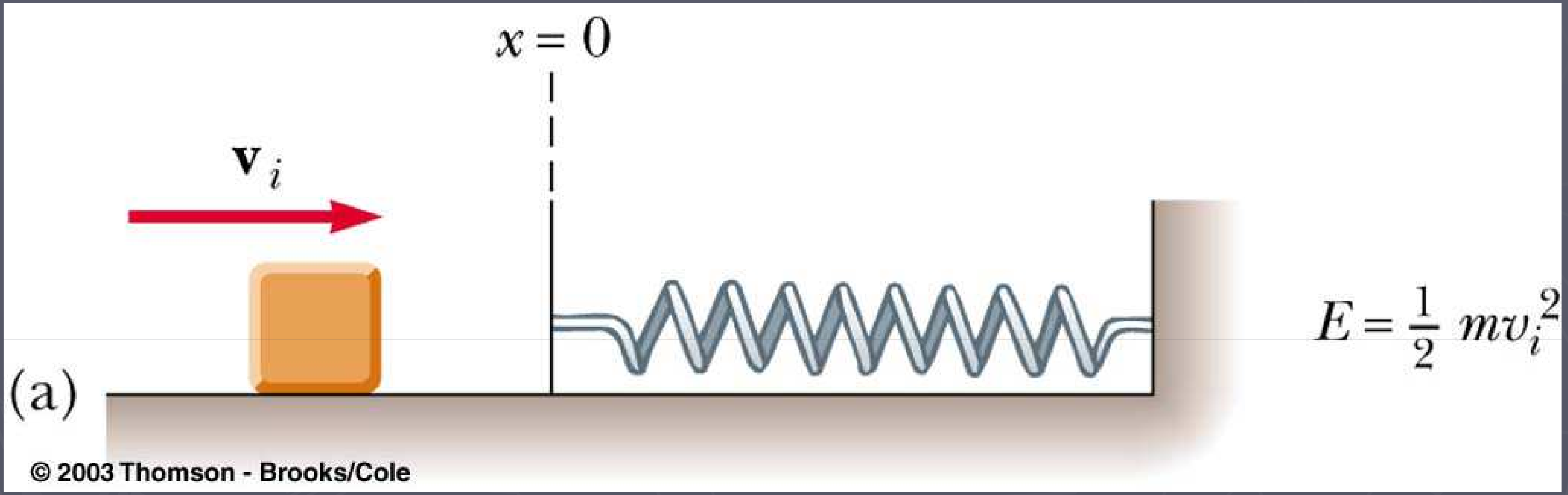
- The energy stored in a stretched or compressed spring or other elastic material is called *elastic potential energy*
  - $Pe_s = \frac{1}{2}kx^2$
- The energy is stored only when the spring is stretched or compressed
- Elastic potential energy can be added to the statements of Conservation of Energy and Work-Energy

Energy in a Spring Mass System

- A block sliding on a frictionless system collides with a light spring
- The block attaches to the spring

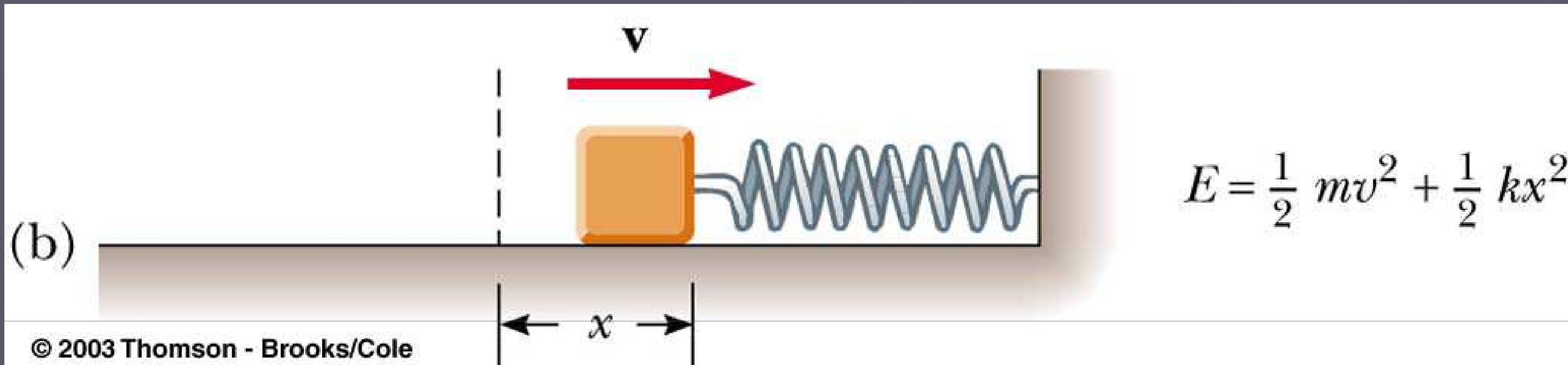


Energy Transformations



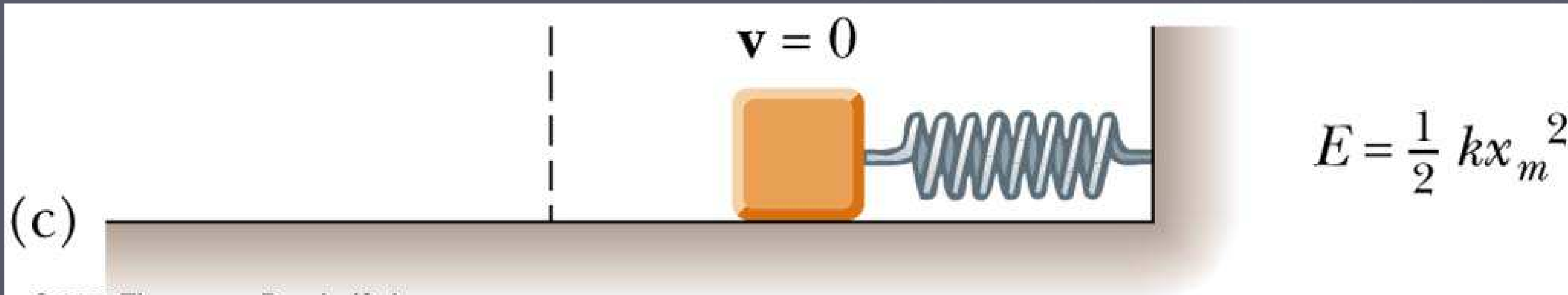
- The block is moving on a frictionless surface
- The total mechanical energy of the system is the kinetic energy of the block

Energy Transformations, 2



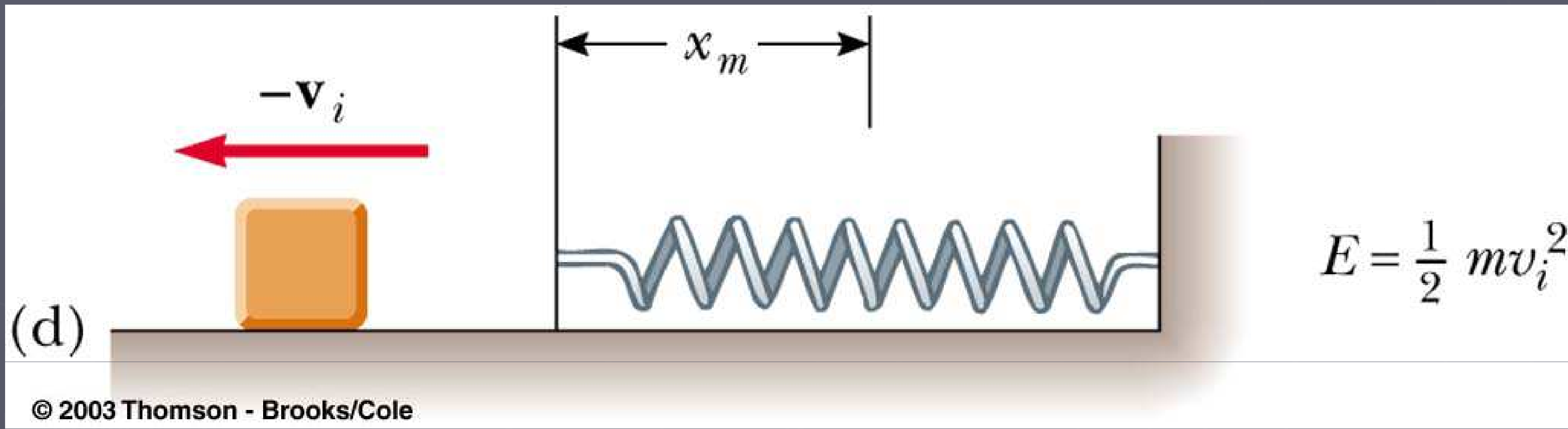
- The spring is partially compressed
- The energy is shared between kinetic energy and elastic potential energy
- The total mechanical energy is the sum of the kinetic energy and the elastic potential energy

Energy Transformations, 3



- The spring is now fully compressed
- The block momentarily stops
- The total mechanical energy is stored as elastic potential energy of the spring

Energy Transformations, 4



- When the block leaves the spring, the total mechanical energy is in the kinetic energy of the block
- The spring force is conservative and the total energy of the system remains constant

Velocity as a Function of Position

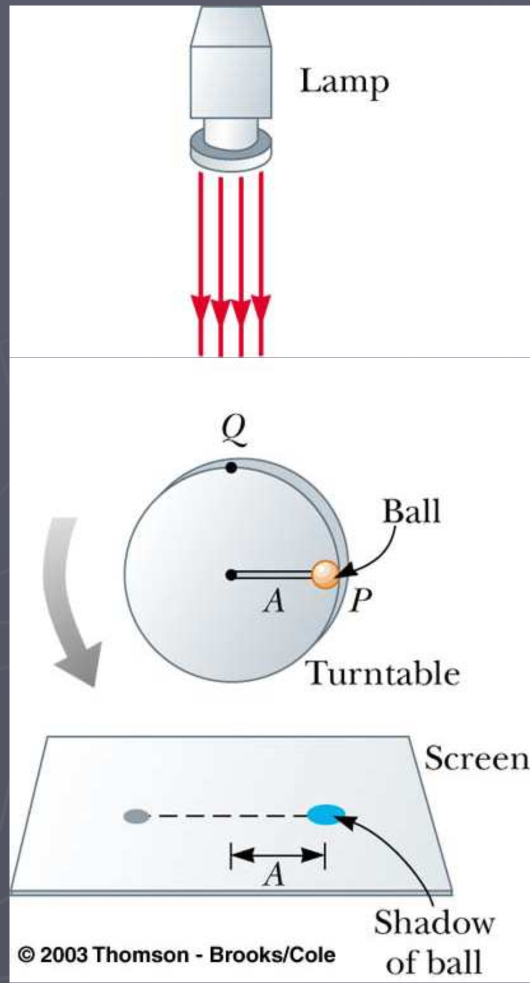
- Conservation of Energy allows a calculation of the velocity of the object at any position in its motion

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

  - Speed is a maximum at $x = 0$
  - Speed is zero at $x = \pm A$
  - The ± indicates the object can be traveling in either direction

Simple Harmonic Motion and Uniform Circular Motion

- A ball is attached to the rim of a turntable of radius A
- The focus is on the shadow that the ball casts on the screen
- When the turntable rotates with a constant angular speed, the shadow moves in simple harmonic motion



Period and Frequency from Circular Motion

- Period $$T = 2\pi\sqrt{\frac{m}{k}}$$
  - This gives the time required for an object of mass m attached to a spring of constant k to complete one cycle of its motion
- Frequency $$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
  - Units are cycles/second or Hertz, Hz

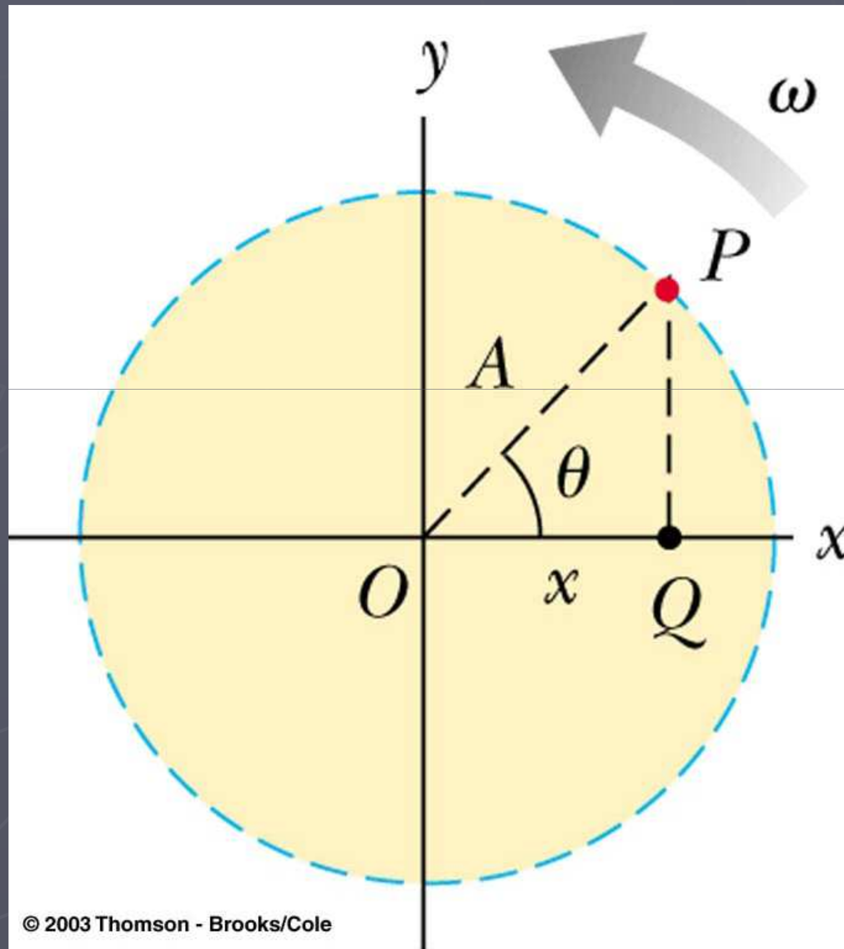
Angular Frequency

- The angular frequency is related to the frequency

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

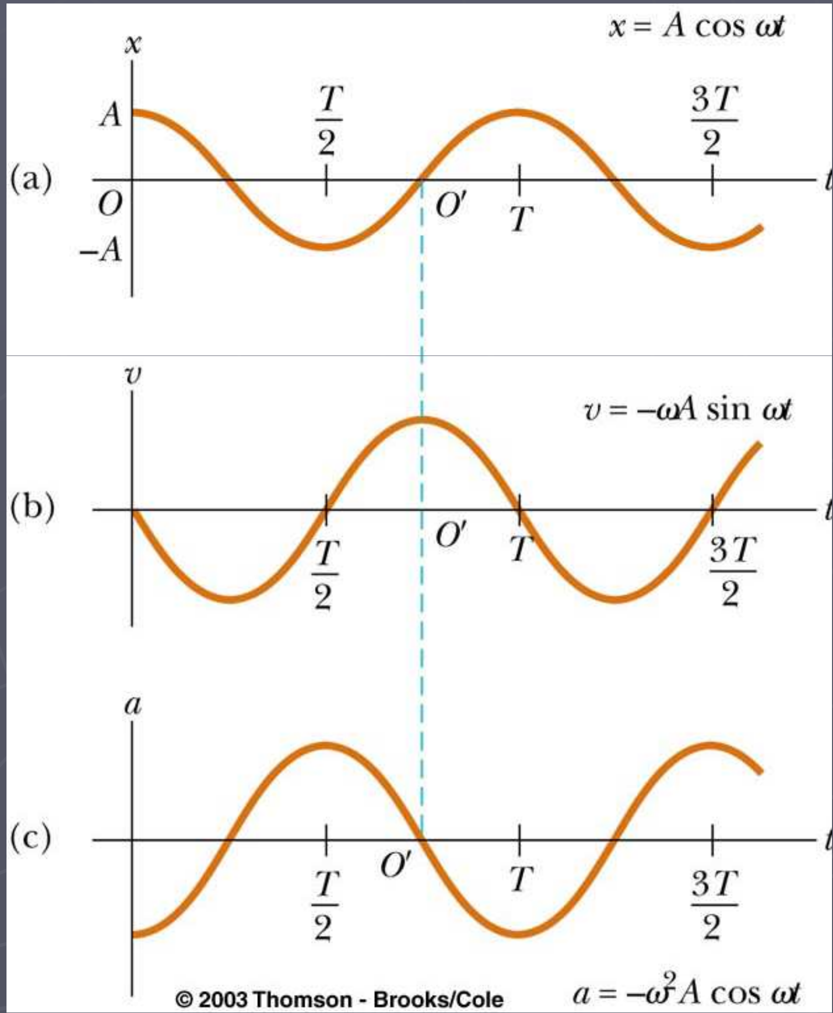
Motion as a Function of Time

- Use of a *reference circle* allows a description of the motion
- $x = A \cos(2\pi f t)$
  - x is the position at time t
  - x varies between +A and -A



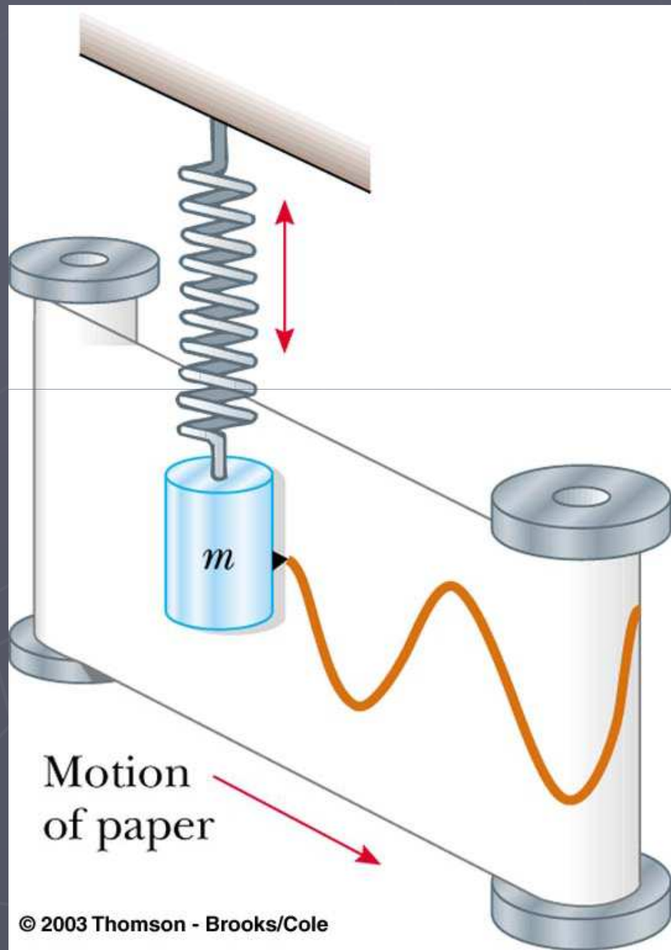
Graphical Representation of Motion

- When x is a maximum or minimum, velocity is zero
- When x is zero, the velocity is a maximum
- When x is a maximum in the positive direction, a is a maximum in the negative direction



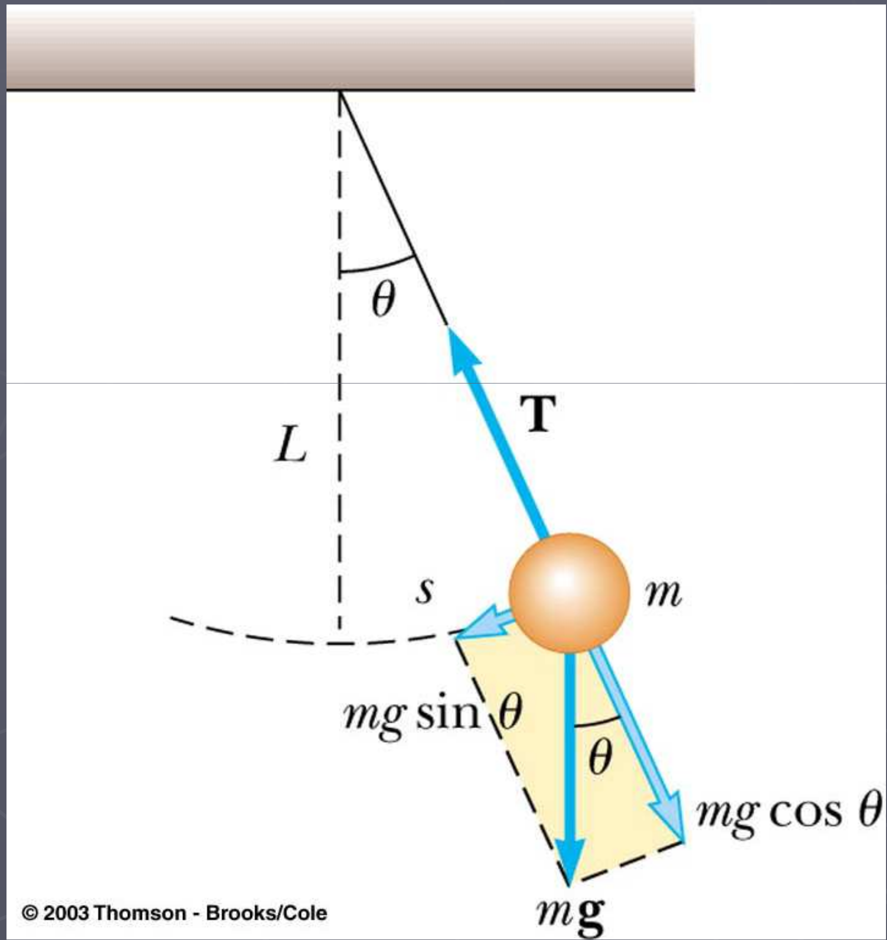
Verification of Sinusoidal Nature

- This experiment shows the sinusoidal nature of simple harmonic motion
- The spring mass system oscillates in simple harmonic motion
- The attached pen traces out the sinusoidal motion



Simple Pendulum

- The simple pendulum is another example of simple harmonic motion
- The force is the component of the weight tangent to the path of motion
  - $F = - m\ g \sin \theta$



Simple Pendulum, cont

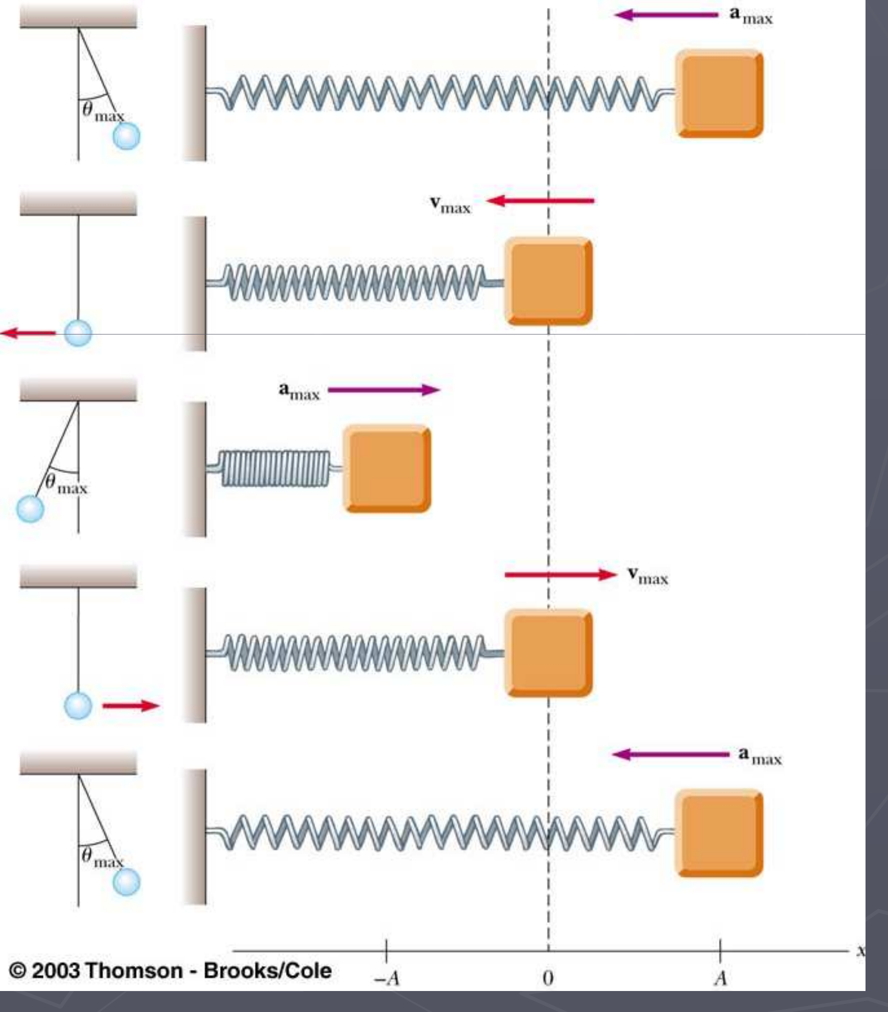
- In general, the motion of a pendulum is not simple harmonic
- However, for small angles, it becomes simple harmonic
  - In general, angles < 15° are small enough
  - $\sin \theta = \theta$
  - $F = - m g \theta$
    - This force obeys Hooke's Law

Period of Simple Pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- This shows that the period is independent of of the amplitude
- The period depends on the length of the pendulum and the acceleration of gravity at the location of the pendulum

Simple Pendulum Compared to a Spring-Mass System

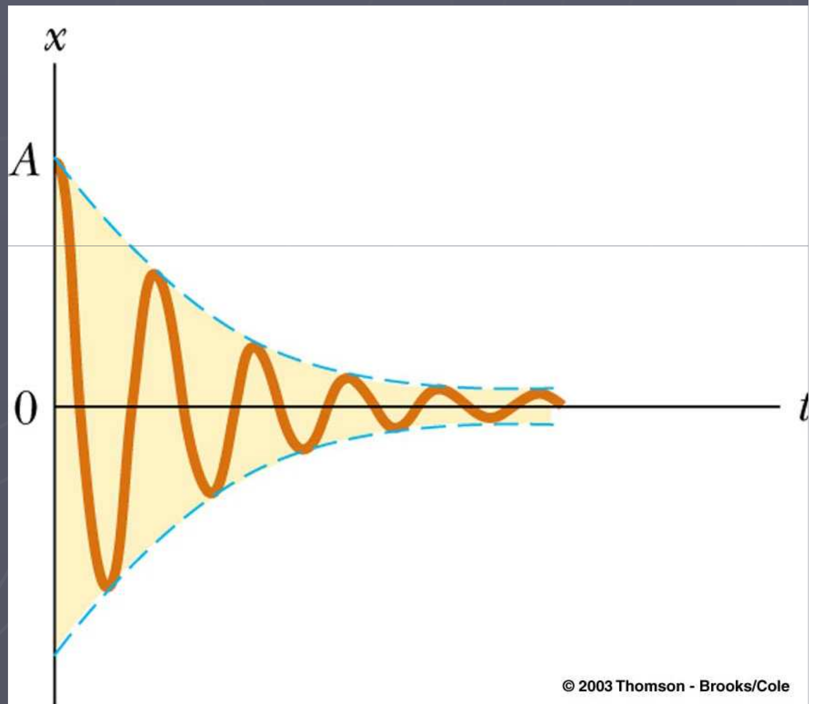


Damped Oscillations

- Only ideal systems oscillate indefinitely
- In real systems, friction retards the motion
- Friction reduces the total energy of the system and the oscillation is said to be *damped*

Damped Oscillations, cont.

- Damped motion varies depending on the fluid used
  - With a low viscosity fluid, the vibrating motion is preserved, but the amplitude of vibration decreases in time and the motion ultimately ceases
    - This is known as *underdamped* oscillation

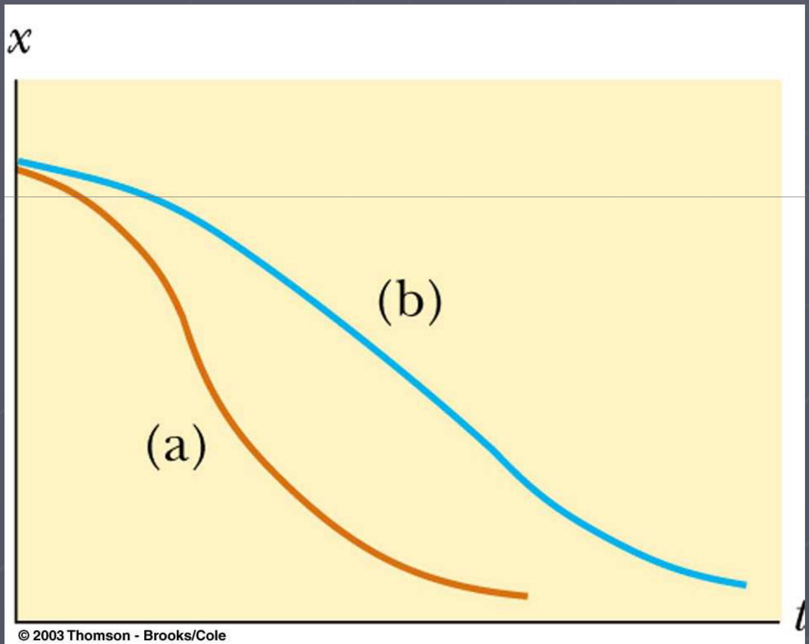


More Types of Damping

- With a higher viscosity, the object returns rapidly to equilibrium after it is released and does not oscillate
  - The system is said to be *critically damped*
- With an even higher viscosity, the piston returns to equilibrium without passing through the equilibrium position, but the time required is longer
  - This is said to be *over damped*

Damping Graphs

- Plot a shows a critically damped oscillator
- Plot b shows an overdamped oscillator

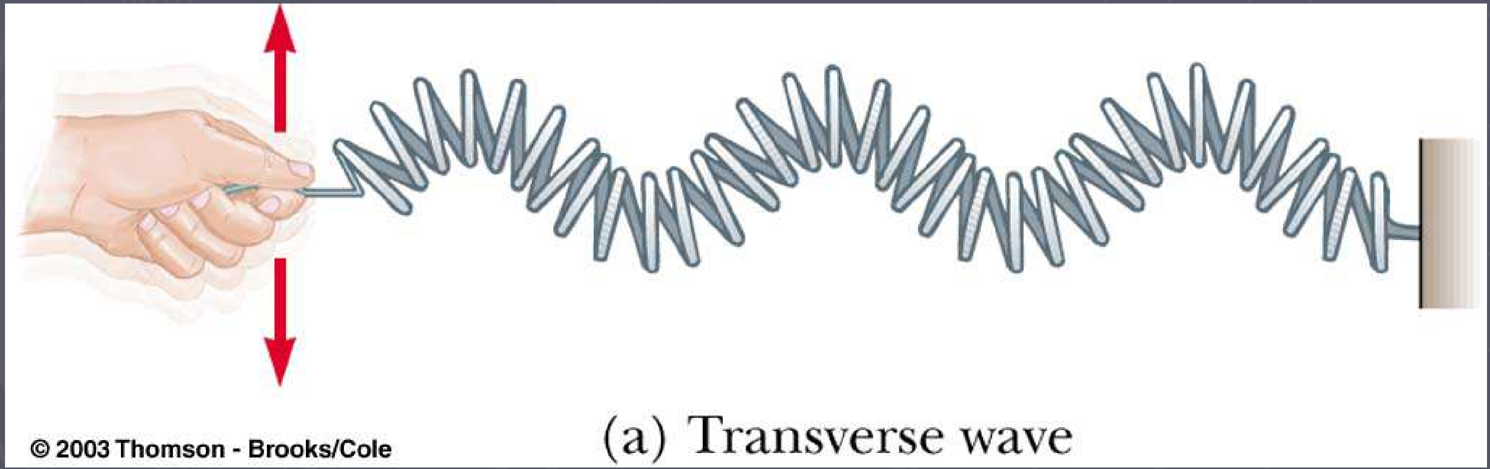


Wave Motion

- A wave is the motion of a disturbance
- Mechanical waves require
  - Some source of disturbance
  - A medium that can be disturbed
  - Some physical connection between or mechanism though which adjacent portions of the medium influence each other
- All waves carry energy and momentum

Types of Waves -- Transverse

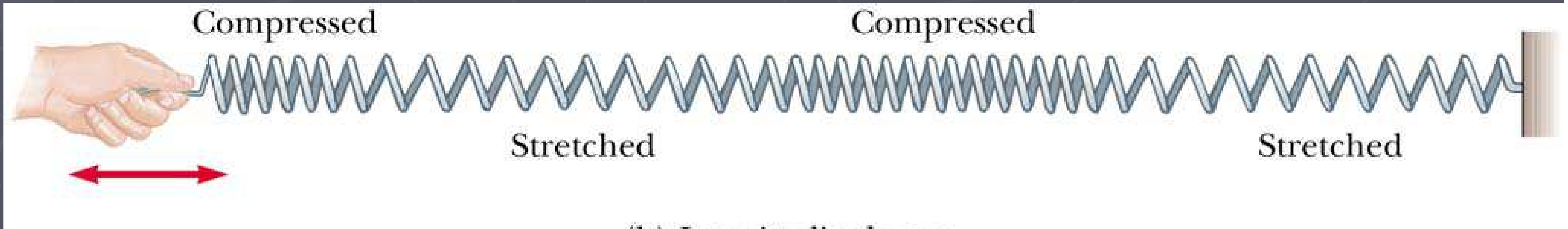
- In a transverse wave, each element that is disturbed moves perpendicularly to the wave motion



(a) Transverse wave

Types of Waves -- Longitudinal

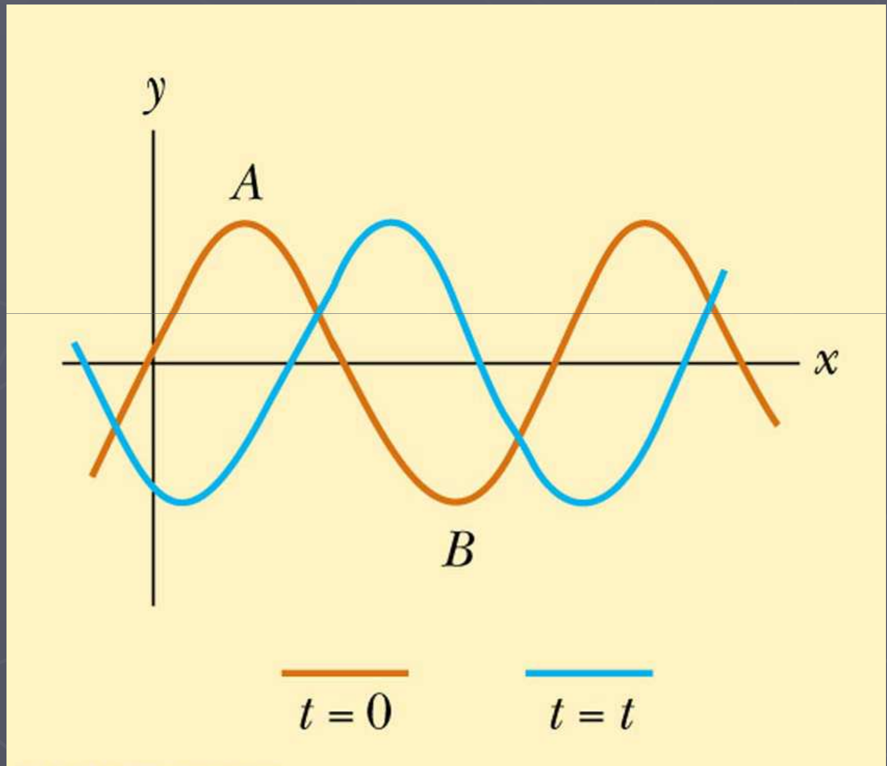
- In a longitudinal wave, the elements of the medium undergo displacements parallel to the motion of the wave
- A longitudinal wave is also called a compression wave



(b) Longitudinal wave

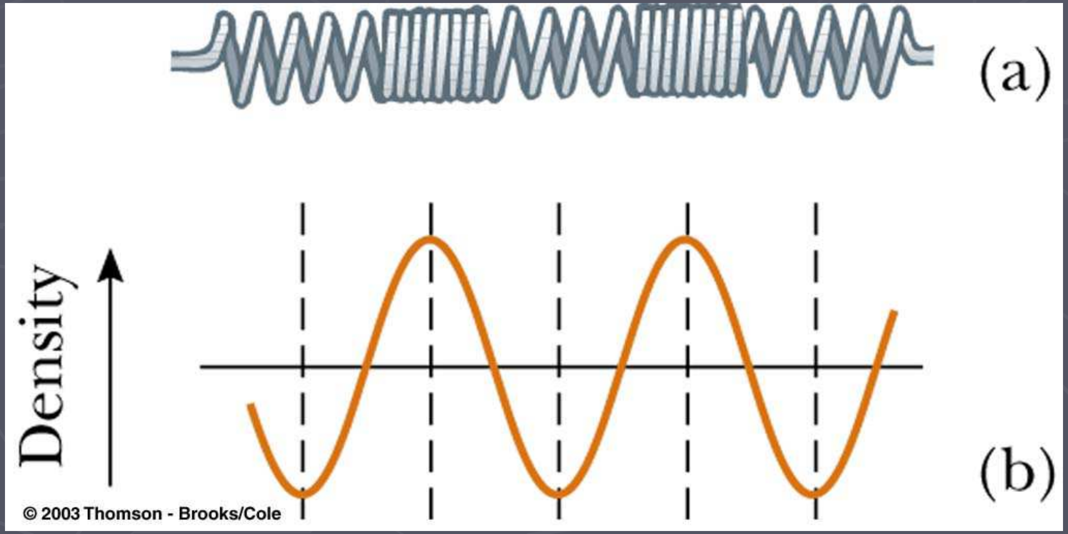
Waveform – A Picture of a Wave

- The red curve is a "snapshot" of the wave at some instant in time
- The blue curve is later in time
- A is a *crest* of the wave
- B is a *trough* of the wave



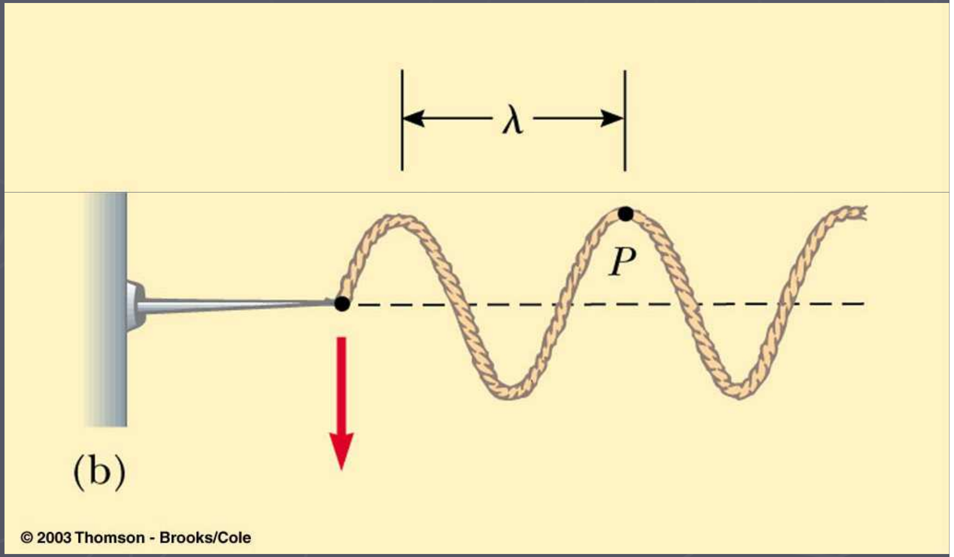
Longitudinal Wave Represented as a Sine Curve

- A longitudinal wave can also be represented as a sine curve
- Compressions correspond to crests and stretches correspond to troughs



Description of a Wave

- Amplitude is the maximum displacement of string above the equilibrium position
- Wavelength, λ, is the distance between two successive points that behave identically



Speed of a Wave

- $v = f\,\lambda$
  - Is derived from the basic speed equation of distance/time
- This is a general equation that can be applied to many types of waves

Speed of a Wave on a String

- The speed on a wave stretched under some tension, F

$$v = \sqrt{\frac{F}{\mu}} \quad \text{where} \quad \mu = \frac{m}{L}$$

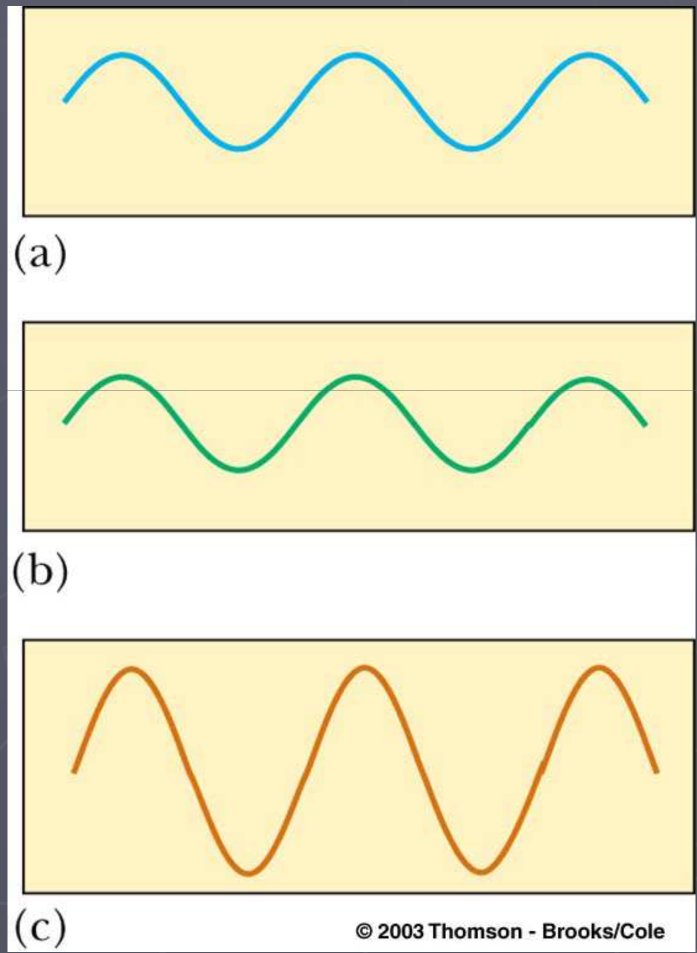
- The speed depends only upon the properties of the medium through which the disturbance travels

Interference of Waves

- Two traveling waves can meet and pass through each other without being destroyed or even altered
- Waves obey the *Superposition Principle*
  - If two or more traveling waves are moving through a medium, the resulting wave is found by adding together the displacements of the individual waves point by point
  - Actually only true for waves with small amplitudes

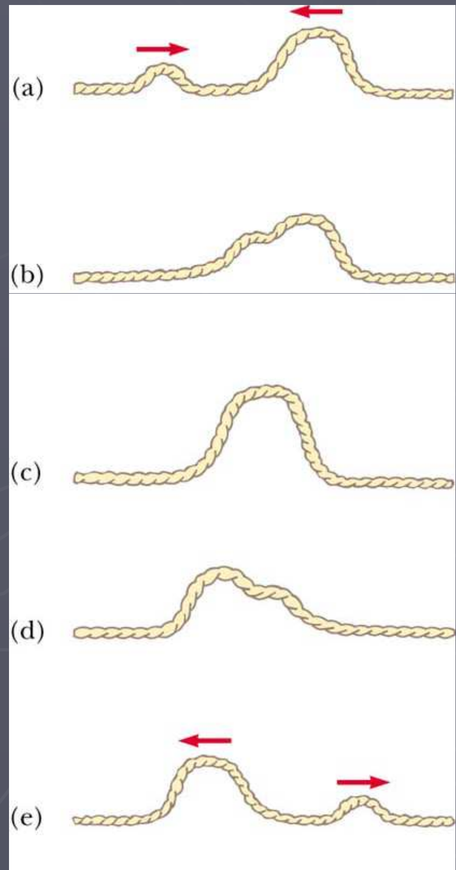
Constructive Interference

- Two waves, a and b, have the same frequency and amplitude
  - Are *in phase*
- The combined wave, c, has the same frequency and a greater amplitude



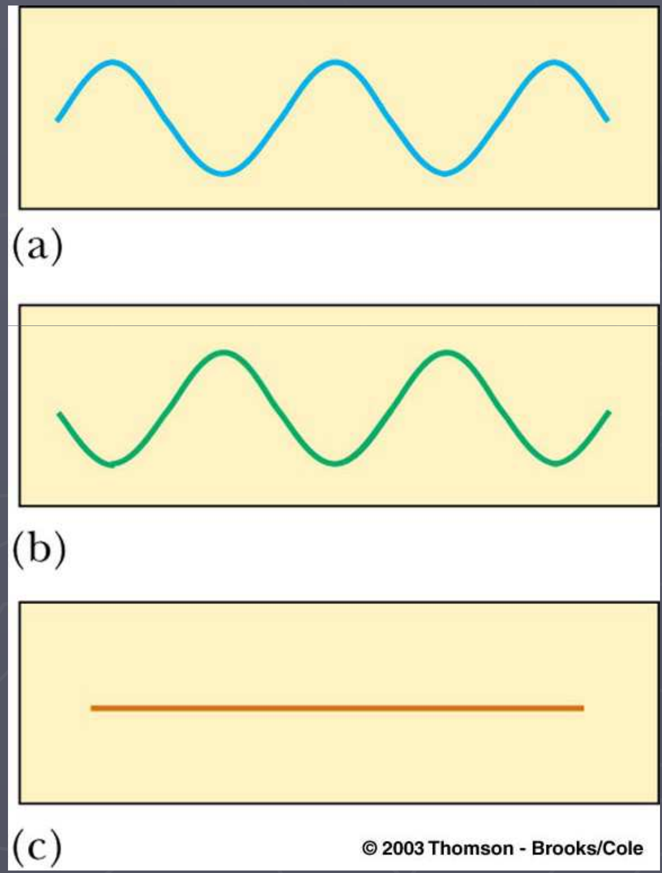
Constructive Interference in a String

- Two pulses are traveling in opposite directions
- The net displacement when they overlap is the sum of the displacements of the pulses
- Note that the pulses are unchanged after the interference



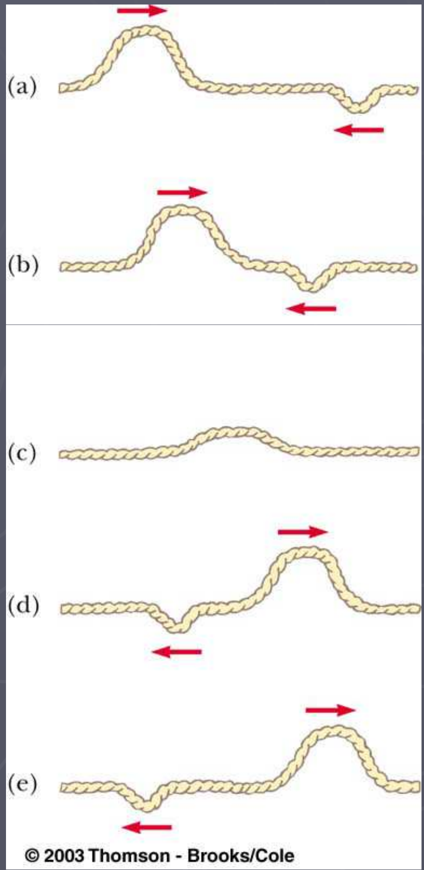
Destructive Interference

- Two waves, a and b, have the same amplitude and frequency
- They are 180° out of phase
- When they combine, the waveforms cancel



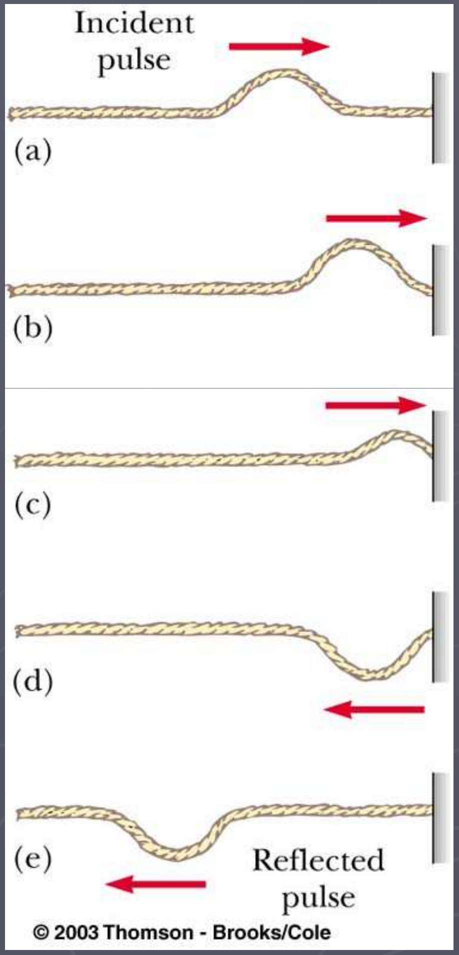
Destructive Interference in a String

- Two pulses are traveling in opposite directions
- The net displacement when they overlap the displacements of the pulses subtract
- Note that the pulses are unchanged after the interference



Reflection of Waves – Fixed End

- Whenever a traveling wave reaches a boundary, some or all of the wave is reflected
- When it is reflected from a fixed end, the wave is inverted



Reflected Wave – Free End

- When a traveling wave reaches a boundary, all or part of it is reflected
- When reflected from a free end, the pulse is not inverted