Applications of Vector Analysis and Fourier Series and Its Transforms

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Vector Analysis

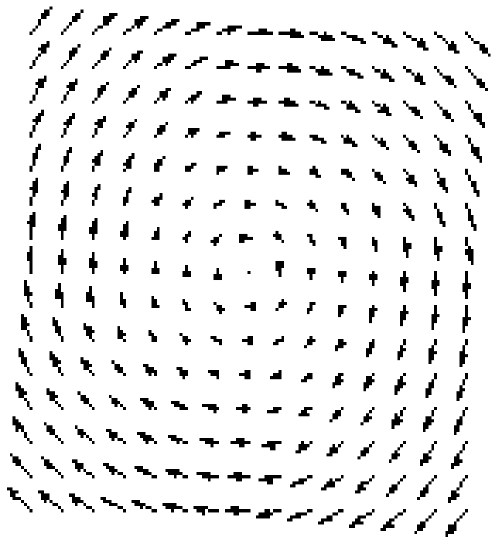
- Engineering, Meteorology , Electromagnetism, Oceanography , Astrophysics, Geology
- V(3D) = ai+bj+ck= [x, y, z]
- Curl of Vector Field: "rotation"

$$\nabla \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix},$$

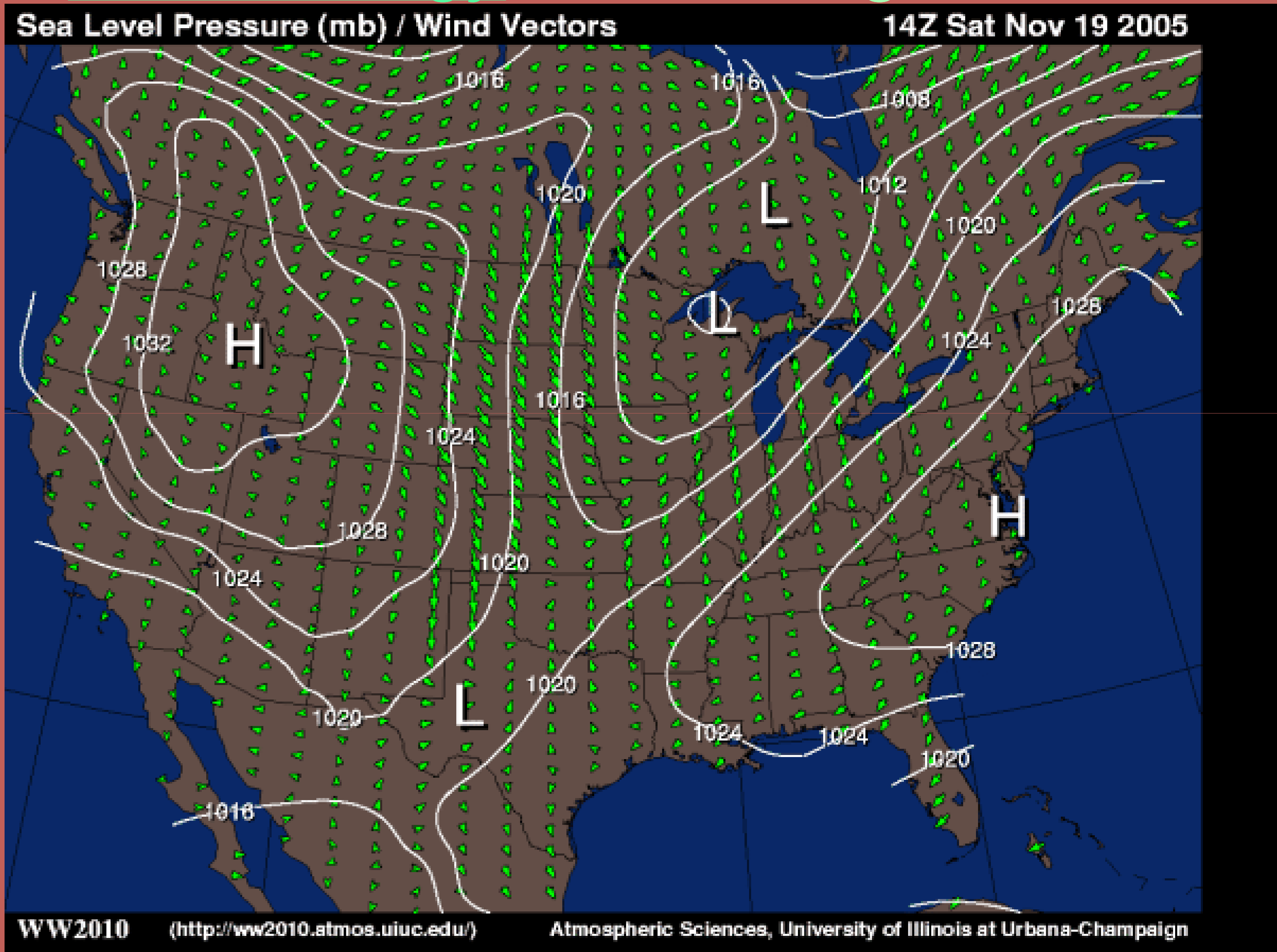
- Directional Derivative

$$\nabla_{\hat{\mathbf{u}}} f = \frac{\partial f}{\partial x} u_x + \frac{\partial f}{\partial y} u_y + \frac{\partial f}{\partial z} u_z.$$

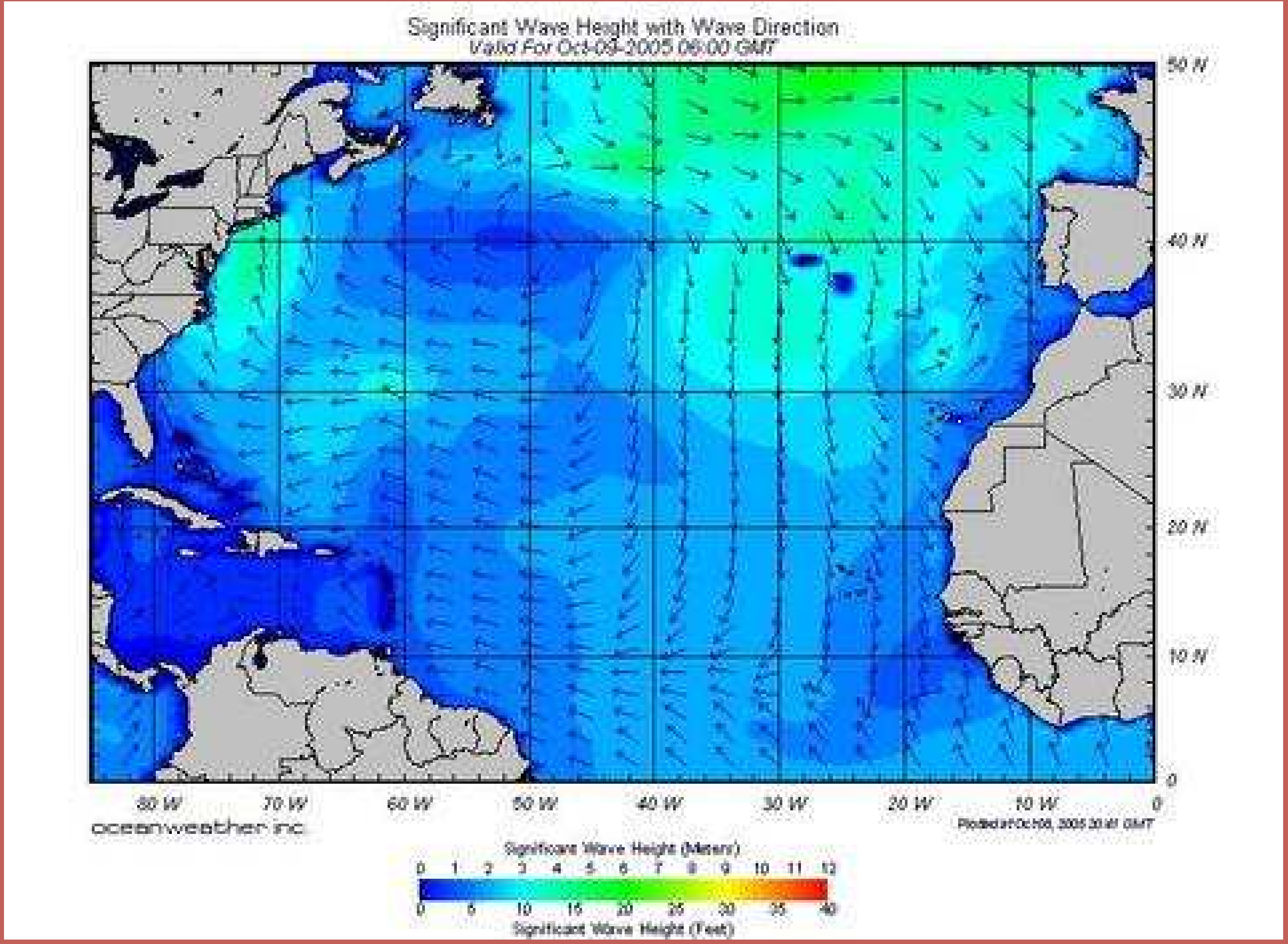
Vector Field



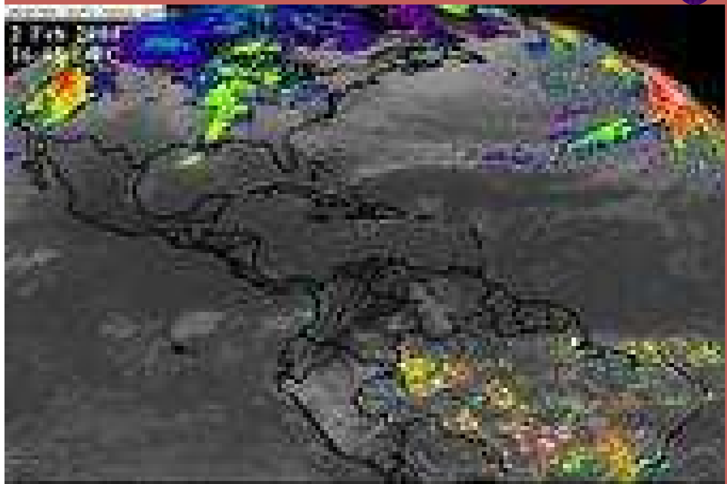
Meteorology : Watching the Wind



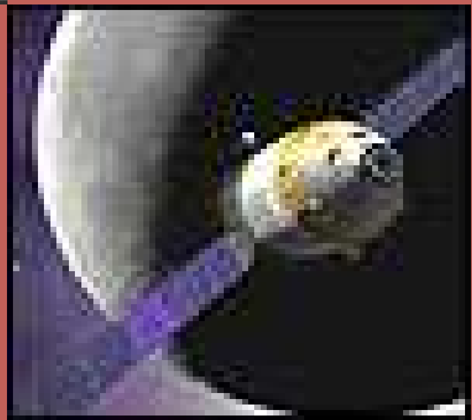
Oceanography : Tracking currents



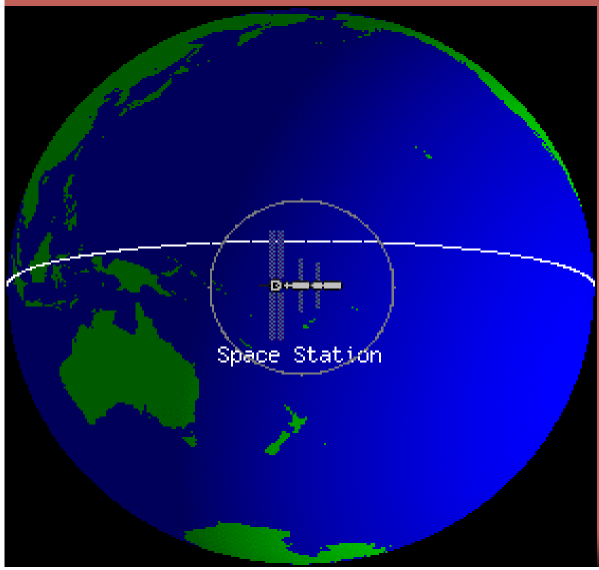
Tracking and launching



- Satellites tracking java program
- http://science.nasa.gov/Realtime/JTrack/3d/JTrack3D.html
- $F_{12} = -G \frac{(m_1 * m_2)}{(R_{21})^2} r \quad G=6.67e\text{-}11$



r is the unit vector



- Space stations tracked
- http://science.nasa.gov/temp/StationLoc.html

Geology : Geophysics
The Movement of the Plates

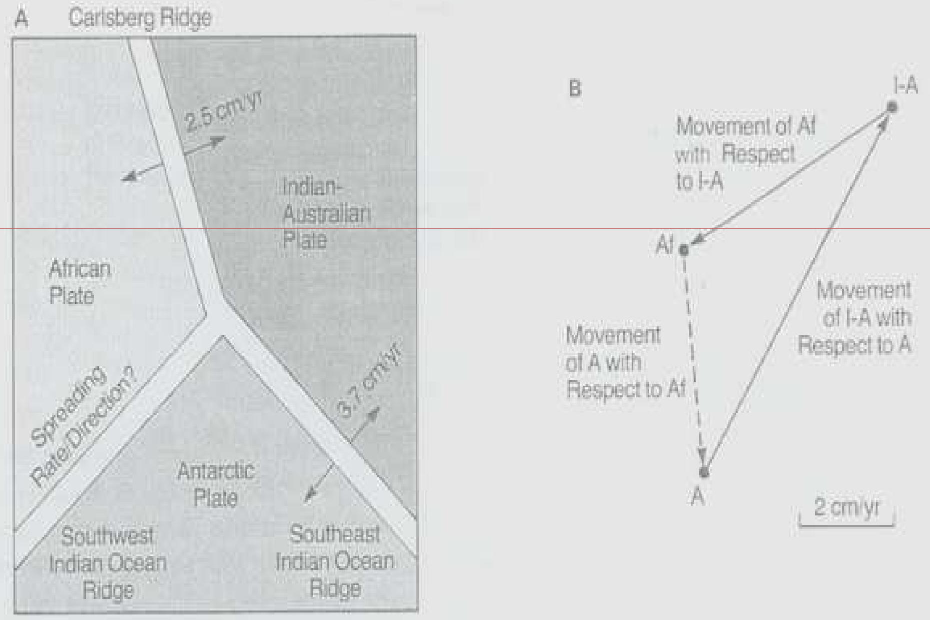
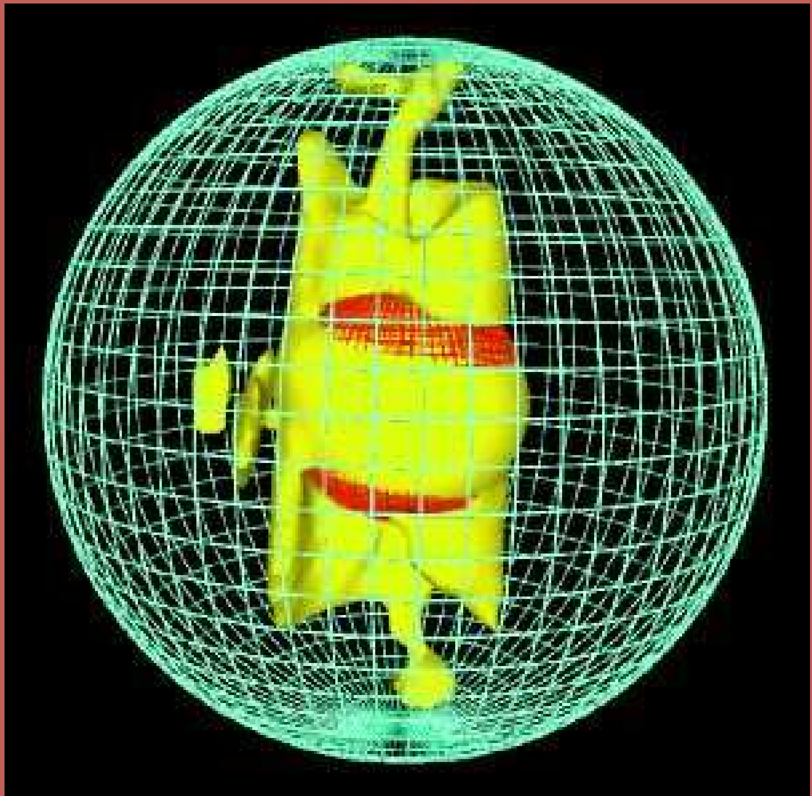


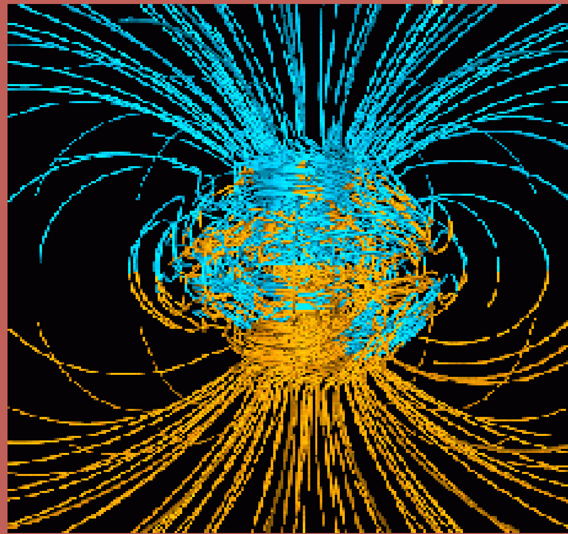
Figure 10.63 Relative motion among plates at the ridge–ridge triple junction between the Indian–Australian, African, and Antarctic plates. (*A*) Configuration and half-spreading rates. (*B*) Vector circuit diagram to determine the relative velocity of the Antarctic plate with respect to the African plate.

Coriolis
effect on Magma

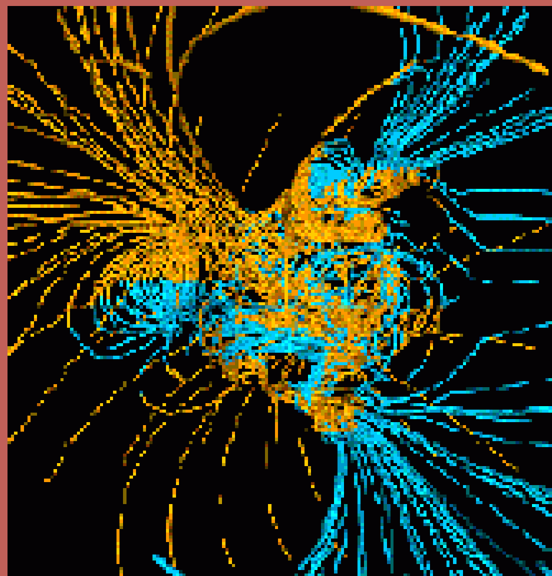


Orange is south
Blue is north

Before flip



During flip

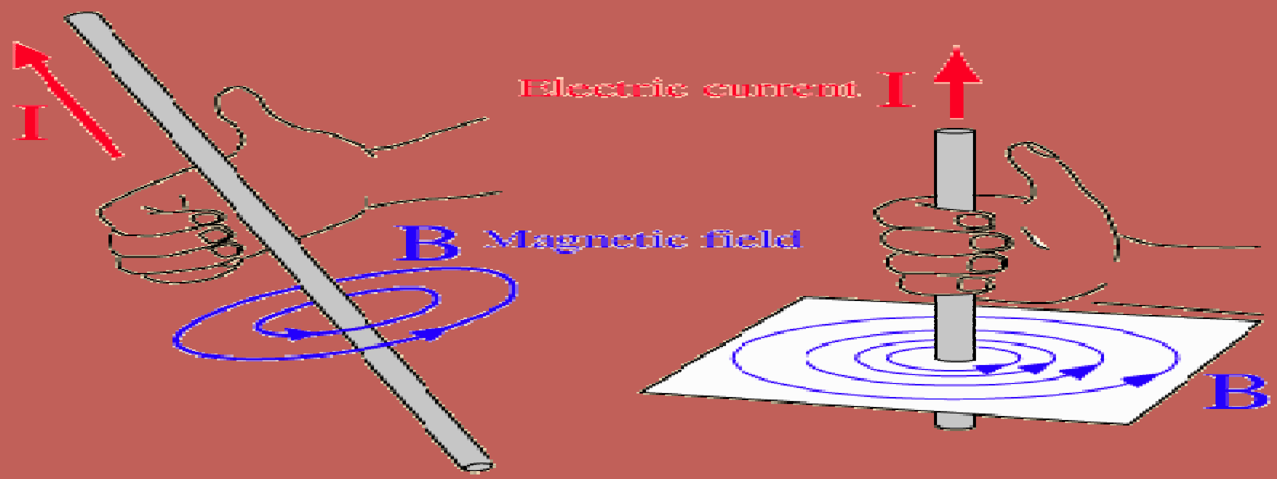


Electromagnetics, Wave theory, and Quantum mechanics

- The Laplacian

$$\nabla^2 \phi = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1 h_3}{h_2}\frac{\partial}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial}{\partial u_3}\right)\right]\phi,$$

- vector of the flow of electrons (which is negative to positive) which is opposite to the flow of current



Fourier Series and Its Transforms

- **The average value of sin(nx) over a period equals the average value of Cos(ns) over a period which equals one half**

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x')\,dx'$$
$$a_n = \frac{1}{L}\int_{-L}^{L} f(x')\cos\left(\frac{n\pi x'}{L}\right)dx'$$
$$b_n = \frac{1}{L}\int_{-L}^{L} f(x')\sin\left(\frac{n\pi x'}{L}\right)dx'.$$

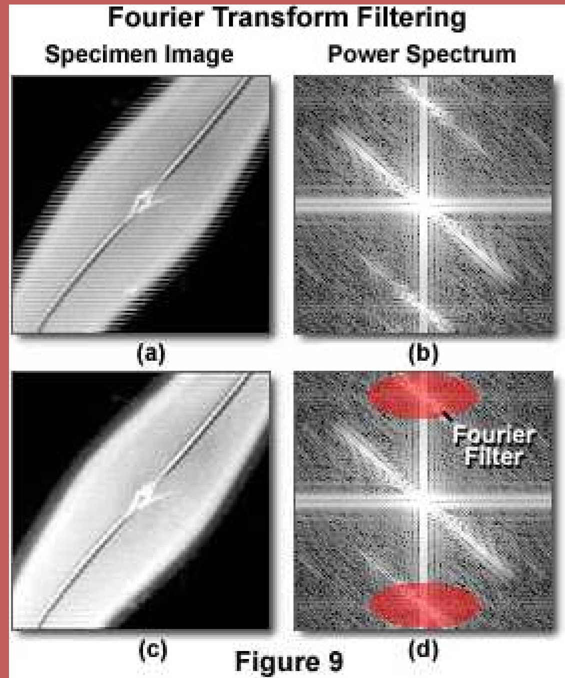
$$f(x) = \sum_{n=-\infty}^{\infty} A_n e^{inx}.$$

- **Fractional Fourier Transform**

$$F_n = \sum_{k=0}^{N-1} f_k\, e^{2\pi i b n k/N}.$$

Fractional Fourier Transforms

- Geophysics: 2 dimensional data of seismic motion :fan-filters
- Finding oil resources
- Image recovery
- Synthesizers



- Differential equations:
- $$f'(x) = \sum_{n=1} (-n a_n \sin nx + n b_n \cos nx)$$

Other **Fourier Transform**

- **Definition of Fourier Transform**

f(x) = 1/(2) g(t) e^(i tx) dt

- **Inverse Identity of Fourier Transform**

g(x) = 1/(2) f(t) e^(-i tx) dt

- **Fast Fourier Transform**

- **Discrete fourier transform**

- discrete <--> discrete

$$G_k = \sum_{j=0}^{N} g_j e^{-2\pi \nu_k i j / f}$$

The future of physics

- Satellites, Microwaves, Radio waves, and electrical devices
- Electromagnetic, Electric fields, Magnetic fields, currents of Flow

depends on fourier and vector techniques

Good sites

- http://mathworld.wolfram.com/
- http://science.nasa.gov/Realtime/JTrack/3d/JTrack3D.html
- http://science.nasa.gov/temp/StationLoc.html

Fast Fourier Transform

- The fast Fourier transform (FFT) is a discrete Fourier transform algorithm which reduces the number of computations needed for points from to , where lg is the base-2 logarithm. If the function to be transformed is not harmonically related to the sampling frequency, the response of an FFT looks like a sinc function (although the integrated power is still correct). Aliasing (leakage) can be reduced by apodization using a tapering function. However, aliasing reduction is at the expense of broadening the spectral response.

Common Fourier Transform

function	$f(x)$	$F(k) = \mathcal{F}_x[f(x)](k)$
Fourier transform--1	1	$\delta(k)$
Fourier transform--Cosine	$\cos(2\pi k_0 x)$	$\frac{1}{2}[\delta(k-k_0)+\delta(k+k_0)]$
Fourier transform--Delta function	$\delta(x-x_0)$	$e^{-2\pi i k x_0}$
Fourier transform--Exponential function	$e^{-2\pi k_0 \lvert x \rvert}$	$\frac{1}{\pi}\frac{k_0}{k^2+k_0^2}$
Fourier transform--Gaussian	$e^{-a x^2}$	$\sqrt{\frac{\pi}{a}}\, e^{-\pi^2 k^2/a}$
Fourier transform--Heaviside step function	$H(x)$	$\frac{1}{2}\left[\delta(k)-\frac{i}{\pi k}\right]$
Fourier transform--Inverse function	$-PV\frac{1}{\pi x}$	$i[1-2H(-k)]$
Fourier transform--Lorentzian function	$\frac{1}{\pi}\frac{\frac{1}{2}\Gamma}{(x-x_0)^2+\left(\frac{1}{2}\Gamma\right)^2}$	$e^{-2\pi i k x_0 - \Gamma\pi \lvert k \rvert}$
Fourier transform--Ramp function	$R(x)$	$\pi i \delta'(2\pi k)-\frac{1}{4\pi^2 k^2}$
Fourier transform--Sine	$\sin(2\pi k_0 x)$	$\frac{1}{2} i[\delta(k+k_0)-\delta(k-k_0)]$