LATIHAN SOAL NILAI AWAL DAN SYARAT BATAS

A. Tuliskan $P(x)$ dan $Q(x)$ dalam persamaan diferensial berikut, kemudian periksa di titik manakah kedua fungsi tersebut termasuk fungsi analitik.

1. $\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + (x^2+1)y = 0.$
2. $\dfrac{d^2y}{dx^2} + 2x^2\dfrac{dy}{dx} + (x+1)y = 0.$

3. $(x-5)\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + \dfrac{3}{x-2}y = 0.$
4. $(x+1)\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + \dfrac{1}{x-1}y = 0.$

5. $\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + (x^2+2)y = 0.$
6. $\dfrac{d^2y}{dx^2} + 3(x+2)\dfrac{dy}{dx} + (x+1)y = 0.$

7. $(2-x)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + \dfrac{1}{x+1}y = 0.$
8. $\dfrac{d^2y}{dx^2} + \dfrac{x}{x-2}\dfrac{dy}{dx} + \dfrac{1}{x(x-2)}y = 0.$

9. $\dfrac{d^2y}{dx^2} + \dfrac{2}{x+1}\dfrac{dy}{dx} + \dfrac{1}{x(x+1)}y = 0.$
10. $\dfrac{d^2y}{dx^2} - \dfrac{3x}{x-1}\dfrac{dy}{dx} + \dfrac{2}{x(x-1)}y = 0.$

B. Tentukan penyelesaian deret pangkat (*power series*) bagi persamaan diferensial dan nilai awal berikut

1. $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2-9)y = 0.$
2. $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2-4)y = 0.$
3. $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + 2y = 0.$

4. $\dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} - 2y = 0.$
5. $(x-1)\dfrac{d^2y}{dx^2} + (2x-1)\dfrac{dy}{dx} + 2xy = 0.$

6. $(x+1)\dfrac{d^2y}{dx^2} + (x+2)\dfrac{dy}{dx} + xy = 0.$
7. $(x+2)\dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} + 3y = 0.$

8. $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 8y = 0, \quad y(0) = -1,\ y'(0) = 2.$
9. $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 4y = 0, \quad y(0) = 3,\ y'(0) = 7.$

10. $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 0, \quad y(0) = 1,\ y'(0) = 0.$
11. $\dfrac{d^2y}{dx^2} - 7\dfrac{dy}{dx} + 10y = 0, \quad y(0) = -4,\ y'(0) = 2.$

C. Selidiki apakah **titik singular** pada tiap-tiap persamaan berikut termasuk **titik singular teratur**.

1. $(x^2-3x)\dfrac{d^2y}{dx^2} + (x+2)\dfrac{dy}{dx} + y = 0.$
2. $(x^2-3x)\dfrac{d^2y}{dx^2} + (x+3)\dfrac{dy}{dx} + 5y = 0.$

3. $(x^2-x)\dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} + y = 0.$
4. $(x^2-2x)\dfrac{d^2y}{dx^2} + (x-3)\dfrac{dy}{dx} + y = 0.$

D. Gunakan **metode Frobenius** untuk menentukan penyelesaian persamaan berikut di titik x=0.

1. $5x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2-2)y = 0.$
2. $3x^2\dfrac{d^2y}{dx^2} + (x+2)\dfrac{dy}{dx} + (x^2-1)y = 0.$

3. $5x^2\dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} + (x^2-2)y = 0.$
4. $2x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2-1)y = 0.$

E. Tentukan bayangan Laplace (Laplace transform) $\mathscr{L}\{f(t)\}$ dari fungsi-fungsi berikut:
1. $f(t) = t^2 - t^3 + 2t^5$
2. $f(t) = 3e^t + e^{-2t} + e^{3t}$
3. $f(t) = e^{2t} + te^{-t} - e^t\sin 5t$
4. $f(t) = 2t + \sin 2t + 2\cos 3t$
5. $f(t) = 2e^t + te^{3t} + e^{2t}\cos 2t$
6. $f(t) = e^{-t} + t^2 e^{3t} + e^{2t}\sin 3t$

F. Tentukan invers bayangan Laplace $\mathscr{L}^{-1}\{F(s)\}$ dari
1. $F(s) = \dfrac{5}{s-3} + \dfrac{2}{s^2+16} + \dfrac{5s}{s^2+4}$
2. $F(s) = \dfrac{7}{s} + \dfrac{2}{s^3} + \dfrac{4}{s^5}$
3. $F(s) = \dfrac{s-4}{s^2-16} + \dfrac{s+5}{s^2-s-6}$

4. $F(s) = \dfrac{s+3}{s^2+2s+5}$
5. $F(s) = \dfrac{s+7}{s^2+4s+13}$

G. Selesaikanlah nilai awal berikut dengan menggunakan Transformasi Laplace
1. $y'-4y = 0;\ y(0) = -1$
2. $y''+9y' = e^{-t};\ y(0) = 3,\ y'(0) = 0.$
3. $y''+3y'+2y = 0;\ y(0) = 0,\ y'(0) = -2.$