11.5 TAMBAHAN UKURAN DESKRIPSI SAMPEL

PENAKSIRAN DARI MATRIKS KESALAHAN

Diberikan matriks $\underset{(pxp)}{\hat{A}} = \left[\hat{a}_1, \hat{a}_2, ..., a_p\right]'$; $\underset{(qxq)}{\hat{B}} = \left[\hat{b}_1, \hat{b}_2, ..., b_q\right]'$. Misalkan $\hat{a}^{(i)}$ dan $\hat{b}^{(i)}$ menotasikan ke-i kolom dari $\hat{A}^{-1}$ dan $\hat{B}^{-1}$ berturut-turut. Karena $\hat{U} = \hat{A}x^{(1)}$ dan $\hat{V} = \hat{B}x^{(2)}$ maka

$$\underset{(px1)}{x^{(1)}} = \underset{(pxp)}{\hat{A}^{-1}} \underset{(px1)}{\hat{U}} \; ; \; \underset{(qx1)}{x^{(2)}} = \underset{(qxq)}{\hat{B}^{-1}} \underset{(qx1)}{\hat{V}} \qquad (10-36)$$

Karena sampel Cov$\left(\hat{U}, \hat{V}\right) = \hat{A}S_{12}\hat{B}$, sampel Cov$\left(\hat{U}\right) = \hat{A}S_{11}\hat{A}' = \underset{(pxp)}{I}$ dan

sampelCov $\left(\hat{V}\right) = \hat{B}S_{22}\hat{B}' = \underset{(qxq)}{I}$

$$S_{12} = \hat{A}^{-1}\left[\begin{array}{cccc|c} \hat{\rho}_1^* & 0 & \cdots & 0 & \\ 0 & \hat{\rho}_2^* & \cdots & 0 & \\ \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & \cdots & \hat{\rho}_p^* & \end{array}\right](\hat{B}^{-1})' = \hat{\rho}_1^*\hat{a}^{(1)}\hat{b}^{(1)'} + \hat{\rho}_2^*\hat{a}^{(2)}\hat{b}^{(2)'} + ... + \hat{\rho}_p^*\hat{a}^{(p)}\hat{b}^{(p)'} \qquad (10\text{-}3$$

$$S_{11} = (\hat{A}^{-1})(\hat{A}^{-1})' = \hat{a}^{(1)}\hat{a}^{(1)'} + \hat{a}^{(2)}\hat{a}^{(2)'} + ... + \hat{a}^{(p)}\hat{a}^{(p)'}$$

$$S_{22} = (\hat{B}^{-1})(\hat{B}^{-1})' = \hat{b}^{(1)}\hat{b}^{(1)'} + \hat{b}^{(2)}\hat{b}^{(2)'} + ... + \hat{b}^{(p)}\hat{b}^{(p)'}$$

Jika r kanonis pertama digunakan maka dimisalkan,

$$\bar{x}^{(1)} = \left[\hat{a}^{(1)} \vdots \hat{a}^{(2)} \vdots \cdots \vdots \hat{a}^{(r)}\right]\begin{bmatrix}\hat{U}_1\\ \hat{U}_2\\ \vdots\\ \hat{U}_r\end{bmatrix} \quad dan \quad \bar{x}^{(2)} = \left[\hat{b}^{(1)} \vdots \hat{b}^{(2)} \vdots \cdots \vdots \hat{b}^{(r)}\right]\begin{bmatrix}\hat{V}_1\\ \hat{V}_2\\ \vdots\\ \hat{V}_r\end{bmatrix} \qquad (10-38)$$

sehingga S_{12} diperkirakan Cov$\left(\bar{x}^{(1)}, \bar{x}^{(2)}\right)$.

Selanjutnya, **penaksiran untuk matriks kesalahannya** adalah

$$S_{11} - \left(\hat{a}^{(1)}\hat{a}^{(1)'} + \hat{a}^{(2)}\hat{a}^{(2)'} + \cdots + \hat{a}^{(r)}\hat{a}^{(r)'}\right) = \hat{a}^{(r+1)}\hat{a}^{(r+1)'} + \cdots + \hat{a}^{(p)}\hat{a}^{(p)'}$$

$$S_{22} - \left(\hat{b}^{(1)}\hat{b}^{(1)'} + \hat{b}^{(2)}\hat{b}^{(2)'} + \cdots + \hat{b}^{(r)}\hat{b}^{(r)'}\right) = \hat{b}^{(r+1)}\hat{b}^{(r+1)'} + \cdots + \hat{b}^{(q)}\hat{b}^{(q)'} \qquad (10\text{-}39)$$

$$S_{12} - \left(\hat{\rho}_1^*\hat{a}^{(1)}\hat{b}^{(1)'} + \hat{\rho}_2^*\hat{a}^{(2)}\hat{b}^{(2)'} + \cdots + \hat{\rho}_r^*\hat{a}^{(r)}\hat{b}^{(r)'}\right) = \rho_{r+1}^*\hat{a}^{(r+1)}\hat{b}^{(r+1)'} + \cdots + \rho_p^*\hat{a}^{(p)}\hat{b}^{(p)'}$$

PROPORSI DARI VARIANS SAMPEL YANG DIKETAHUI

$$\text{Sampel Cov}\left(z^{(1)}, \hat{U}\right) = \text{sampel Cov}\left(\hat{A}_z^{-1}\hat{U}, \hat{U}\right) = \hat{A}_z^{-1}$$

dan $$\text{Sampel Cov}\left(z^{(2)}, \hat{V}\right) = \text{sampel Cov}\left(\hat{B}_z^{-1}\hat{V}, \hat{V}\right) = \hat{B}_z^{-1}$$

$$\bar{A}_z^{-1} = \left[\hat{a}_z^{(1)}, \hat{a}_z^{(2)}, \ldots, \hat{a}_z^{(p)}\right] = \begin{bmatrix} r_{\hat{U}_{1,x_1^{(1)}}} & r_{\hat{U}_{2,x_1^{(1)}}} & \cdots & r_{\hat{U}_{p,x_1^{(1)}}} \\ r_{\hat{U}_{1,x_2^{(1)}}} & r_{\hat{U}_{2,x_2^{(1)}}} & \cdots & r_{\hat{U}_{p,x_2^{(1)}}} \\ \vdots & \vdots & \ddots & \vdots \\ r_{\hat{U}_{1,x_p^{(1)}}} & r_{\hat{U}_{2,x_p^{(1)}}} & \cdots & r_{\hat{U}_{p,x_p^{(1)}}} \end{bmatrix}$$

$$\bar{B}_z^{-1} = \left[\hat{b}_z^{(1)}, \hat{b}_z^{(2)}, \ldots, \hat{b}_z^{(q)}\right] = \begin{bmatrix} r_{\hat{V}_{1,x_1^{(2)}}} & r_{\hat{V}_{2,x_1^{(2)}}} & \cdots & r_{\hat{V}_{q,x_1^{(2)}}} \\ r_{\hat{V}_{1,x_2^{(2)}}} & r_{\hat{V}_{2,x_2^{(2)}}} & \cdots & r_{\hat{V}_{q,x_2^{(2)}}} \\ \vdots & \vdots & \ddots & \vdots \\ r_{\hat{V}_{1,x_q^{(2)}}} & r_{\hat{V}_{2,x_q^{(2)}}} & \cdots & r_{\hat{V}_{q,x_q^{(2)}}} \end{bmatrix} \qquad (10\text{-}40)$$

dimana $r_{\hat{U}_{i,x_k^{(1)}}}$ *dan* $r_{\hat{V}_{i,x_k^{(2)}}}$ adalah koefisien korelasi sampel antara elemen yang ditulis.

Total varians sampel standar dalam himpunan pertama

$$= tr(R_{11}) = tr(\hat{a}_z^{(1)}\hat{a}_z^{(1)'} + \hat{a}_z^{(2)}\hat{a}_z^{(2)'} + \cdots + \hat{a}_z^{(p)}\hat{a}_z^{(p)}) = p$$

Total varians sampel standar dalam himpunan kedua (10-41)

$$= tr(R_{22}) = tr(\hat{b}_z^{(1)}\hat{b}_z^{(1)'} + \hat{b}_z^{(2)}\hat{b}_z^{(2)'} + \cdots + \hat{b}_z^{(q)}\hat{b}_z^{(q)}) = q$$

Kita definisikan kontribusi dari r variasi kanonis yang pertama terhadap total varians sampel standar sebagai

$$tr(\hat{a}_z^{(1)}\hat{a}_z^{(1)'} + \hat{a}_z^{(2)}\hat{a}_z^{(2)'} + \cdots + \hat{a}_z^{(p)}\hat{a}_z^{(p)}) = \sum_{i=1}^{r}\sum_{k=1}^{p} r^2_{\hat{U}_{i,x_k^{(1)}}}$$

dan

$$tr(\hat{b}_z^{(1)}\hat{b}_z^{(1)'} + \hat{b}_z^{(2)}\hat{b}_z^{(2)'} + \cdots + \hat{b}_z^{(q)}\hat{b}_z^{(q)'}) = \sum_{i=1}^{r}\sum_{k=1}^{p} r^2_{\hat{V}_{i,x_k^{(2)}}}$$

Proporsi dari total varians sampel standar dijelaskan dengan r variasi kanonis menjadi

$$R^2_{z^{(1)}\,|\hat{U}_1,\hat{U}_2,\ldots,\hat{U}_r} = \begin{pmatrix} \text{proporsi dari total varians sampel standar dalam} \\ \text{himpunan pertama yang dijelaskan oleh } \hat{U}_1,\hat{U}_2,\ldots,\hat{U}_r \end{pmatrix}$$

dan

$$= \frac{\operatorname{tr}(\hat{a}_z^{(1)}\hat{a}_z^{(1)\prime} + \cdots + \hat{a}_z^{(r)}\hat{a}_z^{(r)\prime})}{\operatorname{tr}(R_{11})} = \frac{\sum_{i=1}^{r}\sum_{k=1}^{p} r^2_{\hat{U}_i, z_k^{(1)}}}{p}$$

$$R^2_{z^{(2)}\,|\hat{V}_1,\hat{V}_2,\ldots,\hat{V}_r} = \begin{pmatrix} \text{proporsi dari total varians sampel standar dalam} \\ \text{himpunan kedua yang dijelaskan oleh } \hat{V}_1,\hat{V}_2,\ldots,\hat{V}_r \end{pmatrix} \qquad (10\text{-}42)$$

$$= \frac{\sum_{i=1}^{r}\sum_{k=1}^{q} r^2_{\hat{V}_i, z_k^{(1)}}}{q}$$

$$\frac{\cdots + \hat{b}_z^{(r)}\hat{b}_z^{(r)\prime})}{\cdots)}$$

Ukuran deskripsi diatas memberikan petunjuk seberapa baik variasi kanonis dan memberikan gambaran nilai tunggal dari matriks kesalahannya, terutama

$$\frac{1}{p}\operatorname{tr}[R_{11} - \hat{a}_z^{(1)}\hat{a}_z^{(1)\prime} - \hat{a}_z^{(2)}\hat{a}_Z^{(2)\prime} - \cdots - \hat{a}_z^{(r)}a_z^{\prime(r)\prime}] = 1 - R^2_{z^{(1)}|\,\hat{U}_1,\,\hat{U}_2,\ldots,\,\hat{U}_r}$$

$$\frac{1}{q}\operatorname{tr}[R_{22} - \hat{b}_z^{(1)}\hat{b}_z^{(1)\prime} - \hat{b}_z^{(2)}\hat{b}_Z^{(2)\prime} - \cdots - \hat{b}_z^{(r)}\hat{b}_z^{(r)\prime}] = 1 - R^2_{z^{(2)}\,|\,\hat{V}_1,\,\hat{V}_2,\ldots,\,\hat{V}_r}$$

11.6 INFERENSI SAMPEL BESAR

Dalam analisis kanonik kita menguji hipotesis dengan metode likelihood. Ketika $\Sigma_{12} = 0$ maka $a'X^{(1)}$ dan $b'X^{(2)}$ memiliki $a'\Sigma_{12}b = 0$ untuk semua vektor a dan b. Cara untuk menguji $\Sigma_{12} = 0$ untuk sampel besar.

Misalkan $x_j = \begin{bmatrix} x_j^{(1)} \\ -- \\ x_j^{(2)} \end{bmatrix}, j = 1,2,\ldots,n$ merupakan sample acak dari populasi

$N_{(p+q)}(\mu,\Sigma)$ dengan $\Sigma = \begin{bmatrix} \underset{(p\times p)}{\Sigma_{11}} & \underset{(p\times q)}{\Sigma_{12}} \\ \underset{(q\times p)}{\Sigma_{21}} & \underset{(q\times q)}{\Sigma_{22}} \end{bmatrix}$

Tes Rasio likelihood dari $H_0 : \Sigma_{12} = 0$ melawan $H_1 : \Sigma_{12} \neq 0$. Tolak H_0 untuk nilai yang lebih besar dari $-2\ln\Lambda = n\ln\left(\frac{|S_{11}||S_{22}|}{|S|}\right) = -n\ln\prod_{i=1}^{p}\left(1-\hat{\rho}_i^{*2}\right)$

Dimana $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ adalah penaksir tak bias dari Σ.

Untuk n yang besar,

Tolak H_0 jika $-\left(n-1-\frac{1}{2}(p+q+1)\right)\ln\prod_{i=1}^{p}\left(1-\hat{\rho}_i^{*2}\right) > \chi^2_{pq}(\alpha)$

Jika hipotesis :

$H_0^{(k)}: \rho_1^* \neq 0, \rho_2^* \neq 0, ..., \rho_k^* \neq 0, \rho_{k+1}^* \neq 0, = ... = \rho_p^* \neq 0$

$H_1^{(k)}: \rho_i^* \neq 0$, untuk beberapa $i \geq k+1$

Tolak H_0 jika $-\left(n-1-\frac{1}{2}(p+q+1)\right)\ln\prod_{i=k+1}^{p}\left(1-\hat{\rho}_i^{*2}\right) > \chi^2_{(p-k)(q-k)}(\alpha)$