Differentiation

MATHEMATICS FOR SCIENCE

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by Syam

The derivative

- There are three notations for the derivative of $y = f(x)$ with respect to x
- They are

$$y' = f'(x) \qquad D_x y \qquad \frac{d}{dx}(y) = \frac{dy}{dx}$$

The rules for finding Derivatives

1) $\frac{d}{dx}c = 0$ contantfunction

2) $\frac{d}{dx}x^n = nx^{n-1}$ powerfunctionrule

3) $\frac{d}{dx}\left(cf(x)\right) = cf'(x)$ constant- multiple rule

The rules for finding Derivatives

4) $$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

sum - differencerule

5) $$\frac{d}{dx}\left[f(x) . g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

Productrule

6) $$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

Quotient rule

Finding marginal-revenue function from average-revenue function using the product rule

$$R = PQ \qquad P = 15\text{-}Q \qquad R = (15\text{-}Q)Q$$

$$\frac{dR}{dQ} = Q\frac{d}{dQ}P + P\frac{d}{dQ}Q$$

$$= Q(-1) + (15 - Q)(1) = 15 - 2Q$$

$$R = 15Q - Q^2$$

$$\frac{dR}{dQ} = 15 - 2Q$$

Derivatives of Trigonometric Functions

1) $\frac{d}{dx}(\sin x) = \cos x$ 2) $\frac{d}{dx}(\cos x) = -\sin x$

3) $\frac{d}{dx}(\tan x) = \sec^2 x$ 4) $\frac{d}{dx}(\sec x) = \sec x \tan x$

5) $\frac{d}{dx}(\csc x) = -\csc x \cot x$

The Chain Rule

Imagine trying to find the derivative of

$$F(x) = (2x - 4x + 1)^{60}$$

Let

$$g(x) = y = 2x^2 - 4x + 1 \quad \text{and}$$

$$f(y) = z = y^{60}$$

then

$$F(x) = z = f(y) = f(g(x)) = (f \circ g)(x)$$

The Chain Rule

Let $z = f(y)$ and $y = g(x)$

If f and g is differentiable at x and f is differentiable at y, then the composite function $f \circ g$ defined by $(f \circ g)(x) = f(g(x))$ is differentiable at x and

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

or $$D_x z = (D_y z)(D_x y)$$

or $$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

Implicit Differentiation

Let $y^3 + 7y = x^3$,

it cannot be solved for y in terms of x.

Assume $y = y(x)$, write

$$\left(y(x)\right)^3 + 7y(x) = x^3$$

Applying the Chain Rule,

$$\frac{d}{dx}(y^3) + \frac{d}{dx}(7y) = \frac{d}{dx}x^3$$

$$3y^2\frac{dy}{dx} + 7\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx}(3y^2 + 7) = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{3y^2 + 7}$$

Differentials

Let $y = f(x)$ be a differentiabel function of the independent variabel x.

dx, called the differential of the independent variable x.

dy, called the differential of the independent variable y.

then we have:

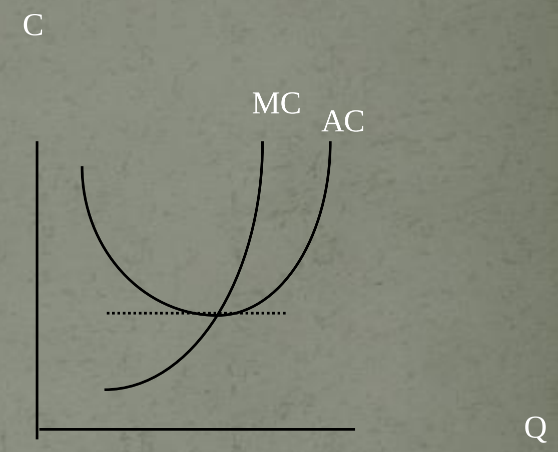
$$dy = f'(x)dx$$

$$\frac{dy}{dx} = f'(x)$$

interpret the derivative as a quotient of two differentials

Relationship between marginal-cost and average-cost functions

Find the minimum average cost



$TC = C(Q)$ total cost

$MC = C'(Q)$ marginal cost

$AC = C(Q)/Q$ average cost

$$\frac{d}{dQ}\frac{C(Q)}{Q} = \frac{Q \cdot C'(Q) - C(Q) \cdot 1}{Q^2}$$

$$= \frac{1}{Q}\left[C'(Q) - \frac{C(Q)}{Q}\right] = \frac{1}{Q}\left[MC - AC\right]$$

$$\text{if } \frac{d}{dQ}\frac{C(Q)}{Q} = 0, \text{ then AC} = \text{MC}$$

Inverse-function Rule

Let $y = f(x)$, and $dy/dx = f'(x)$

where y is a strictly monotonic function of x

then $x = f^{-1}(y)$ and

9) $dx/dy = f^{-1'}(y) = \dfrac{1}{dy/dx}$

Inverse-function rule

- A monotonic function is one in which

a given value of x yield a unique value of y

and a given value of y will yield a unique value of x.

(Recall the definition of a function, p.17, one y for each x)

- $Q = f(P)$ $\qquad$ $P = f^{-1}(Q)$
- $Q_s = b_0 + b_1 P$ $\qquad$ $P = -b_0/b_1 + (1/b_1)Q_s$

(where $b_1 > 0$)

- $dQ_s/dP = b_1$ $\qquad$ $dP/dQ_s = 1/b_1 = 1/dQ_s/dP$

Inverse-function rule

If the function $y = f(x)$ represents a one-to-one mapping,

i.e., a different value of x will always yield a different value of y,

the function will have an inverse function

$y = f(x)$

$dy/dx = f'(x)$

Solve for x (one equation, one unknown)

$x = f^{-1}(y)$.

$$dx/dy = f^{-1\prime}(y) = \frac{1}{dy/dx} = \frac{1}{f'(x)} \qquad [9]$$

Inverse-function rule

- This property of one to one mapping is unique to the class of functions known as monotonic functions:
- Recall the definition of a function, p. 17,
  function: one y for each x
  monotonic function: one x for each y (inverse function)
- if $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$ monotonically increasing
  $Q_s = b_0 + b_1 P$ supply function (where $b_1 > 0$)
  $P = -b_0/b_1 + (1/b_1)Q_s$ inverse supply function
- if $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$ monotonically decreasing
  $Q_d = a_0 - a_1 P$ demand function (where $a_1 > 0$)
  $P = a_0/a_1 - (1/a_1)Q_d$ inverse demand function