ELEMENTARY PUPILS' STRATEGIES OF MENTAL COMPUTATION IN INDONESIA AND AUSTRALIA

Tatang Herman [*]

Abstract. This study investigated the strategies of mental arithmetic used by pupils in performing mental computation tasks. Forty-one fifth grade pupil in one of Victorian (Melbourne Australia) elementary school and sixty-three fifth grade pupils in one elementary school in Bandung (Indonesia) participated in this study. All of the pupils were tested in a written mental computation test. The test contains 40 items for additions, subtractions, multiplications, and divisions. Twelve children were chosen, based the test result, as subjects of the research. They were classified as 4 high level, 4 middle level, and 4 low level pupils in mental computation. The subjects were individually interviewed concerning their strategies in mental computation. This study found that there were some similar strategies used by pupils in Indonesia and Australia. Elementary pupils in Australia used variety ways and more efficient strategies in performing mental computation, whereas Indonesian pupils' strategies in mental arithmetic were heavily influenced by written algorithm which is still predominate in the elementary mathematics classes.
Keyword: mental computation, mental arithmetic, thinking strategies.

1. INTRODUCTION

One major goal of elementary school mathematics in the most of counties around the world is to help children to develop number sense. Number sense can be described as a good intuition about number and their relationships, which develop gradually as a result of exploring numbers (Howden, 1989). Mentioned in the *Principle and Standard for School Mathematics* that number sense is the ability to decompose number naturally, use particular number like 100 or ½ as referents, understand the base-ten number system, use the relationships among arithmetic operations to solve problems, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers (National Council of Teachers of Mathematics, 2000).

Many studies found that the development of number sense is not something that occurs naturally for most children during regular instructions, but can be developed with appropriate activities (Hope, 1989; Markovits, Hershkowits, & Brukheimer, 1989; Thomson, 1989; McIntosh, Reys, & Reys, 1992). It is demonstrated by of

[*] *Department of Mathematics Education, Faculty of Mathematics and Science Education, Indonesia University of Education*

some of these researchers that the most appropriate activities that encourage children to make sense of numbers involve practicing mental computation activities. Sowder (1990) argues that “gaining skill at mental computation through reflection on what the numbers to be computed really mean is one of the best ways of developing number sense.” (p.18).

In Indonesia, three alternatives for completing computation — using written algorithms; using computational aids such as abacus, calculators or computers; and computing mentally — are a central issue in school mathematics programs in recent years. Written algorithms, often called paper-and-pencil procedures, have long dominated the elementary school curriculum. In the information boom, however, paper-and-pencil procedures are much less important than in the previous times. Calculators and computers can take over the computational tasks for almost all kinds of occupations (Usnick, Lamphere, & Bright, 1995), because of their accuracy and efficiency.

The emergence of calculators and computers has change the role of computational skills in society. In everyday computational situations, such as shopping in supermarkets or shops, measuring things, and balancing account in banks, the calculation process is increasingly being done by calculators or computers. In the view of the decreasing use of paper-and-pencil computation in society, the low ability of pupils in mental computation has been reported by researchers. For example, Hope & Sherrill (1987) reported that 45 percent of the 17-year-olds sampled by the third mathematics national assessment in the United States (NAEP 1983) were unable to multiply 90 x 70 ‘in the head’. In NAEP 1985-86, Kouba, Carpenter, & Swafford (1989) found that more than fifty percent of grade 7 students couldn’t complete 4.3 – 0.53 correctly. Reys, Reys, & Hope (1993) supported this by claiming that 72 percent pupils in the second grades couldn’t answer ’42 plus 30’ correctly, and 99 percent of pupils in the fifth grades answered 75 +85 + 25 + 2000 incorrectly. Somerset (1996) did a diagnostic survey of basic number skills in Indonesia junior high school which was found that the easiest item (23 + 9 + 168) was answered correctly by nearly 90 percent of pupils and the most difficult (47.1 – 0.65), by about 33 percent.

Even though the importance of the number sense has been placed as one of the curriculum goals in the elementary schools around the world (e.g. National Council of Teachers of Mathematics, 2000; Australian Educational Council, 1991; Shuard, et. al., 1991; Puskur (2001); Nohda, Shimizu, and Tsubota, 2000), the information related to this area has been very limited (Herman, 1996). Because of this limitation and also the similarities of some curriculum goals, do elementary pupils in Indonesia possess similar number sense performance with those of pupils in developed countries? Through this research which was done in Indonesia and Australia, the question would be answered.

2. SOME RELEVANT STUDIES

There is strong evident that children perform calculation in various ways (Thornton et. al., 1995). Even when they begin to enter elementary school, original and creative strategies for operations usually appear (Cooper et. al., 1993). In particular, children have been able to invent cognitive strategies and these strategies have been closer to mental rather than to paper-and-pencil procedures (Bebout, 1990). In conclusion, children can posses their ability to perform calculation because of their creativity and invention of their thinking.

Referring back to many findings of studies about children's skills in mental computation such as Bebout (1990), Carpenter and Mose (1984), Coper et. al., (1993), and Thornton et. al., (1995), it is argued that children invent various strategies in performing mental mental computation based on their own experience with numbers. Children use such strategies due to their familiarity with number and Operation properties, instead of replicating what teachers taught. Strong support is claimed by Carraher (1988), and Pettito and Ginsburg (1981) that children's informal experience which is got outside school often produces complex efficient strategies in computing mentally.

Efficiency in calculating mentally are recognized as an important aspect in determining mental computation skills. Efficiency here is concerned with working economically or quickly without wasting time. This is a consequence of the strategies which are used in performing such problems (e.g. Reys, 1986; Hope, 1987; and Thornton et. al., 1995).

Hope and Sherrill (1987) also identified fifteen skilled and fifteen unskilled mental calculator students in grade 11 and 12 through a screening test of mental computation abilities. From interviewing the students, they argued that unskilled students used strategies more suited to written algorithms than mental computation, and skilled students used strategies based upon the number properties suggested by the factor in the task (e.g. distribution, factoring, and retrieval). The ability to change numbers to a simpler form to calculate instantly really helped the skilled students in computing mentally.

Hope (1987) found that the highly skilled mental calculator girl could calculate rapidly and correctly because of her ability to transform numbers into one or more factors and apply them in more head-manageable strategies. Proficient mental calculators use efficient strategies and produce acurate answers because of their familiarity with number and operation properties. They use a variety of non-standard strategies in performing mental computation by using simplication of the numbers and ordering them for appropriate operation to compute with their heads. According to Herman (1996), understanding how to use numbers and operations in various formats is an important key to be able to choose sophisticated strategies in solving computational problem mentally.

3. RESEARCH DESIGN

Methods of research in this study include both qualitative and quantitative approaches. In the qualitative aspect, data was collected which measured pupils' abilities in mental computation by using a mental computation screening test. By this kind of methods, the study was expected to gather complete information for describing pupils' performance in number sense.

Subject of the Study

The participants in the mental computation test of the study were forty one pupils from two classes of grade fifth pupils in a Victorian government elementary school in the eastern area of suburban Melbourne, Australia and sixty three pupils from two classes of grade fifth pupils in government elementary school in Bandung, Indonesia. Twelve pupils (six from each school) were chosen, based the test result, as subjects of the research.

Instruments

Two instruments were designed to be used with pupils in fifth grade: a mental computation test, and an interview. The mental computation test was constructed in order to identify pupils' abilities in mental computation, because there were no existing data describing such abilities which was 'useful'. The interview was used to gain deeper insight into the kinds of strategies pupils used in mental computation and factors that possibly effected strategies.

a. Mental Computation Test

The test was designed for group administration. It contained 40 items for the four operations (addition, subtraction, multiplication, and division), including whole numbers, fractions, decimals, and percentages. The items were mostly on multiplication tasks for each type of number, because it is expected that multiplication can invite a greater variety of invented strategies than the other operations. The test items were chosen to represent he scope of topics listed in the current mathematics curriculum for fifth grade pupils for both countries, Indonesia and Australia. By doing this, it was intended that the content of the test would represent the computational outcomes prescribed in the documents. Therefore, it was saved to assume that the test would measure what pupils should be able to do from the point of view of the expected curriculum. The test included only straight computational items and it was possible to answer each question using a variety of common mental methods. This was expected to encourage pupils to perform the items mentally rather than using other methods. There was also no item in story context, so that pupils could focus exclusively on the required computation avoiding any decision related to choice of operations.

b. Interview

A structured interview was provided in the study. The interview concerned with pupils' strategies in mental computation. Fifteen items of interview were directly drawn from the mental computation test. The items represented all the four operations, including whole numbers, fractions, decimals, and percent. Each interview item was followed by seven questions about pupils' thinking strategies in performing that item.

Collecting Data and Analysis Procedures

The first step in collecting data was to conduct the mental computation test. Each item was presented visually through use of an overhead projector. The presenter read aloud each item's number but did not read the test item. This was done because there are many ways in mathematics to read number sentences that can affect pupils' interpretation of operation (Anghileri and Jonhson, 1988). All items of the test were given one at a time and the time allowed for each item was strictly controlled through a stopwatch. The time allowed for each item ranged from 13 to 18 seconds depending on its level of difficulty. Pupils were asked to answer each item in a specially constructed answer sheet that provided only a space for the written answer for each item, thereby discouraging the pupils to write anything else on the paper.

Pupils scores of mental computation test were ranked in descending order and stored into three groups of mental computation abilities: high, middle, and low level abilities. Twelve pupils were chosen as subjects of the study: six pupils from fifth grade from one of Victorian, (Melbourne) Australia elementary school and six fifth grade pupils in one elementary school in Bandung, Indonesia. They were classified as 4 high level, 4 middle level, and 4 low level pupils in mental computation.

The subjects were individually interviewed concerning their strategies in mental computation. The interview of each subject was conducted by the researcher in a vacant room during regular school days. All the interviews were recorded on tapes and summarized in document scripts analysis. All of the interviews which had been tape-recorded were transcribed and categorized in such a way that was appropriate for interpretation.

4. RESEARCH FINDINGS AND DISCUSSION

In order to assist analysis and discussion, the variety of pupils' procedures in computing mentally is differentiated into two group of strategies: the first on addition and subtraction, and the second on multiplication and division. Pupils' strategies have been summarized into categories for each group.

Elementary Pupils' Strategies in Indonesia

a. Strategies on Addition and Subtraction

For the addition and subtraction items which were asked in the interview, there were found two strategies and categorized as *mental image of paper-and-pencil* and *grouping by hundreds, tens, ones, etc.* The methods of *mental image of paper-and-pencil* were like paper-and-pencil algorithms but were done mentally without paper and pencil. The process of calculation was usually performed digit by digit from right to left. The transcripts presented in Extract 1 illustrates the *mental image of paper-and-pencil* strategy.

Extract 1

Researcher : Can you work out in your head and tell me the answer for twenty three plus ninety nine?
Weni : One hundred….., twenty….., two.
Researcher : Are you sure that your answer is correct?
Weni : I think so, one hundred twenty two.
Researcher : How would you explain to a friend what you did?

Weni : *Well, three plus nine is twelve, we write two and put one. And two plus nine, eleven then plus one become twelve, Twelve means one hundred and twenty two.*

This strategy was used by all subjects for almost all the items asked. Of the all addition and subtraction items asked to the subjects 91.6% categorized as the *mental image of paper-and-pencil* strategy.

The *grouping by hundreds, tens, ones, etc.* strategy was used by separating the numbers into part-whole relationships such as hundreds, tens, ones, or decimals/factions. The calculation was done by adding or subtracting the relevant part-whole relationships from left to right or from right to left, and then joining the parts again at the end of the process. One example of the strategy is illustrated in Extract 2 below.

Extract 2

Researcher : *Can you tell me the answer for five point three plus two point eight?*

Agus : *Eight point one* (after 10')

Researcher : *Tell me again exactly what you did!*

Agus : *Yes, three plus eight, one point one, and then added to five and two which is eight point one.*

This strategy was used only one subject from the high level pupil of mental computation to the several items asked. Of the all strategies explained, the strategy of *grouping by hundreds, tens, ones, etc.* was used only 6.3%. Not all of the pupil's answers of this strategy were correct, even though they could mention their ways of thinking. There was only one addition and subtraction item that couldn't be explained by pupil of the low level in mental computation. The number and percentage of categorized strategies on addition and subtraction were shown in the Table 1 below.

Table 1

The Number and Percentage of Categorized Strategies on Addition and Subtraction

No.	Categorized Strategies	Total	Percentage
1	Mental image of paper and pencil	44	91.6%
2	Grouping by hundreds, tens, ones, etc.	3	6.3%
3	Couldn't explain	1	2.1%

b. Strategies on Multiplication and Division

To the multiplication and division items were found some strategies which were grouped into 4 category of strategies*: mental image of paper and pencil*, *partial results, double or half*, and *using basic operations fact*. The methods of written algorithms again became the most strategies used by pupils for the multiplication and division items (66,7%).

The *partial results* strategy used only by small number and mostly by high level pupil in mental computation (14,3%). Basically, this strategy transformed one or more of the factor into a series of sums (essentially using distributive law), differences, products, or quotients. For example, to solve 7 x 25 was done by thinking 7 x (20 + 5) = (7 x 20) + (7 x 5) = 140 + 35 = 175. This strategy was explained by pupils as illustrated in the following excerpts.

Extract 3

Researcher	:	*Can you tell me the answer for forty nine times eight?*
Iwan	:	*Three hundred ninety….,six. Sorry, three hundred ninety two.*
Researcher	:	*Are you sure that your answer is correct?*
Iwan	:	*Sure, three hundred ninety two.*
Researcher	:	*Could you think aloud as you went along?*
Iwan	:	*Well, I did forty times eight plus nine times eight.*
Researcher	:	*Is there another way to do that?*
Iwan	:	*Maybe yes.*
Researcher	:	*Can you tell me that another way?*
Iwan	:	*I don't know.*

The strategy of *double or half* was only used to the item contains percent, e.g. 50% of 48. This method was preferred only by high level pupil in mental computation or 2.4% of the all strategies mentioned. While *using basic operations fa*cts was mentioned by two pupils or about 9.5% of the all discovered strategies. The process of calculation for this strategy was done by simplifying the item by ignoring the zero for a while, or by computing the numbers one-by-one digit considering their place value. The calculation was generated by one or two or more combination of operations using basic number facts.

The number and percentage of categorized strategies on multiplication and division were shown in the Table 2 below.

Table 2

The Number and Percentage of Categorized Strategies on Multiplication and Division

No.	Categorized Strategies	Total	Percentage
1	Mental image of paper and pencil	28	66.7%
2	Partial results	6	14.3%
3	Double or half	1	2.4%
4	Using basic operation facts	4	9.5%
3	Couldn't explain	3	7.1%

Elementary Pupils' Strategies in Australia

a. Strategies on Addition and Subtraction

There were seven strategies of addition and subtraction used by Australian elementary pupils. The strategies were categorized as *grouping by hundreds, tens, ones, etc*.; *rounding and adjust*; *decomposition*; *hold one addend constant*; *retrieval of numbers*; *change to other equivalent numbers*; and *mental image of paper and pencil*. To the all strategies used by Australian elementary pupils, there were two strategies which were also used by Indonesian elementary pupils, i.e. *mental image of paper and pencil* and *grouping by hundreds, tens, ones, etc*. Only low level pupils of mental computation used the strategy of *mental image of paper and pencil* and than was only 2.1% from the all discovered strategies in the Australian elementary school, whereas the strategy of *grouping by hundreds, tens, ones, etc*. reached 33.3%.

The strategies used by the Australian pupils but didn't use by the Indonesian pupils were mentioned as follows.

Rounding and adjust. The strategy was initiated by firstly rounding one or more addends or subtrahends into a multiple of tens, doing the operation and then adjusting the numbers in the last step. This strategy was mentioned only by high level pupils in mental computation and reached 29.2% of the all strategies explained by pupils. The following Extract 4 illustrates the kind of strategy.

Extract 4

Researcher : *Can you tell me the answer for twenty three plus ninety nine?*
Erica : *One hundred and twenty two* (in 5 seconds)
Researcher : *Are you sure that your answer is correct?*
Erica : *Sure!* (very confident).

Researcher : *Can you tell me how you did that?*
Erica : *Well, I rounded ninety nine to a hundred, and then plus twenty three and took away one.*

Decomposition. This thinking strategy was quite efficient by breaking up one of the addends, minuend, or subtrahend and then arranging the operation in several steps. The pupils really expressed the number in simpler components as constituent elements. For example, to solve 140 – 61 was done by (140 – 40) – 21 = 100 – 21 = 79. This way is mentioned by the following excerpts.

Extract 5

Researcher : *Can you work out in your head and tell me the answer for one hundred twenty six plus one hundred ninety nine?*
Kim : *Three hundred and twenty five* (in 13 seconds)
Researcher : *Tell me again exactly what you did!*
Kim : *Well, one hundred twenty six is one hundred twenty five plus one and then plus one hundred ninety nine that is three hundred and twenty five.*

Hold one addend constant. This strategy was done by holding one of the addends or subtrahends fixed or unchanging. The procedure tries to simplify calculation procedure by decomposing the other numbers and the process of operation was usually done more than once. For instant, to solve 293 – 99 was done by thinking 293 – 90 = 203 – 9 = 194.

Retrieval of numbers. The process of finding an answer was done by recalling directly from memory without using any calculating procedure. Several answers were retrieved from their sense of number rather than calculating. For instance, one pupil answered for $4\frac{1}{2} - 2\frac{1}{4}$ was "two and a quarter" quickly, because the pupil had already known that $2\frac{1}{4}$ was half of $4\frac{1}{2}$. Pupil's answers for such strategy were usually accompanied by the phrases such as 'I know that…', 'just added together, that's easy'.

Change to other equivalent numbers. Both the addends or minuend and subtrahend were substituted by the other equivalent number in order to make computation process simpler to do mentally. To alter the numbers that should be calculated into certain equivalent numbers was used by a certain pupil as calculative aid. The strategy was only found from the pupil who had categorized as high level ability on mental computation.

The number and percentage of categorized strategies on addition and subtraction were shown in the Table 3 below.

Table 3

The Number and Percentage of Categorized Strategies on Addition and Subtraction

No.	Categorized Strategies	Total	Percentage
1	Grouping by hundreds, tens, ones, etc.	16	33.3%
2	Rounding and adjust	14	29.2%
3	Decomposition	10	20.8%
4	Hold one addend constant	3	6.2%
5	Retrieval of numbers	2	4.2%
6	Change to other equivalent numbers	1	2.1%
7	Mental image of paper and pencil	1	2.1%
8	Couldn't explain	1	2.1%

b. Strategies on Multiplication and Division

Regarding multiplication and division tasks faced to Australian pupils, there were found nine strategies to compute mentally. The strategies were *partial results*, *rounding factors and adjust*, *using basic operation facts*, *double or half*, *changing division to multiplication*, *change to another equivalent numbers*, *repeated addition or subtraction*, *skip counting*, and *mental image of paper and pencil*. To the all strategies of multiplication and division used by Australian elementary pupils, there were four strategies which were also used by Indonesian elementary pupils, i.e. *partial results, using basic operation facts, double or half,* and *mental image of paper and pencil*.

The strategies of multiplication and division used by the Australian pupils but didn't use by the Indonesian pupils were mentioned as follows.

Rounding factors and adjust. This strategy was done by one of the factors was rounded up (or down) to the multiple of two, five, or ten. The calculation was, then, adjusted by subtraction or addition related to the rounding judgment. For example, to calculate 49 x 8 was done by (50 – 1) x 8 = (50 x 8) – (1 x 8) = 400 – 8 = 392. The following Extract 6 pictures the strategy.

Extract 6

Researcher : *Can you tell me the answer for seven times twenty five?*
Andrew : *One hundred and seventy five* (in 6 seconds)
Researcher : *Could you think aloud as you went along?*
Andrew : *Well, I did eight times twenty five took away twenty five*

Changing division to multiplication. Some pupils said that division was more difficult than multiplication. Because of that they change division task to the multiplication form, using principle that 'divisor times quotient is dividend'. The following is the excerpts.

Extract 7

Researcher	:	*Can you tell me the answer for three hundred divided by five?*
Andrew	:	*Sixty* (in 14 seconds)
Researcher	:	*Tell me again exactly what you did!*
Andrew	:	*Yes, we know that three hundred is produced of five, ten, and six. So, three hundred divided by five is ten times six, sixty.*

Change to another equivalent numbers. Both the factors or dividend and divisor were altered by the other equivalent number in order to make the calculation simpler to do mentally. As with addition and subtraction, this strategy was also only used by the same pupil who was high level ability in mental computation. For instance, to compute 28 x 50, pupil would change to 14 x 100 which was simpler to do in the head.

Repeated addition or subtraction. The strategy was only applicable to multiplication tasks. The calculation was done by adding repeatedly the quantity (one factor) as many times as the other factor. For example, to solve 7 x 25 was done by 25 + 25 = 50; 50 + 50 = 100; 100 + 50 = 150; 150 + 25 = 175. This kind of strategy was only used by low level ability of mental computation.

Skip counting. The stategy involves counting quickly and jumping from one number to another by fixed intervels. The process of calculation was similar to the repeated addition strategy but in this method pupils generally used their fingers to facilitate computing mentally. The transcript below illustrates the strategy.

Extract 8

Researcher	:	*Can you work out in your head and tell me the answer for three hundred divided by five?*
Andrew	:	*Sixty* (in 20 seconds)
Researcher	:	*Can you tell me again exactly what you did!*
Andrew	:	*Yeap. Five, and I ..., five, ten, fifteen, twenty, twenty five, thirty, thirty five, forty, forty five, fifty (while count and fold his fingers), that made ten. I made that twice which was a hundred, then I did three times that three hundred. I got sixty frm ten, twenty, and sixty.*

The number and percentage of categorized strategies on multiplication and division were shown in the Table 4 below.

Table 4

The Number and Percentage of Categorized Strategies on Multiplication and Division

No.	Categorized Strategies	Total	Percentage
1	Partial results	12	28.6%
2	Rounding factors and adjust	8	19.1%
3	Using basic operation facts	4	9.5%
4	Double or half	3	7.1%
5	Changing division to multiplication	3	7.1%
6	Change to another equivalent numbers	6	14.3%
7	Repeated addition ar subtraction	1	2.4%
8	Skip counting	1	2.4%
9	Mental image of paper and pencil	2	4.7%
10	Couldn’t explain	2	4.7%

c. Pupils’ Strategies and Their Number Sense Performance

The findings of the study show that there were many strategies of mental computation used by elementary pupils. Totally there were 22 strategies used by elementary pupils in Indonesia and Australia. In the one hand, only six strategies used by Indonesian pupils consisted two strategies for addition and subtraction, and four strategies for multiplication and division. In the other hand, Australian pupils used 16 variety of strategies, consisted 7 strategies for addition and subtraction, and 9 strategies for multiplication and division.

Needless to say that elementary pupils in Indonesia generally used the strategy of mental image of paper and pencil (91.7%) for addition and subtraction tasks and (66.7%) for multiplication and division. This would heavily be influenced by written algorithms which is still predominate in the elementary mathematics classes in Indonesia. However, in Australia this kind of strategy was only used by small number of pupils (2.1%) for addition and subtraction tasks, and (4.7%) for multiplication and division tasks. The most interesting thing is that high level pupils in mental computation ability in Indonesia prefered to compute using this kind of strategy which was used by small numbers of elementary pupils in Australia and belong to low level ability of mental calculation. This means that it is not sophisticated to compute 126 + 199 mentally by using paper-and-pencil procedure rather than to think that 126 + 199 = 125 + 200.

A strategy may be said to be efficient if it means the pupil can reform a number to one which would be easier to compute with mentally. Efficient pupils were capable of changing a number to another form rapidly after the question was asked, so that the number appears in new form that is manageable to compute with quickly in their heads. The tricks they used to reform numbers into more manageable ones included rounding, distribution, or factoring, avoiding carrying problems and avoiding using long (repeated) operations so the task is completing using digit by digit operations (Herman, 1996). These types of strategies were only used by the highly skilled pupils in mental computation.

5. CONCLUSION

The findings of this study show that knowledge of number properties and knowledge of numerical operations are very important for solving mental computation tasks. This knowledge has been used by pupils in various mental computation strategies. Pupils' strategies in performing the tasks reflect their understanding and familiarity with number as well as their understanding and familiarity with operations.The high pupils performance of number sense generally used efficient strategies by showing facility in taking advantages of number and operations.

This research found that elementary pupils in Indonesia used less efficient strategies than those of elementary pupils in Australia in performing mental computation tasks. With regard to these findings and some investigations which were done by Hope (1989), Sowder (1991), and Herman (1996), it is implied that elementary pupils in Australia possesed better number sense performance rather than that of elementary pupils in Indonesia.

References

Anghileri, J., and Jonhson, D. C. (1988). Arithmetic operation on whole number: Multiplication and division. In T. R. Post (Ed.) *Teaching Mathematics in Grades K-8* (pp.148-187) Massachusetts: Allyn and Bacon.

Bebout, H. (1990). Children symbolic representations of addition and subtraction word problem. Carpenter, P. T., and Moser, J. M. (1984) The acquisition of addition and subtraction concepts grades ones through three. *Journal for Research in Mathematics Education*, 21(2), 123-131.

Carpenter, P. T., and Moser, J. M. (1984) The acquisition of addition and subtraction concepts grades ones through three. *Journal for Research in Mathematics Education*, 18(4), 11-28.

Carraher, T. P. (1988). Street mathematics and school mathematics. Preceeding 12th international conference: *Psychology of Mathematics Education*. Hungary, 20-25 July.

Cooper, T., Heirdfield, A., and Irons, C. (1993). Mental computation strategies for addition and subtraction algorithms. In B. Atweh (Eds.), *Contexts in Mathematics Education*. Proceeding of sixtennth annual conference of the Mathematics Education Research of Australia (MERGA). Brisbane, July, 9-13.

Herman, T. (1996). *Pupils' strategies in mental computation*. Unpublished master thesis. Melbourne: Deakin University.

Hope, J. A. (1989). Promoting number sense in school. *Arithmetic Teacher*, 36(6), 12-16.

Hope, J.A., and Sherrill, J.M. (1987). Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education*, 18(2), 98-111.

Howden, H. (1989). Teaching number sense. *Arithmetic Teacher,* 36(6), 6-11.

Kouba, V.L, Carpenter,T. M., and Swafford, J.O. (1989). Number and operations. In M.M. Lindquist (Ed.), *Result fron the Fourth Mathematics Assessment of Educational Progress* (pp.64-93). Reston, VA: National Council of Teachers of Mathematics.

Markovits, Z., Hershkowits, R., and Brukheimer. (1989). Research into practice: Number sense and nonsense. *Arithmetic Teacher*, 36(6), 53-55.

McIntosh, A., Reys, B.J., and Reys, R.E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, 3, 2-14.

National Council of Teachers of Mathematics. (2000). *Principles and Standard for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Nohda, N., Shimizu, S., & Tsubota, K. (2000). *School Mathematics in Japan*. Makuhari: Japan Society of Mathematical Education Research Section.

Pettito, A. L., and Ginsburg, H. P. (1981). Mental arithmetic in Africa and America: Strategies, principles, and explanation. *International Journal of Psychology*, 17, 81-102.

Reys, B.J. (1986). Estimation and mental computation: It's "about" time. *Arithmetic Teacher*, 34(1), 22-23.

Reys, B.J., Reys, R. E., and Hope, J. A. (1993). Mentan computation: A snapshot of second, fifth, and seventh grade student performance. *School Science and Mathematics*, 93(6), 306-315.

Somerset, A. (1996). *JSE Mathematics: Diagnostic survey of basic number skills*. Jakarta: Ministry of Education and Culture.

Sowder, J. (1990). Mental computation and number sense. *Arithmetic Teacher*, 37(7), 18-20.

Sowder, J. (1991). *Teaching computational in ways that promote number sense*. Paper presented at a conference of challenging children to think when they compute: Dev eloping number sense, mental computation, and estimation. Queensland University of Technology, Brisbane, 9-11 August.

Thomson, C. S. (1989). Number sense and numeration in grade K-8. *Arithmetic Teacher*, 37(1), 22-24.

Thornton, C. A., Jones, G. A., Neal, J. L. (1995). The 100s chart: A stepping stone to mental mathematics. *Teaching Children Mathematics*, 1(8), 480-483.

Usnick, V.E., Lamphere, P. and Bright, G.W. (1995). Calculators in elementary school mathematics instruction. *Scool Science and Mathematics*, 95(1), 11-18.