VECTOR ANALYSIS

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Vectors and Scalars

- **A vector** is a quantity having both **magnitude and direction**, such as displacement, velocity, force, and acceleration.
- **A scalar** is a quantity having **magnitude** but **no direction**, e. g. mass, length, time, temperature, volume, speed and any real number.

Vectors and Scalars

- **Graphically a vector** is represented by an **arrow** OP (Fig. 1) defining the direction, the magnitude of the vector being indicated by the length of arrow.
- **Magnitude of vector** is determined by arrow, using precise **unit**.

Symbol and Notation of Vector

- Vector is denoted by bold face type such as **A** or it can be represented by $\vec{A}$
- The magnitude is denoted by A or $|\vec{A}|$
- Vector is drawn by arrow. Tail of arrow show position of **Origin** or **initial point** while the head of arrow show **terminal point** or **terminus.**

Graphic of Vector

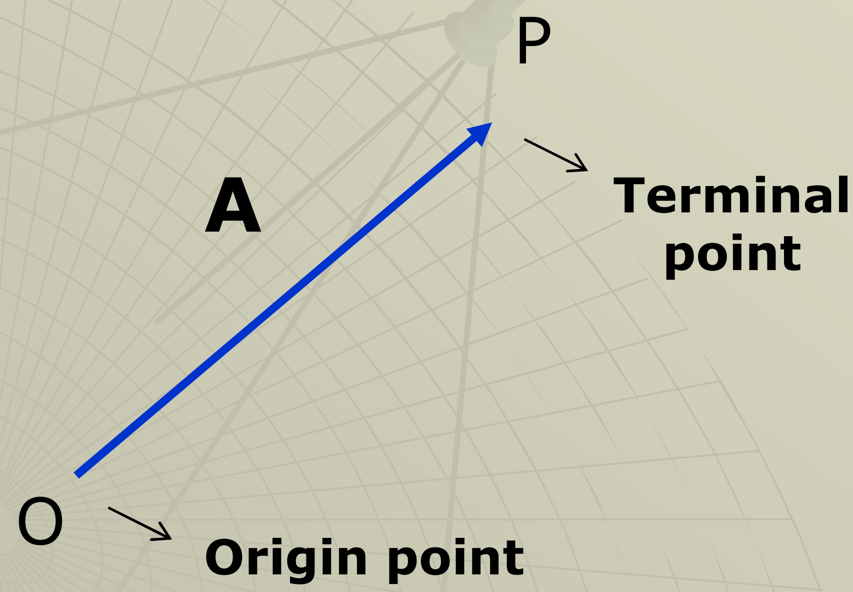


Figure 1

Definition

- Two vector **A** and **B** are **equal**, if they have the **same magnitude and direction** regardless of the position of their initial points.

Thus **A = B** in Fig. 2

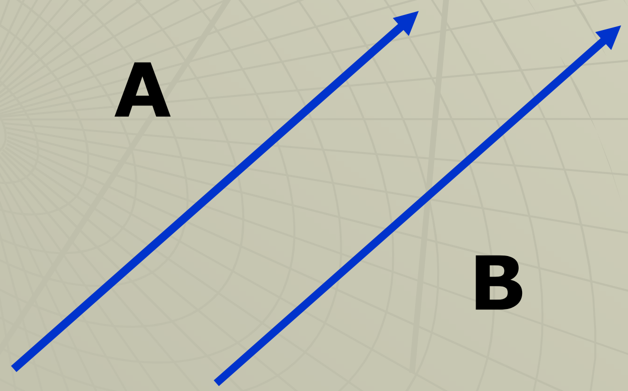


Figure 2

Definition

- **A vector** having **direction opposite** to that of vector **A** but having the **same magnitude** is denoted by –**A**.

A = - B

B = - A

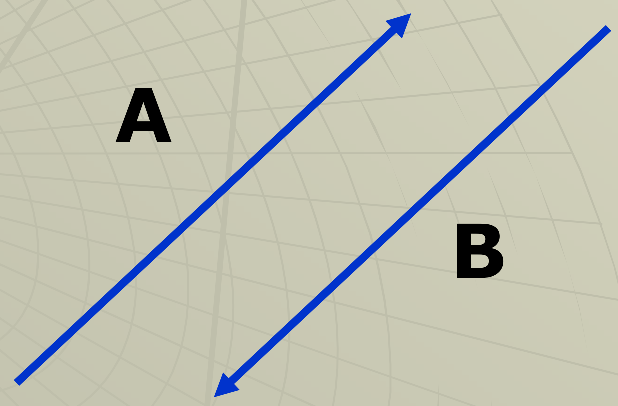


Figure 3

Resultant of Vector

Definition:

- The sum or resultant of vector **A** and **B** is a vector **C** formed by placing the initial point of **B** on the terminal point of **A** to the terminal point of B (Figure 4)

Definition

- The Sum of Vector

C = A + B → VECTOR

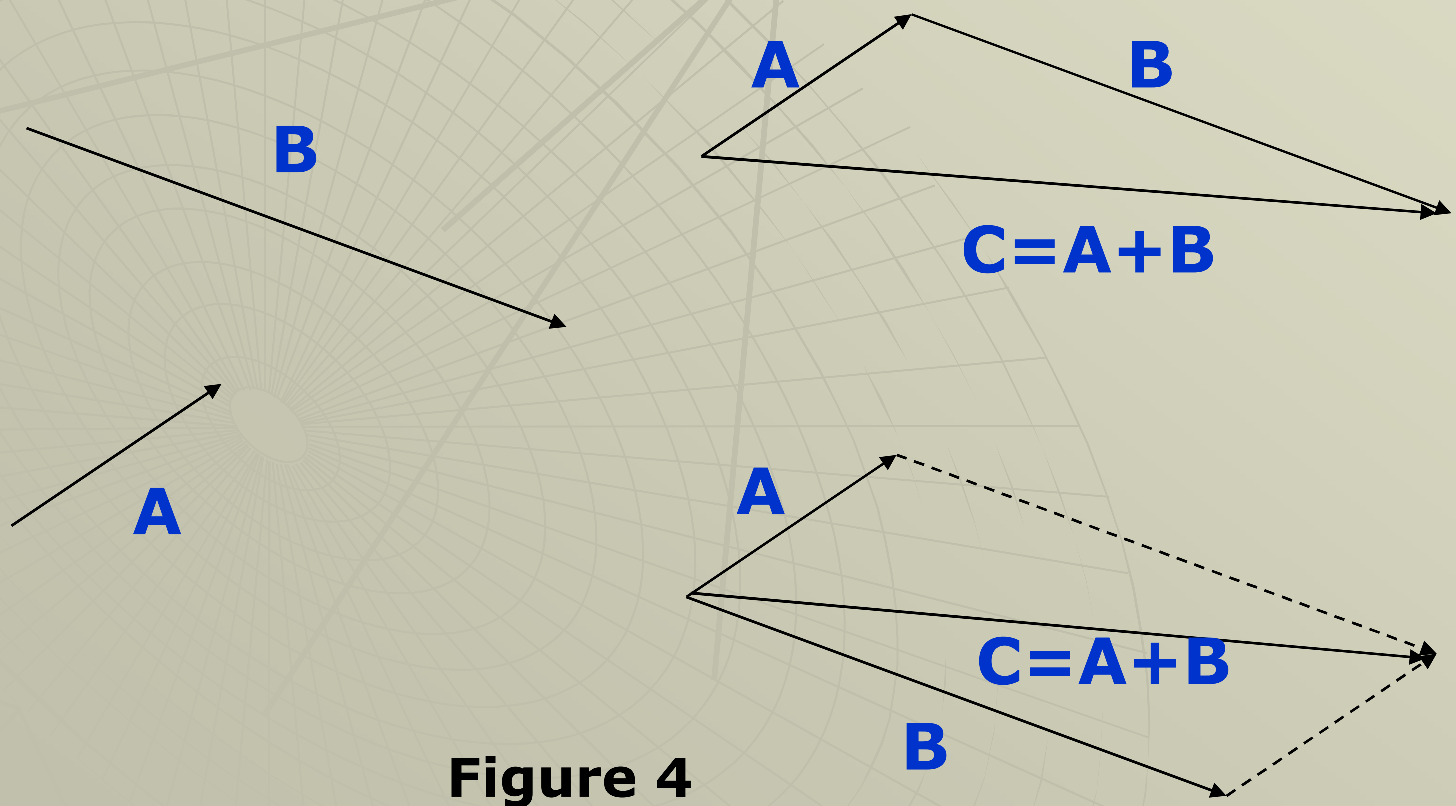
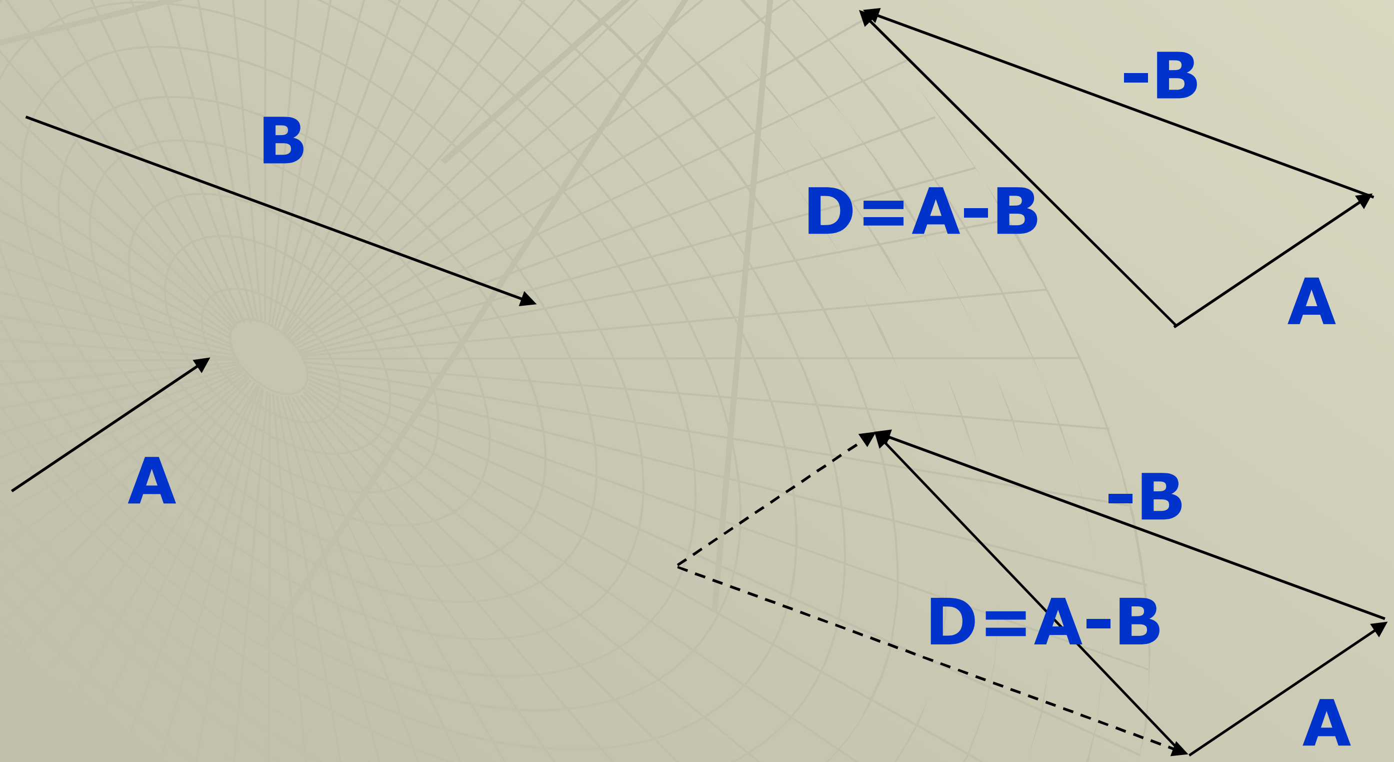


Figure 4

The difference of vector

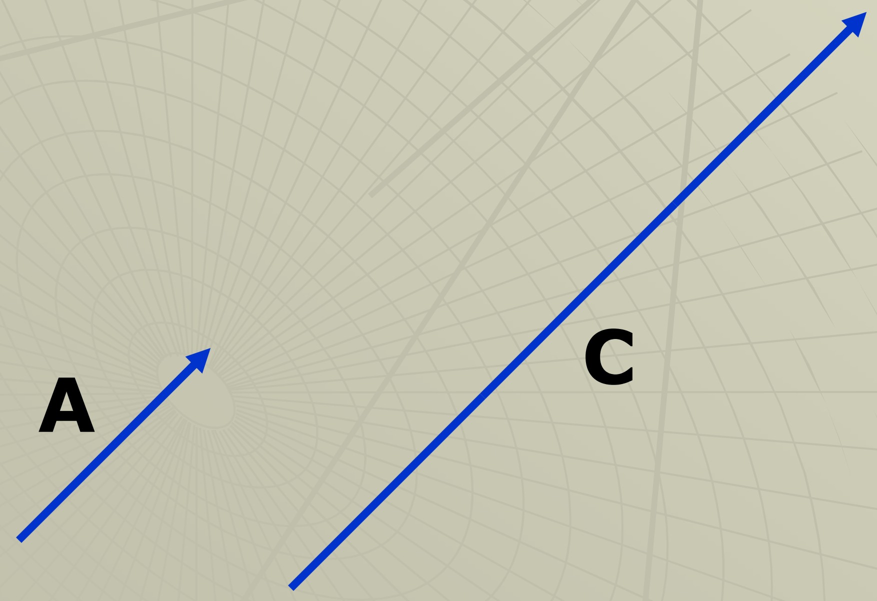
- The difference of vector **A** and **B**, represented by:

  D = A - B = A + (-B) → VECTOR



Definition

- The product of vector **A** by scalar *m* is a vector *m***A** → Vector



If, $m = 3$

$\mathbf{C} = 3\mathbf{A}$

Laws of Vector Algebra

If **A, B, C** are vectors and m, n are scalars.

1. **A+B=B+A** → Commutative Law for Addition
2. **A+(B+C)=(A+B)+C** → Associative Law for Addition
3. m**A**=**A**m → Commutative Law for Multiplication
4. m(n**A**)=mn(**A**)=n(m**A**) → Associative Law for multiplication
5. (m+n)**A** =m**A**+n**A** → Distributive Law
6. m(**A+B**) =m**A**+m**B** → Distributive Law

A Unit Vector

- A Unit Vector is a vector having unit magnitude, if **A** is a vector with magnitude A≠0, then **A**/A is a unit vector having the same direction as **A**.
- Any vector **A** can be represented by a unit vector **a** in the direction of **A** multiplied by the magnitude of **A**. In symbols, **A** = A**a**

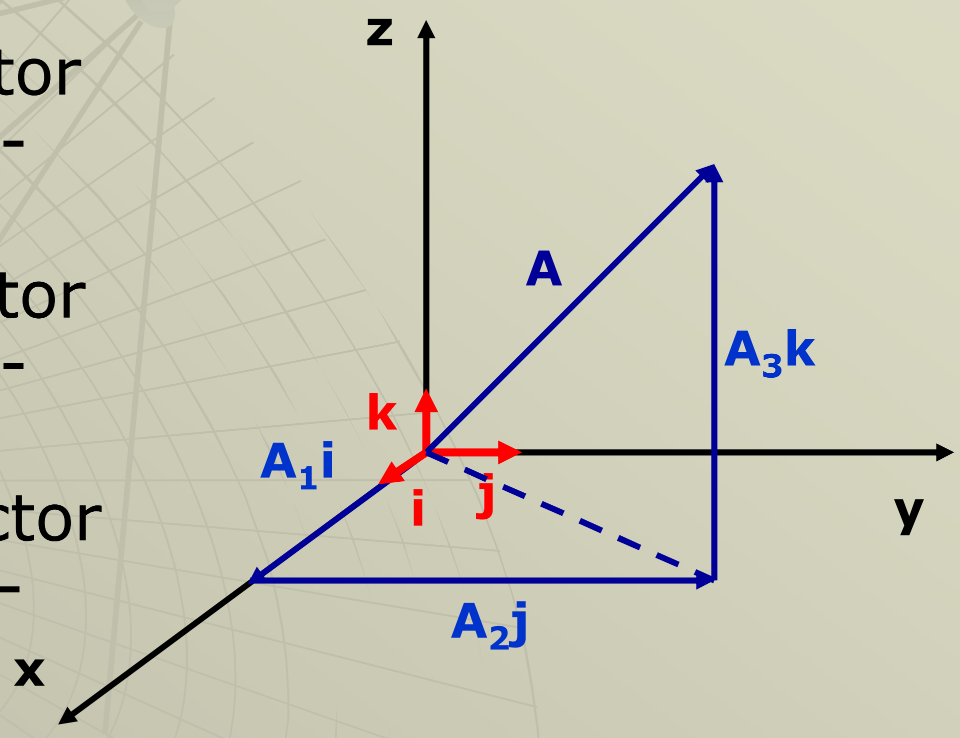
Components of a Vector

$A = A_1 i + A_2 j + A_3 k$

$A_1 i$ = component of vector A in the direction of x-axis

$A_2 j$ = component of vector A in the direction of y-axis

$A_3 k$ = component of vector A in the direction of z-axis



Addition of Vector

$$A = A_1 i + A_2 j + A_3 k$$
$$B = B_1 i + B_2 j + B_3 k$$

$$C = A + B = (A_1 i + A_2 j + A_3 k) + (B_1 i + B_2 j + B_3 k)$$

$$C = A + B = (A_1 + B_1) i + (A_2 + B_2) j + (A_3 + B_3) k$$

$$C = A - B = (A_1 i + A_2 j + A_3 k) - (B_1 i + B_2 j + B_3 k)$$

$$C = A - B = (A_1 - B_1) i + (A_2 - B_2) j + (A_3 - B_3) k$$

Vector Multiplication with scalar

$$A = A_1 i + A_2 j + A_3 k$$

$$B = B_1 i + B_2 j + A_3 k$$

$$D = 3A = 3(A_1 i + A_2 j + A_3 k)$$

Magnitude of Vector

Phytagoras Teorema :

$(OP)^2 = (OQ)^2 + (QP)^2$

but

$(OQ)^2 = (OR)^2 + (RQ)^2$

so

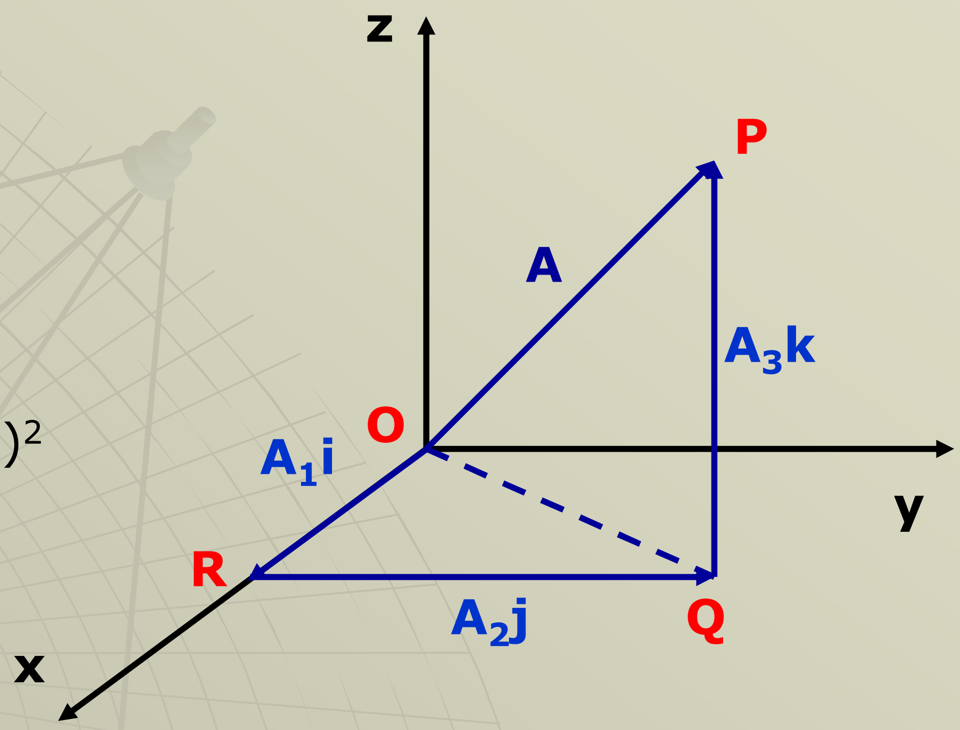
$(OP)^2 = (OR)^2 + (RQ)^2 + (QP)^2$

or

$A^2 = A_1^2 + A_2^2 + A_3^2$

or

$$A = \sqrt{A_1^2 + A_2^2 + A_3^2}$$



Example:

Known r_1= 2i+4j-5k and r_2 = i+2j+3k

a. Determine vector resultant r_1 and r_2 !

b. Determine unit vector in vector resultant direction !

Answer :

a. $R = r_1 + r_2 = (2i+4j-5k) + (i+2j+3k) = 3i + 6j - 2k$

b. $|R| = |(3i + 6j - 2k)| = \sqrt{9+36+4} = \sqrt{49} = 7$

$$r = \frac{R}{|R|} = \frac{3i + 6j - 2k}{7}$$

Ceck magnitude of unit vector = 1

Solved Problems

1. State which of the following are scalars and which are vectors.

(a) Weight (vector)
(b) Calorie (scalar)
(c) Specific heat (scalar)
(d) Momentum (vector)
(e) Density (scalar)
(f) Energy (scalar)
(g) Volume (scalar)
(h) distance (scalar)
(i) speed (scalar)
(j) magnetic field intensity (vector)

2. Represent graphically:

(a) A force of 10 N in a direction 30° north of east.

(b) A force of 15 N in a direction 30° east of north.

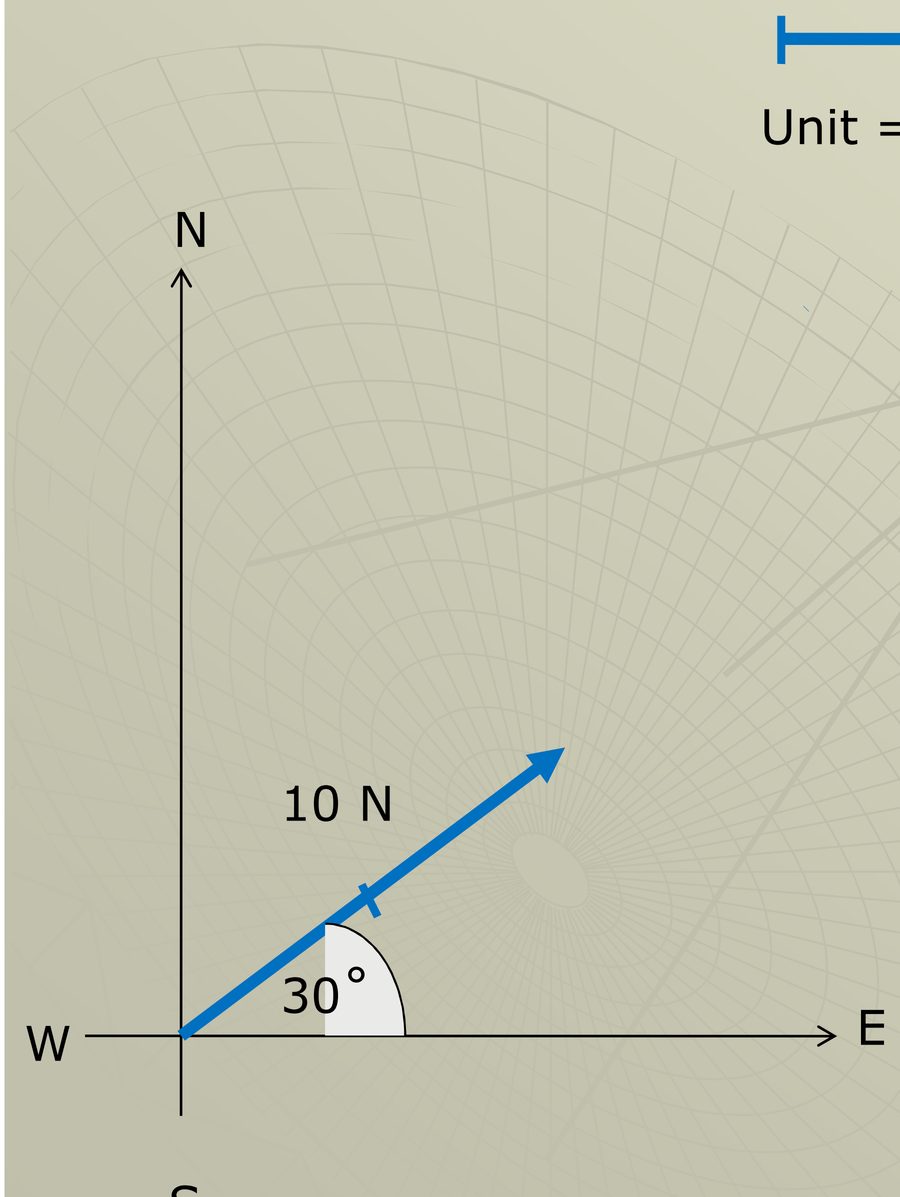


Figure (a)

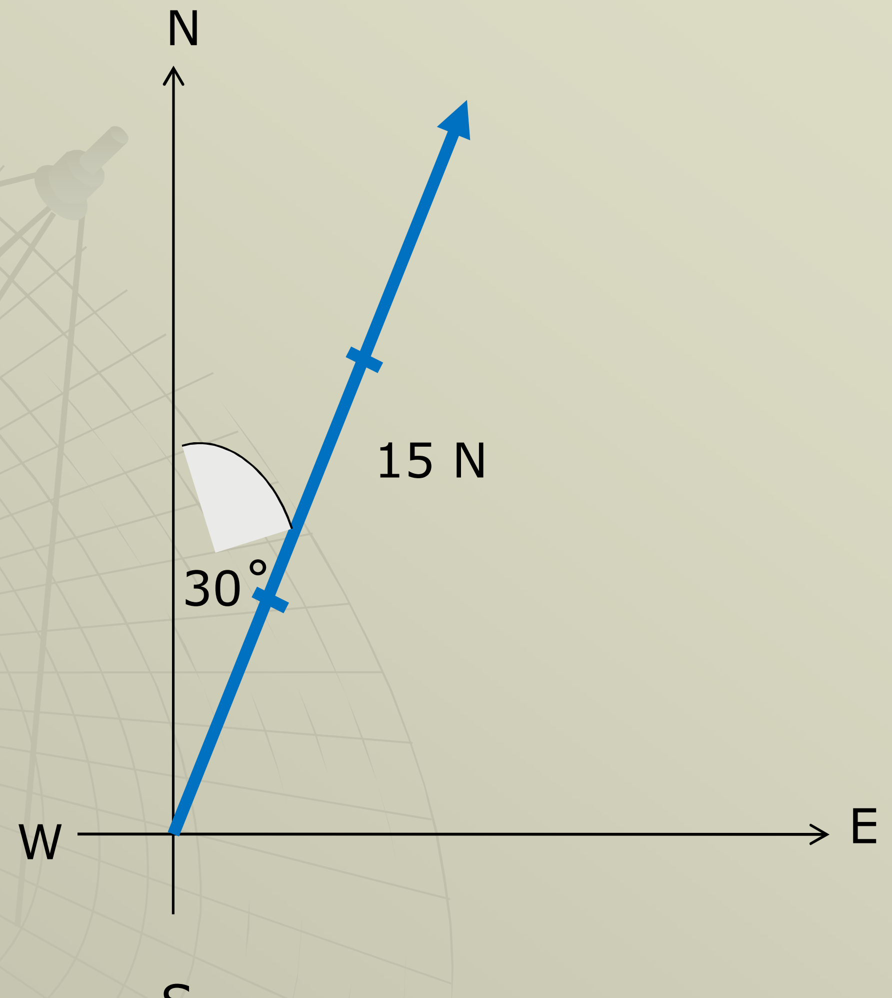


Figure (b)

to be continued...

Thanks...

Perkalian Titik

(Dot Product)

Dot poduct antara **A** dan **B**
Atau perkalian skalar didefinisikan :

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$

θ Adalah sudut terkecil yang diapit **A** dan **B**

Secara fisis dot product adalah proyeksi suatu vektor terhadap vektor lainnya, sehingga sudut yang diambil adalah sudut yang terkecil

Perkalian Titik

(Dot Product)

$$A\,.\,B = (A_1 i + A_2 j + A_3 k).(B_1 i + B_2 j + B_3 k)$$

$$= (A_1 i).(B_1 i + B_2 j + B_3 k) + (A_2 j).(B_1 i + B_2 j + B_3 k)$$

$$+ (A_3 k).(B_1 i + B_2 j + B_3 k)$$

$$= A_1 B_1 (i.i) + A_1 B_2 (i.j) + A_1 B_3 (i.k)$$

$$+ A_2 B_1 (j.i) + A_2 B_2 (j.j) + A_2 B_3 (j.k)$$

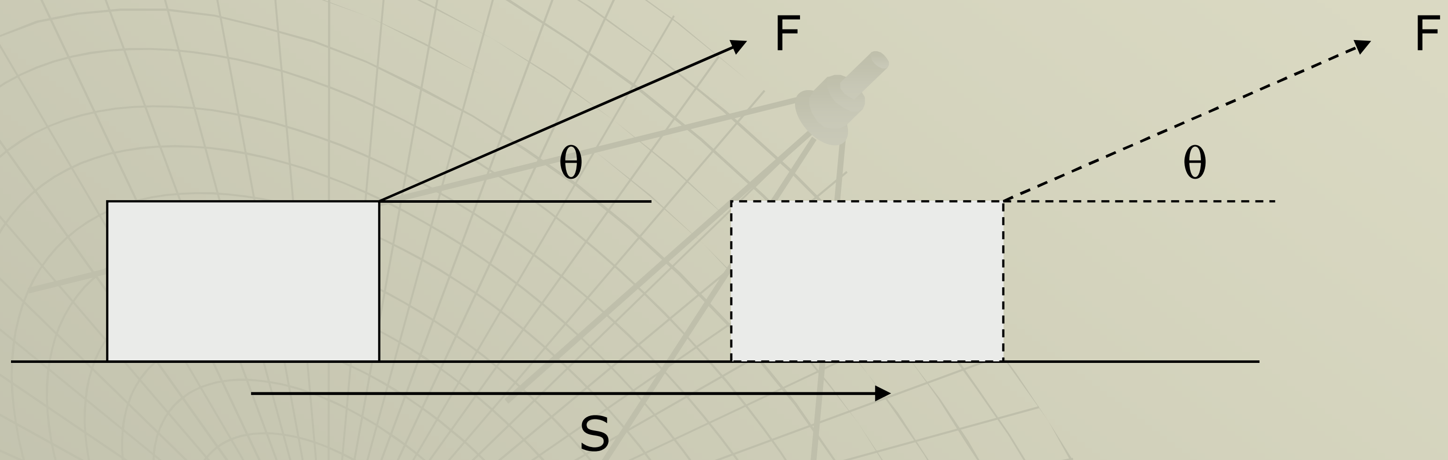
$$+ A_3 B_1 (k.i) + A_3 B_2 (k.j) + A_3 B_3 (k.k)$$

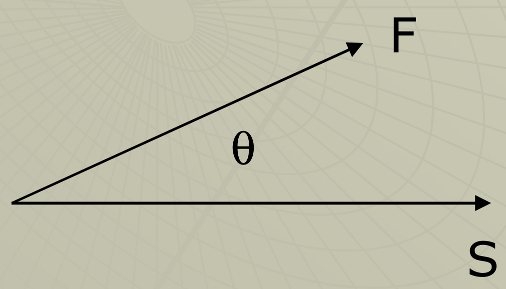
$$A.B = A_1 B_1 + A_2 B_2 + A_3 B_3$$

$$i.i = |i||i| \cos 0^o = 1 \qquad j.j = |j||j| \cos 0^o = 1 \qquad k.k = |k||k| \cos 0^o = 1$$

$$i.j = j.i = |i||j| \cos 90^o = 0 \qquad j.k = k.j = |j||k| \cos 90^o = 0 \qquad i.k = k.i = |k||i| \cos 90^o = 0$$

Contoh dot product dalam Fisika





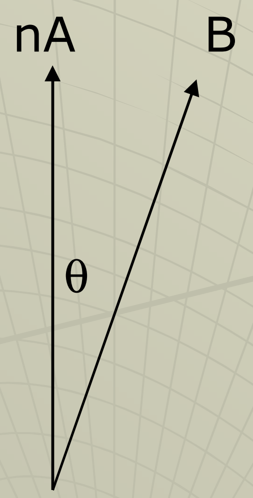
$W = FS\cos\theta = \mathbf{F}\cdot\mathbf{S}$

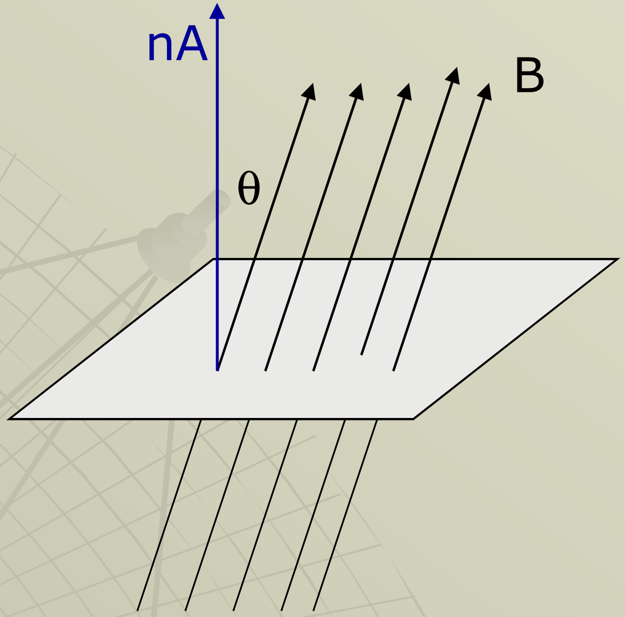
W = usaha

F = Vektor gaya

S = Vektor perpindahan

Contoh dot product dalam Fisika





$\phi = BA \cos\theta = \mathbf{B} \cdot \mathbf{A}$

ϕ = Fluks magnetik

B = Medan magnetik

A = arah bidang

Catatan :

Bidang adalah vektor memiliki luas dan arah. Arah bidang adalah arah normal bidang di suatu titik.

Normal = tegak lurus

Perkalian Silang

(Cross Product)

Cross poduct antara **A** dan **B**
Atau perkalian vektor didefinisikan :

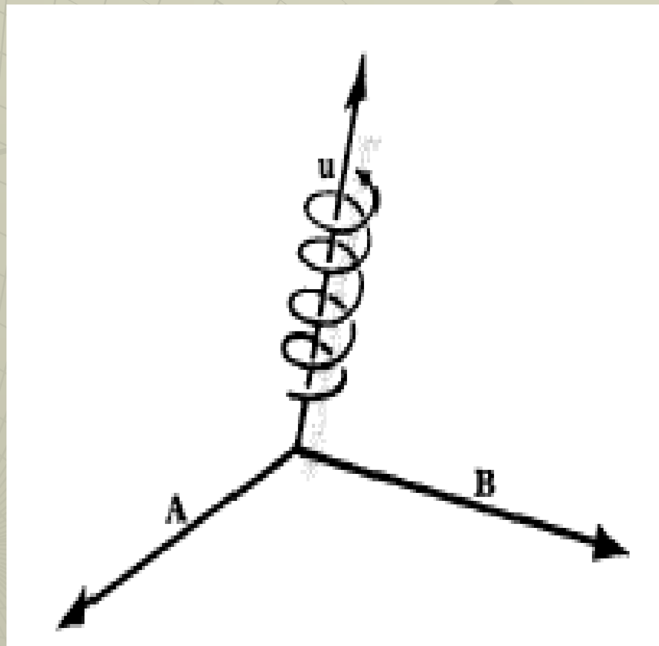
$$\mathbf{A} \times \mathbf{B} = AB \sin\theta\, \mathbf{u}$$

θ Adalah sudut terkecil yang diapit **A** dan **B**

Hasil perkalian silang antara vektor A dan vektor B adalah sebuah vektor C yang arahnya tegak lurus bidang yang memuat vektor A dan B, sedemikian rupa sehingga A, B, dan C membentuk sistem tangan kanan (sistem skrup)

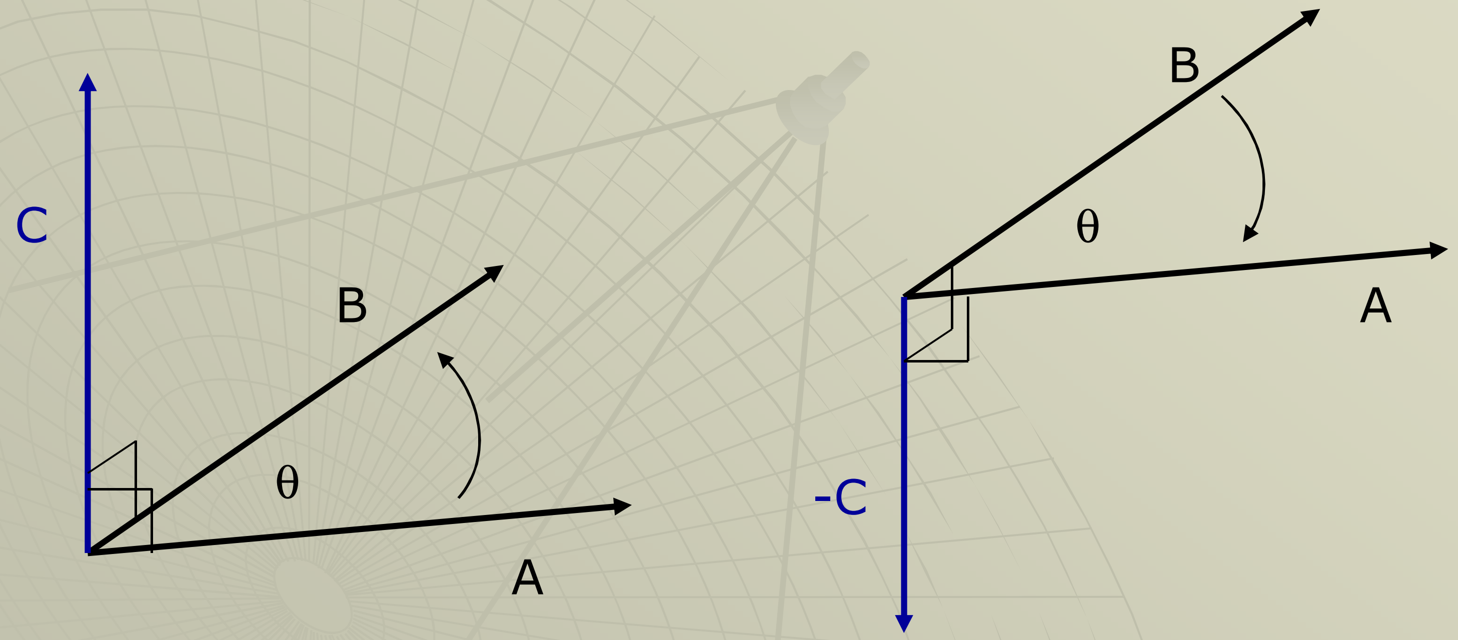
Perkalian Silang

(Cross Product)



Perkalian Silang

(Cross Product)



C = A x B

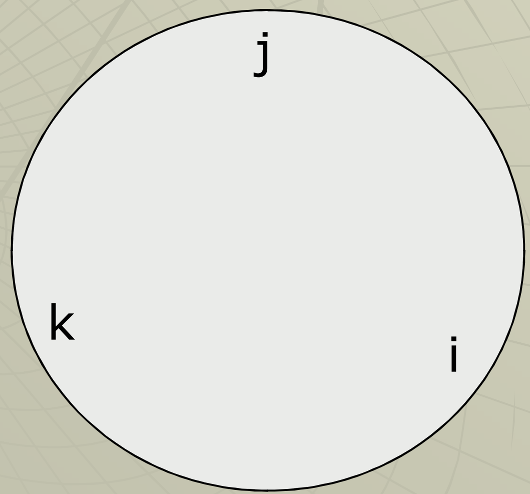
-C = B x A

Pada sistem koordinat tegak lurus

$$i \times i = 0 \qquad j \times j = 0 \qquad k \times k = 0$$

$$i \times j = k \qquad j \times k = i \qquad k \times i = j$$

$$j \times i = -k \qquad k \times j = -i \qquad i \times k = -j$$



Perkalian silang

(Cross Product)

$$A \times B = (A_1 i + A_2 j + A_3 k) \times (B_1 i + B_2 j + B_3 k)$$

$$= (A_1 i) \times (B_1 i + B_2 j + B_3 k) + (A_2 j) \times (B_1 i + B_2 j + B_3 k)$$

$$+ (A_3 k) \times (B_1 i + B_2 j + B_3 k)$$

$$= A_1 B_1 (i \times i) + A_1 B_2 (i \times j) + A_1 B_3 (i \times k)$$

$$+ A_2 B_1 (j \times i) + A_2 B_2 (j \times j) + A_2 B_3 (j \times k)$$

$$+ A_3 B_1 (k \times i) + A_3 B_2 (k \times j) + A_3 B_3 (k \times k)$$

$$= A_1 B_1 (0) + A_1 B_2 (k) + A_1 B_3 (-j)$$

$$+ A_2 B_1 (-k) + A_2 B_2 (0) + A_2 B_3 (i)$$

$$+ A_3 B_1 (j) + A_3 B_2 (-i) + A_3 B_3 (0)$$

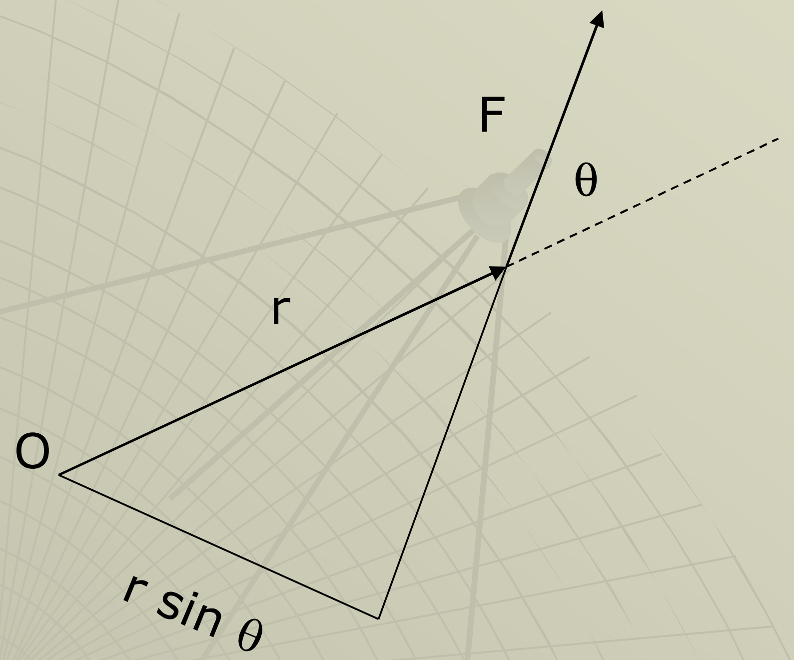
Perkalian silang
(Cross Product)

$$A \times B = A_1B_1(0) + A_1B_2(k) + A_1B_3(-j)$$
$$+ A_2B_1(-k) + A_2B_2(0) + A_2B_3(i)$$
$$+ A_3B_1(j) + A_3B_2(-i) + A_3B_3(0)$$

$$A \times B = (A_1B_2 - A_2B_1)\, k + (A_3B_1 - A_1B_3)\, j$$
$$+ (A_2B_3 - A_3B_2)\, i$$

$$A \times B = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

Contoh perkalian silang dalam Fisika

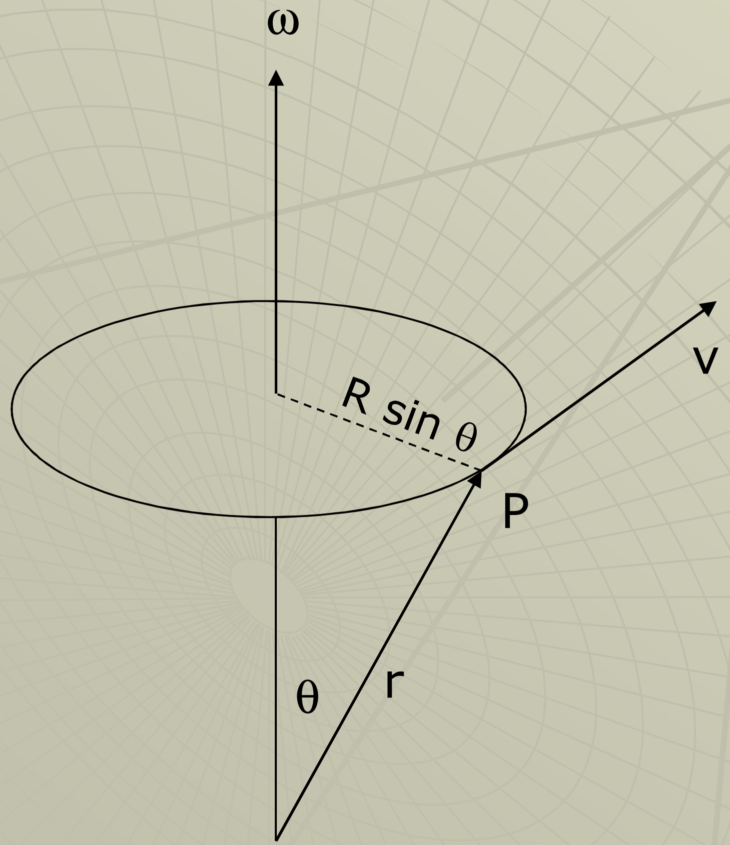


$$\tau = \vec{r} \times \vec{F} = r\,F\sin\theta = \vec{r}\times\vec{F}$$

Contoh Soal

Jika gaya **F = 2i - j + 3k** bekerja pada titik (2,-1,1), tentukan torsi dari **F** terhadap titik asal koordinat

Gerak melingkar



$$\vec{v} = \vec{\omega} \times \vec{r}$$

Perkalian tiga vektor

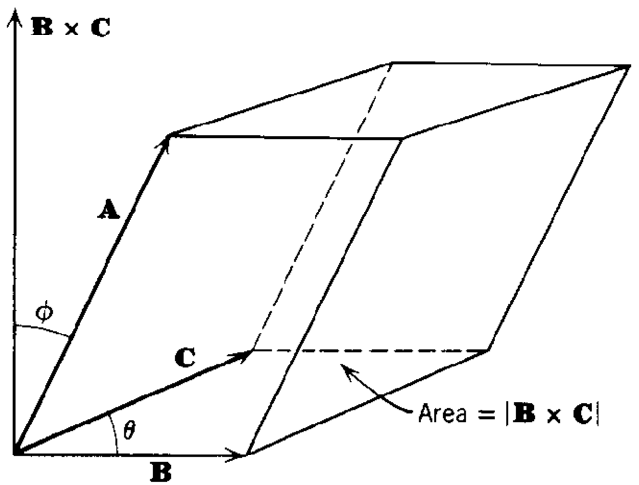


FIGURE 3.1

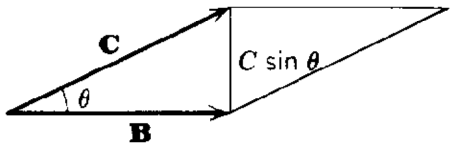
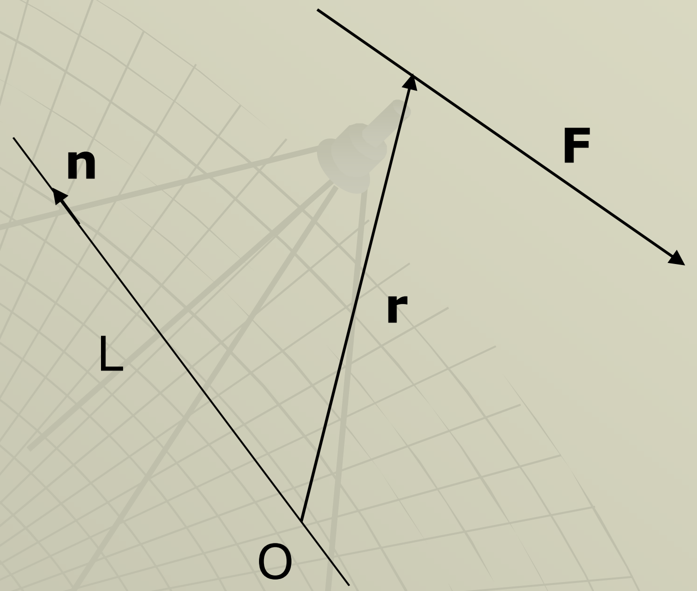


FIGURE 3.2

$$\left|\vec{B}\right|\left|\vec{C}\right|\sin\theta\left|\vec{A}\right|\cos\phi = \left|\vec{B}\times\vec{C}\right|\left|\vec{A}\right|\cos\phi = \vec{A}\bullet\left(\vec{B}\times\vec{C}\right)$$

Aplikasi Perkalian Skalar Tiga Vektor



Komponen torsi terhadap garis L :

$$\tau_{II} = \hat{n} \bullet \vec{\tau} = \hat{n} \bullet \left(\vec{r} \times \vec{F} \right)$$

Contoh Soal

Jika gaya **F = i + 3j – k** bekerja pada titik (1,1,1), tentukan komponen torsi dari **F** terhadap garis **r** = 3**i** + 2**k** + (2**i** - 2**j** + **k**)t.

Solusi:

Pertama kita tentukan vektor torsi terhadap sebuah titik pada garis yaitu titik (3,0,2). Torsi tersebut adalah τ = **r** x **F** dimana **r** adalah vektor berasal dari titik pada garis ke titik dimana **F** bekerja, yaitu dari (3,0,2) ke (1,1,1), sehingga **r** = (1,1,1) - (3,0,2) = (-2,1,-1). Dengan demikian vektor torsi τ :

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Contoh Torsi:

$$\mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & -1 \\ 1 & 3 & -1 \end{vmatrix} = 2\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}.$$

- Torsi untuk garis adalah **n**.(**r**x**F**) dimana **n** adalah vaktor satuan sepajang garis, dengan **n** = 1/3(2**i**-2**j**+**k**).
- Kemudian torsi untuk garis adalah

  n.(**r**x**F**) = 1/3(2**i**-2**j**+**k**).(2**i**-3**j**-7**k**)=1

Aplikasi Tripel Scalar Product

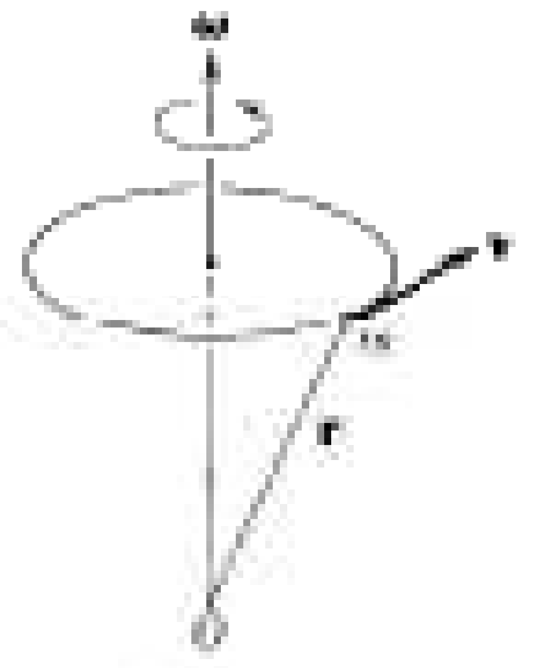
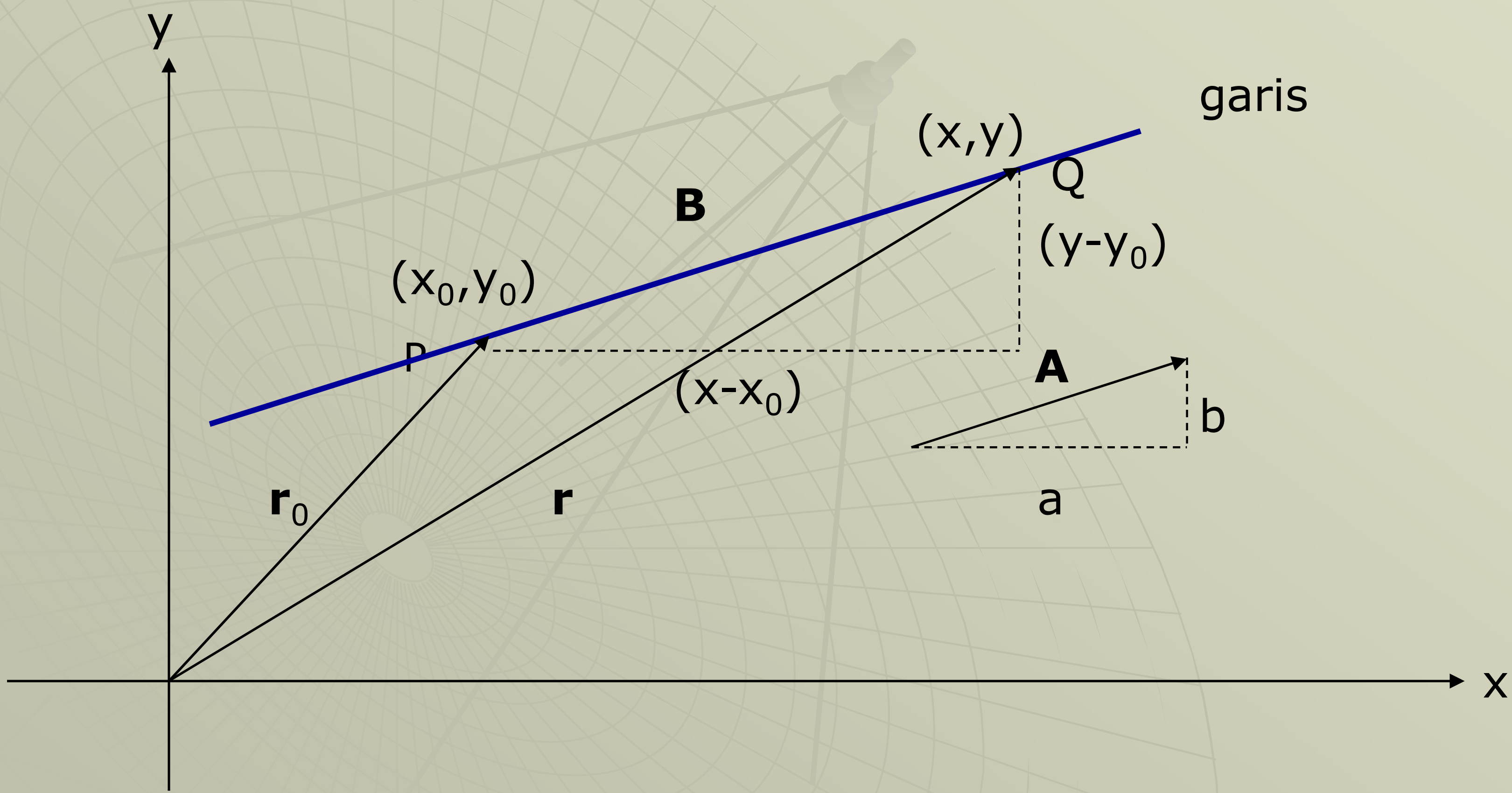


FIGURE 3.8

- Aplikasi Tripel Scalar Product salah satunya pada momentum linear

PERSAMAAN GARIS LURUS DAN PERSAMAAN BIDANG

Persamaan Garis Lurus



Definisi Garis

Apakah garis itu?
Garis adalah deretan titik-titik secara kontinu

Dari gambar :
B = r – $\mathbf{r_0}$
dan
A // B (Perbandingan setiap komponen akan sama

dimana
B = $(x\mathbf{i}+y\mathbf{j})-(x_0\mathbf{i}+y_0\mathbf{j})$
= $(x-x_0)\mathbf{i}+(y-y_0)\mathbf{j}$
dan
A = $a\mathbf{i}+b\mathbf{j}$

sehingga

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}\rightarrow 2D$$

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}\rightarrow 3D$$

Disebut persamaan garis lurus simetris

(x_0,y_0,z_0) adalah suatu titik yang dilalui garis a,b,c.
Komponen vektor arah.

Dari gambar di atas juga :

$$\mathbf{r} = \mathbf{r_0} + \mathbf{B}$$

dan

$$\mathbf{B} = t\mathbf{A}$$

sehingga

$$\mathbf{r} = \mathbf{r_0} + \mathbf{A}t$$

$$= (x_0, y_0, z_0) + (a, b, c)t$$

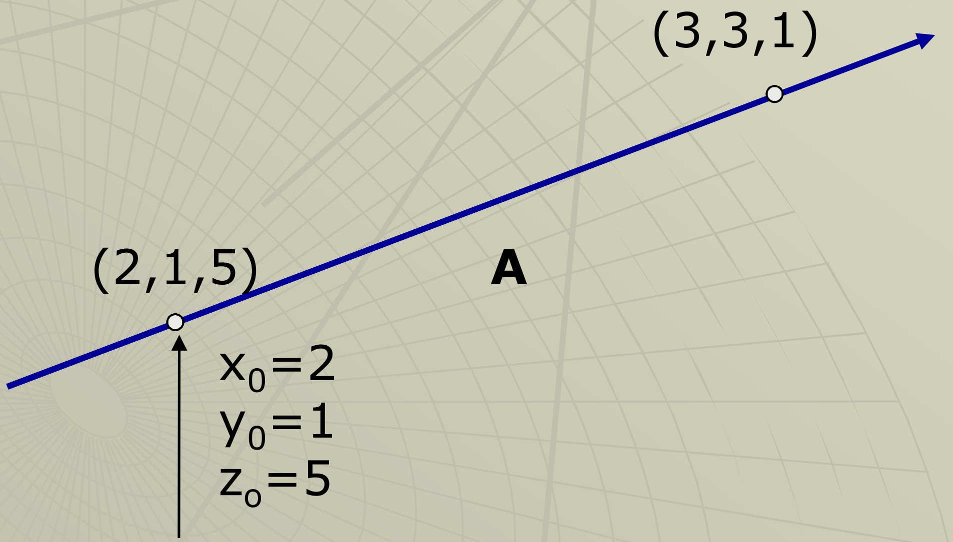
atau

$$\mathbf{r} = \mathbf{i}x_0 + \mathbf{j}y_0 + \mathbf{k}z_0 + (a\mathbf{i} + b\mathbf{j} + z\mathbf{k})t$$

Disebut persamaan garis lurus parametrik

Contoh

Tentukan persamaan garis lurus parametrik dan simetrik yang melalui titik (2,1,5) dan titik (3,3,1)!



Solusi

A = (3,3,1) – (2,1,5)

= (1,2,-4)

A = **i**+2**j**-4**k**

a = 1, b = 2, c = -4

Sehingga :

r = (2,1,5) + (1,2,-4)t

atau

r = 2**i**+**j**+5**k**+(**i**+2**j**-4**k**)t → Persamaan garis parametrik

Titik yang dilalui: 2**i**+**j**+5**k**

Arah garis: (**i**+2**j**-4**k**)

Lanjutan...

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}$$

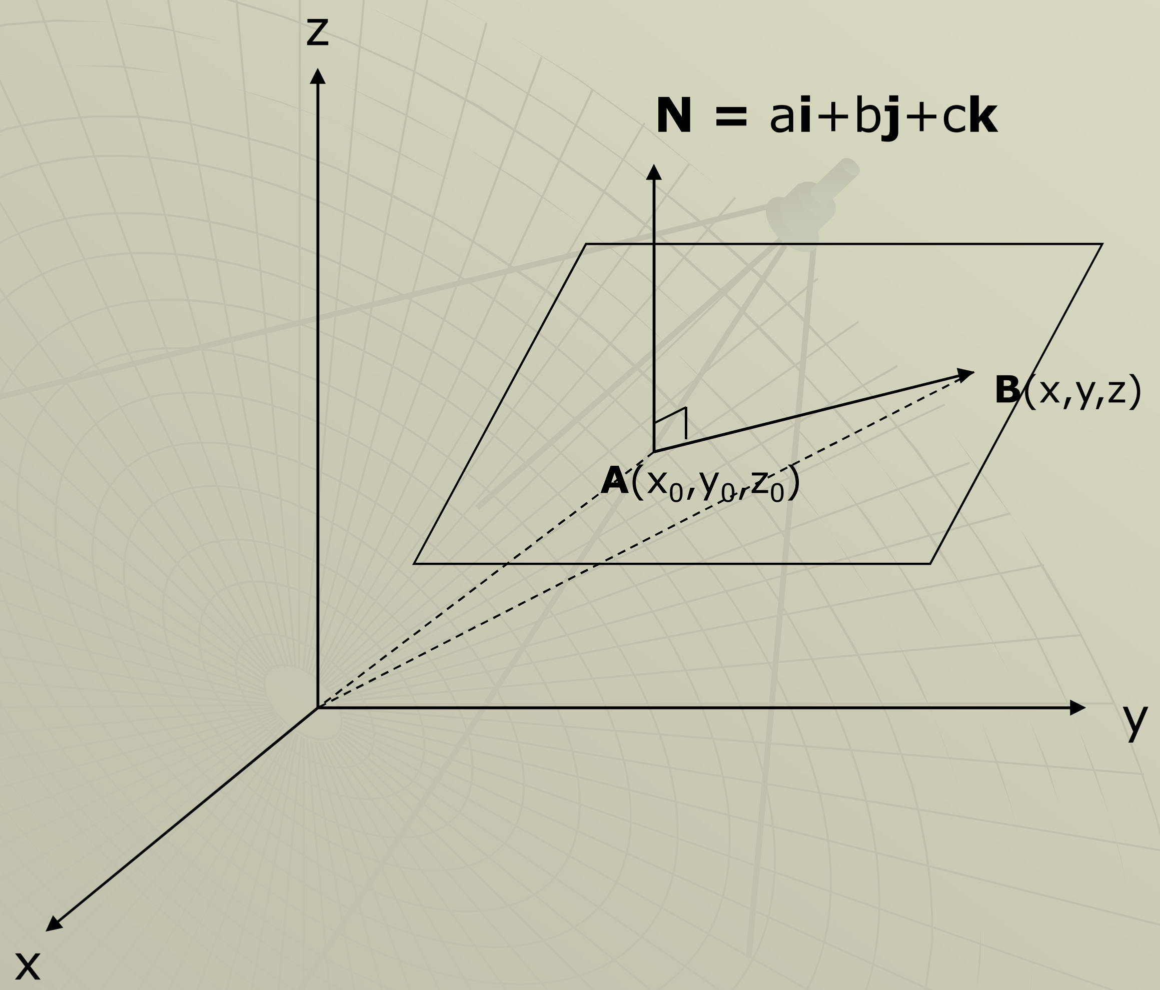
$$\frac{x-2}{1}=\frac{y-1}{2}=\frac{z-5}{-4}$$

$$x-2=\frac{y-1}{2}=\frac{z-5}{-4}$$ → Persamaan Garis Simetrik

Latihan Soal

1. Cari suatu persamaan garis lurus melalui **(3,2,1)** dan sejajar dengan vektor (3**i**-2**j**+6**k**)!
2. Cari persamaan garis lurus yang melalui titik (3,0-5) dan sejajar dengan garis **r** = (2,1,-5) + (0,-5,1)t !

Persamaan Bidang



$\mathbf{AB}=(x-x_0)\mathbf{i}+(y-y_0)\mathbf{j}+(z-z_0)\mathbf{k}$

$\mathbf{N} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$

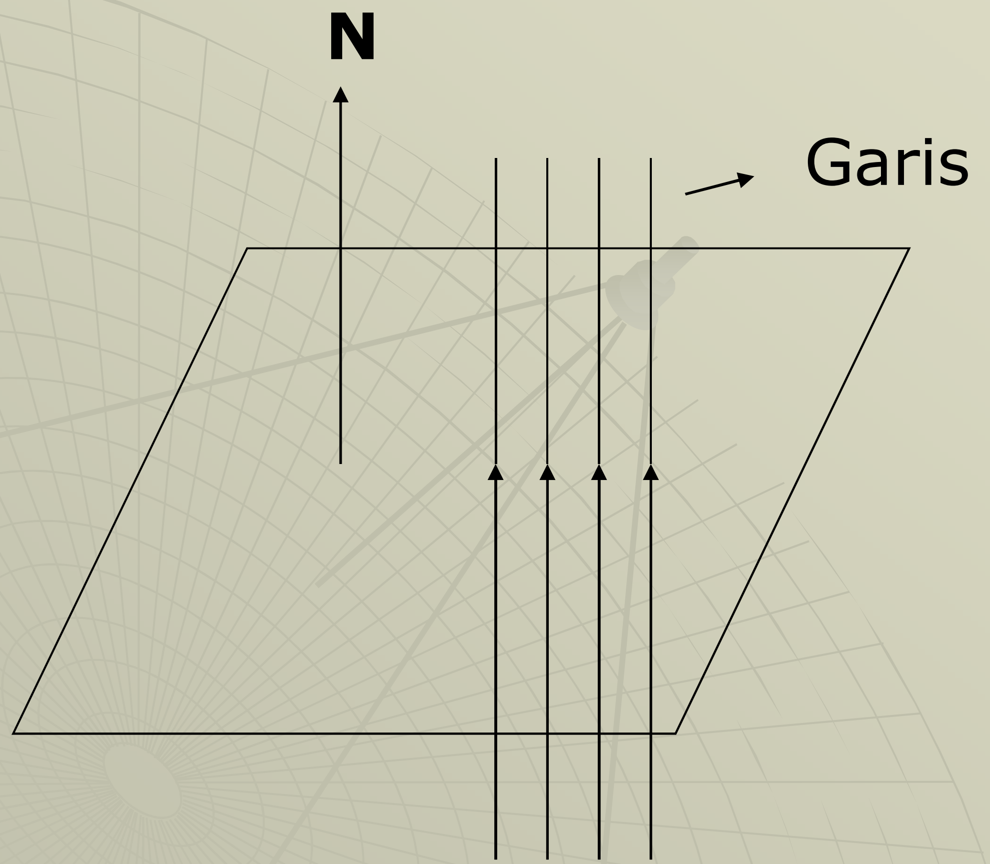
- Lakukan dot product antara **AB** dan **N**
- $\mathbf{N \cdot AB} = N \cdot AB \cdot \cos 90^o = 0$
- $(a\mathbf{i}+b\mathbf{j}+z\mathbf{k})\cdot[(x-x_0)\mathbf{i}+(y-y_0)\mathbf{j}+(z-z_0)\mathbf{k}]=0$
- $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
- $ax+by+cz=ax_0+by_0+cz_0$

Yang diperlukan minimal:

1. Vektor normal bidang (**N**)
2. Suatu titik pada bidang

   Jika diketahui 3 titik pada bidang bisa juga.

- Catatan: Jika suatu garis sejajar dengan arah bidangnya, maka $\theta=0$.

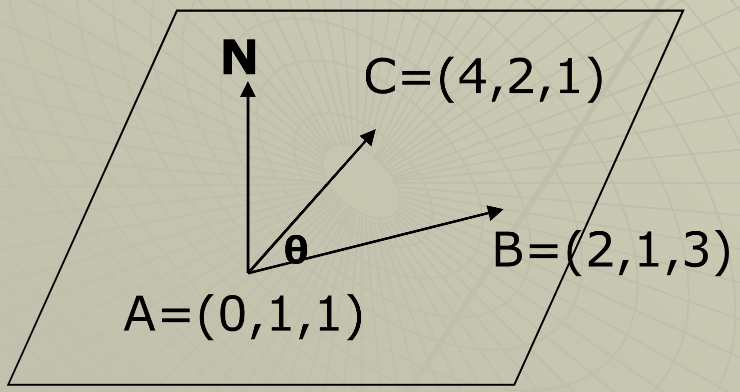


Catatan: Arah bidang selalu tegak lurus terhadap bidang

Contoh Soal:

1. Tentukan persamaan bidang yang mencakup 3 titik

A=(0,1,1); B=(2,1,3); C=(4,2,1)



AB=B-A
AB=(2,1,3)-(0,1,1)
AB=(2,0,2)

AC=C-A
AC=(4,2,1)-(0,1,1)
AC=(4,1,0)

- **N=ABxAC**
- **N**=(2,0,2)x(4,1,0)
- **N**= $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 2 \\ 4 & 1 & 0 \end{vmatrix}$
- **N**=0+8**j**+2**k**+0-2**i**+0
- **N**=-2**i**+8**j+**2**k** →**a**=-2, **b**=8, **c**=2

Lanjutan… Solusi

- Titik yang ditinjau **A**=(0,1,1)
- $x_0=0$; $y_0=1$; $z_0=1$
- $ax+by+cz= ax_0+by_0+cz_0$
- $-2x+8y+2z=8+2$
- $-2x+8y+2z=10$

Latihan Soal:

1. Cari persamaan bidang melalui titik (1,-1,0) dan sejajar dengan garis $\mathbf{r}=(5\mathbf{i}+\mathbf{j}-2\mathbf{k})+(2\mathbf{i}-\mathbf{j}+\mathbf{k})\mathbf{t}$!