DETERMINANT

OLEH

ADE ROHAYATI


PERMUTASI:
- GENAP
- GANJIL
INVERSI/
PEMBALIKAN
DETERMINANT
SUATU MATRIKS
HASIL KALI
ELEMENTER
BERTANDA
HASIL KALI
ELEMENTER

DETERMINAN MATRIKS

SYARAT → MATRIKS BUJUR SANGKAR (jumlah baris = jumlah kolom)

NILAI DETERMINAN → SKALAR

NOTASI → det (A) atau |A|

det (A) = 0 → MATRIKS SINGULAR

Matriks berordo 2 x 2

Jika A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Maka, det (A) = |A| = $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Contoh :

A = $\begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$

Maka |A| = $\begin{vmatrix} 4 & 5 \\ 6 & 7 \end{vmatrix} =$ 4.7 - 6.5 = 28 - 30 = -2

Matriks berordo 3 x 3

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\text{Det }(A) = \left|\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\right|\begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

$$= aei + bfg + cdh - ceg - afh - bdi$$

Matriks berordo 3 x 3

$$A = \begin{pmatrix} 2 & 1 & 4 \\ 4 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix}$$

$$\text{Det (A)} = \left| \begin{array}{ccc|cc} 2 & 1 & 4 & 2 & 1 \\ 4 & 2 & 1 & 4 & 2 \\ 5 & 1 & 1 & 1 & 1 \end{array} \right.$$

$= 2.2.1 + 1.1.1 + 4.4.1 - 5.2.1 - 1.1.2 - 1.4.1$

$= 12 + 5 + 16 – 40 – 2 - 12$

$= - 21$

MATRIKS ADJOINT

Matriks Adjoint adalah transpose dari matriks kofaktornya.

Jika $matriks\ kofaktor\ A = (c_{ij})$

dengan $C_{ij} = (-1)^{i+j} . |M_{ij}|$ maka $Adj\,A = (C_{ji})$

Dimana : C_{ij} = Kofaktor dari elemen a_{ij}

M_{ij} = Minor dari elemen a_{ij}

Jika

$$matriks\ kofaktor\ A = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

maka $adj(A) = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$

Contoh :

Tentukan matriks adjoint dari

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

Jawab :

$$\boxed{C_{ij} = (-1)^{i+j} . \left|M_{ij}\right|}$$

$$C_{11} = (-1)^{1+1} . \left|M_{11}\right| = 1 . \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} = 5 - 4 = 1$$

$$C_{12} = (-1)^{1+2} . \left|M_{12}\right| = (-1) . \begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = 4 - 6 = -2$$

$$C_{13} = (-1)^{1+3} . \left|M_{13}\right| = 1 . \begin{vmatrix} 4 & 5 \\ 3 & 2 \end{vmatrix} = 8 - 15 = -7$$

$$C_{21} = (-1)^{2+1} . \left|M_{21}\right| = (-1) . \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1 - 6 = -5$$

$$C_{22} = (-1)^{2+2}.|M_{22}| = 1.\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 2 - 9 = -7$$

$$C_{23} = (-1)^{2+3}.|M_{23}| = (-1).\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1$$

$$C_{31} = (-1)^{3+1}.|M_{31}| = 1.\begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = 2 - 15 = -13$$

$$C_{32} = (-1)^{3+2}.|M_{32}| = (-1).\begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} = 4 - 12 = -8$$

$$C_{33} = (-1)^{3+3}.|M_{33}| = 1.\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} = 10 - 4 = 6$$

didapat

$C_{11} = 1$ $\quad C_{21} = -5$ $\quad C_{31} = -13$

$C_{12} = -2$ $\quad C_{22} = -7$ $\quad C_{32} = -8$

$C_{13} = -7$ $\quad C_{23} = 1$ $\quad C_{33} = 6$

$$matriks\ kofaktor\ A = \begin{bmatrix} 1 & -5 & -13 \\ -2 & -7 & -8 \\ -7 & 1 & 6 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

INVERS MATRIKS

NOTASI → A^{-1}

RUMUS UMUM → $A^{-1} = \frac{1}{|A|} \text{adjoint}(A)$

INVERS MATRIKS 2X2

misal $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

1. Tentukan matriks kofaktornya dengan rumus

$$\boxed{C_{ij} = (-1)^{i+j} . |M_{ij}|}$$

$$C_{11} = (-1)^{1+1} . |M_{11}| = 1.d = d$$

$$C_{12} = (-1)^{1+2} . |M_{12}| = (-1).c = -c$$

$$C_{21} = (-1)^{2+1}.|M_{21}| = (-1)\,b = -b$$

$$C_{22} = (-1)^{2+2}.|M_{22}| = 1.a = a$$

Matriks kofaktor $A = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$

Maka adj $A = \begin{bmatrix} d & -b \\ & \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|}\,\mathrm{adjoint}\,(A)$$

Jadi, $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Sifat:

$A.A^{-1} = A^{-1}.A = I$

$(AB)^{-1} = B^{-1}.A^{-1}$

contoh :

misalkan $A = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$

maka $A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A^{-1} = \frac{1}{4.3 - 2.5} \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix} = \frac{1}{2} . \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} & -1 \\ -\frac{5}{2} & 2 \end{bmatrix}$$

MATRIKS SINGULAR → **det (A) = 0**

↓

tidak punya invers / balikan

Mengapa?

Misalkan A matriks singular, maka det (A) = 0

$$A^{-1} = \frac{1}{|A|}(\text{adjoint}A) = \frac{1}{0}(\text{adjoint}A)$$

tidak terdefinisi

Maka, matriks singular tidak mempunyai invers

SEKIAN

WASSALAMUALAIKUM

Wr. Wb.

WASSALAMUALAIKUM

Wr. Wb.