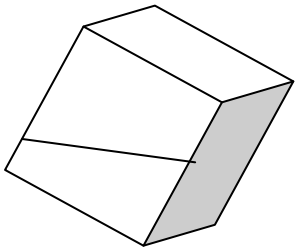
THE LESSON PLAN

Purpose of the Task and Preparation

1. The Task : water-flask problem
2. Subjects : Thirty-nine sixth-grade students in Fukuzawa Elementary School in Setagaya Ward- Tokyo.
3. Purpose : To have student discover various relations implicit in the problem situation where flask containing water is tilted; furthermore, if possible, to have students formulate the relation they discover into mathematical expressions and to explain logically the formulated relations.
4. Instruments and materials :
    a. Ten water flask
    b. Ten beaker for pouring water
    c. A blank sheet of paper for each student
    d. Ten blank transparency sheets for presentations by groups on the overhead projector (Note : we used a science laboratory because it was more convenient than an ordinary classroom for the use of beakers, flask, and water and for discussion)

Sequence of Presentation and Allocation of Time

The teacher used two forty-minute periods for the lesson. Two periods were used because other group members had found from their experience that more than one period was necessary. In most instances, one period allows insufficient time for discussion and a summary of all the students' finding, many of which are very interesting. Two periods, however, provided enough time for students explore the problem situation and discuss most, if not all, of their findings.

The First Period			
Teacher's Presentation and Direction	**Student's Activities**	**Remarks**	**Cummulative Time in Minutes**
1. When we tilt the flask with water in it while fixing one edge of the base on the table, we see that shape and size of various parts are changing. Find out as many relations among the parts as possible, and write them down	1. Understanding the question	1. Explain the problem by using a real flask with water 2. Use the figure as a poster to make sure the students undertand the problem 	**5**
2. Write down what you have noticed on the blank worksheet.	2. Trying to find out various rules (individual work)	3. Distribute sheets to each student. 4. Collect sheets on which students have written their findings	25
3. Within each group, discuss what you have found. The leader of each group should record group's observations.	3. Discussing within groups, and discovering various rules (group work)	5. Distribute new sheets to each group	40
The Second Period			
1. Please present the results of your group discussion	1. Groups take turns presenting their results.	1. List every response from the groups on a poster.	20
2. Let's group together similar findings	2. Rules are grouped from various viewpoints	2. Have students group findings carefully so as not to duplicate or omit any.	30

3. We know the rule that *a* + *b* is constant, where *a* and *b* are the lengths of the sides shown in figure. Can we explain the figure rule?	3. Students methodically consider why the property of the sum's being constant is true.	3. Assign *a* and *b* to the bases of the shaded trapezoid as in figure	40
4. The teacher gives the reason, if necessary.	4. Students listen to the explanation		
5. We can put in order other rules from different points of view.	5. Students summarize their finding		

Summary of Individual and Group Observations of Thirty-nine Students

Category Of Observations (findings)	Number of Rules	Students' Observations (Rules)	Group Number										Number of Students Making Observations
			1	2	3	4	5	6	7	8	9	10	
Constant Sum	1	a + b is constant.	*	*	*		*			*		*	6
	2	The sum of the lengths of the edges above the water surface is constant				*							1
Variation	3	One edge decreases by the amount the other increases						*	*				
	4	When one edge increases, the other decreases											
	5	The length of the edges vary											
	6	The length of the edge of the water surface becomes greater											
	7	When one edge becomes 0, the other edge becomes twice its original											
Range	8	The limit of the length of an edge is 15 cm											
Shape of water surface	9	The water surface (upper) and the base are rectangles											
	10	The water surface is a rectangles or quadrangle											
	11	The shape of the base is constant											

	12	The shape of the side plane changes from trapezoid to triangle											
	13	The side view is a trapezoid											
	14	The shape of the water surface changes											
	15	In some instances, the base becomes smaller											
Area	16	The total area of the side faces does not change											
	17	The are of water surface changes											
	18	The area of water surface becomes larger											
	19	The area, except that of the water surface does not change											
	20	The total surface area changes											
	21	The area of the base is smaller than that of the upper face when tilted											
Volume	22	The volume does not change											
	23	When the water forms prism, the volume equals (base area) x (height)											

Others	24	There is a fixed point, when viewed horizontally.											
	25	The weight of the water does not change											
	26	The angle changes											
	27	The sum of the angles of the side planes is constant											
	28	The surface is level											
	29	The form of the water is a quadrangular prism											
	30	The form of the water changes from a cuboid to a triangles prism											
	31	The form of the water changes											