GUIDED REINVENTION IN MATHEMATICAL MODELING

(A Case Study in Supervising Students of the Department of Mathematics Education, Indonesia University of Education)*

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A. Introduction

Realistic Mathematics Education (RME) is a theory of mathematics education that offers a pedagogical and didactical philosophy on learning and teaching mathematics (Treffers, 1987; Gravemeijer, 1994; Bakker, 2004). This theory emerged in The Netherlands in 1970s, and it has been developed and used there, also in other countries (De Lange, 1996), especially in Indonesia (Sembiring, 2008). One of the principles of RME is guided reinvention (Gravemeijer, 1994).

Guided reinvention should be used in learning and teaching mathematics based on RME approach. However, based on our observation in several mathematics classes—where the teachers implement RME approach, guided reinvention is still not clear-cut used. Therefore, in this poster, we will give an example how guided reinvention is implemented based on RME approach.

B. Realistic Mathematics Education

RME is shaped by Freudenthal's view on mathematics (Freudenthal, 1991), namely mathematics should always be meaningful to students and should be seen as a human activity. The term 'realistic' means that the problem situations should be 'experientially real' for student. This means the problem situations could be problems that can be encountered either in daily life or in abstract mathematical problems as long as the problems are meaningful for students.

There are five tenets of RME according to Treffers (1987) and Bakker (2004), which we summarize as follows:

a. *Phenomenological exploration or the use of meaningful contexts*. A rich and meaningful context or phenomenon, concrete or abstract, should be explored to support students in developing intuitive notions that can be the basis to build awareness.
b. *Using models and symbols for progressive mathematization.* A variety of context problems, models, schemas, diagrams, and symbols can support the development of progressive mathematization gradually from intuitive, informal, context-bound notions towards more formal mathematical concepts.
c. *Selfreliance: students' own constructions and strategies*. It is assumed that what students do in the learning processes is meaningful to them. Students are given the freedom to come up with their own construction and strategies in solving mathematical problems.
d. *Interactivity*. The learning process is part of an interactive instruction where individual work is combined with consulting fellow students, group discussion, class discussion, presentation of one's own strategies, evaluation of various strategies on various levels and explanation by the teacher. Hence, students can learn from each other either in groups or in whole-class discussion.
e. *Intertwinement*. It is important to consider an instructional sequence and its relation to other related topics.

C. Guided Reinvention

According to Treffers (1987), if students progressively mathematize their own mathematical activity, then they can reinvent mathematics under the guidance of the teacher or the instructional design. This is actually the guided reinvention as a RME principle, which states that students should experience the learning mathematics as a process similar to the process by which mathematics was invented (Gravemeijer, 1994; Bakker, 2004). An example, on how this principle is implemented, is described in the following section.

D. How Is Guided Reinvention Implemented?

In this section we present our past experience in implementing guided reinvention, when supervising students of the Department of Mathematics Education, Indonesia University of Education, in mathematical modeling based on RME approach. The goal of this supervision was to make students understand how models in mathematics are invented—where they will write undergraduate theses regarding mathematical modeling based on RME approach.

As a starting point of our discussion with the students, we used the context of our university floor, such as the following (See Figure 1).

Figure 1.

From the Figure 1, students were asked to observe and find out the pattern of the floor. From the observation, they found the following pattern (See Figure 2).

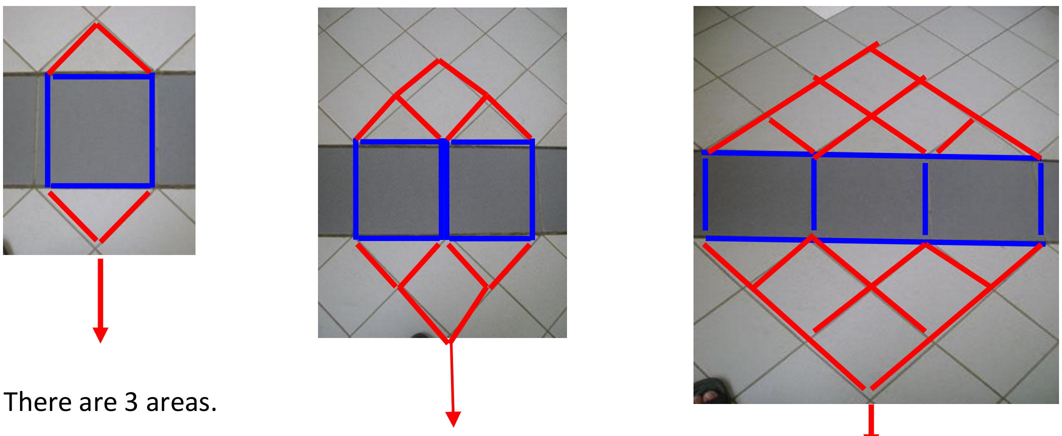

Figure 2.

Based on the pattern in Figure 2 above, students then predict that the pattern follows the following number sequence:

3, 8, 15, 24, ….

So, when they were asked how many areas of the fourth and fifth patterns, for examples, they predicted that they would be 24 and 35 respectively, because they found the following rule (Figure 3).

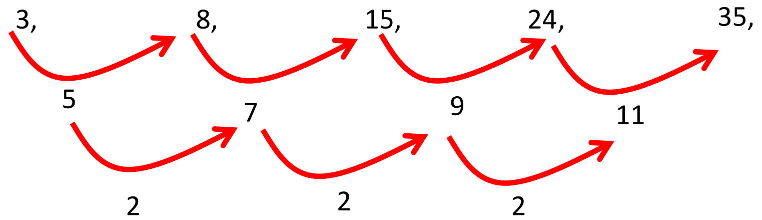

Figure 3.

Then, when students were asked to find, for example, how many number of areas for the 100th pattern, they found that it is not practical to find it out by continuing the rule in Figure 3. Therefore, after several moments, one of them made a conjecture that the formula to find a number of areas of the n^{th} pattern is $n^2 + 2n$. So, the number of areas for the 100th pattern is $100^2 + 2.\ 100 = 10200$. The following is the reason why such conjecture was made.

$$3 = 1.1 + 2 = 1^2 + 2$$

$$8 = 2.2 + 4 = 2^2 + 4$$

$$15 = 3.3 + 6 = 3^2 + 6$$

$$24 = 4.4 + 8 = 4^2 + 8$$

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$$An = n.\ n + 2n = n^2 + 2n.$$

Although, all students accepted this reason, it is actually not a mathematical proof. Hence, to guide students to find the mathematical proof, we gave a clue to them, namely to think about arithmetic sequence.

At first, we asked students to think the following arithmetic sequence:

$$5, \qquad 7, \qquad 9, \qquad 11, \ldots.$$

Thinking about this sequence, one of the students could directly find out the formula for n^{th} term, namely $Un = 2n + 3$ [They remember their secondary school mathematics knowledge]. Then, we asked again to think the sequence in Figure 3.

It was not easy for them to make an intertwinement between arithmetic sequence formula and a possible formula for the sequence in Figure 3. However, after we gave a clue by giving a question: what kind of function is the arithmetic sequence formula, and they knew that it is a linear function (because there is a stage between numbers), they realize that the sequence in Figure 3 has a possible formula which follow a quadratic function (because there are two stages between numbers) . So, they wrote the possible formula as $An = an^2 + bn + c$. One of the students solved this problem as follows.

For $n = 1$, we have $a + b + c = 3$

For $n = 2$, we have $4a + 2b + c = 8$

For $n = 3$, we have $9a + 3b + c = 15$.

Solving this system of equation, the student found $a = 1$, $b = 2$ and $c = 0$. Therefore, $An = n^2 + 2n$. This formula is the same as what they had predicted before. And, they finally understand that this formula is the model for the number of areas of the floor in our university. Further, they realized that this kind of learning-teaching situation really makes-sense for them.

E. Conclusion

We do hope that the example, on how guided reinvention is implemented above, can be useful for teachers who implement learning teaching mathematics at schools based on RME approach.

F. References

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