Sistem Tiga-massa yang Tereksitasi Secara Parametrik

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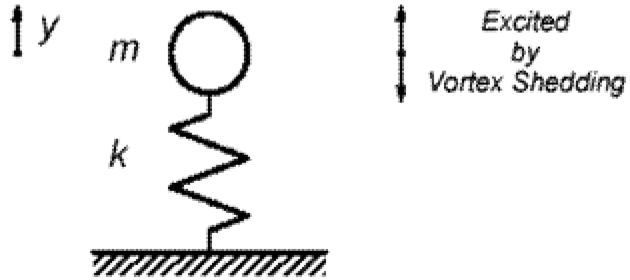
JURDIKMAT FPMIPA UPI BANDUNG

LATAR BELAKANG

Motivasi:

- Meredam atau mengurangi vibrasi yang sebabkan oleh arus induksi
- Studi terhadap sistem dua massa (two-degrees of problem system) yang self-excited dan tereksitasi secara parametrik menunjukkan dinamik yang kompleks dan menarik.

One-degree of freedom



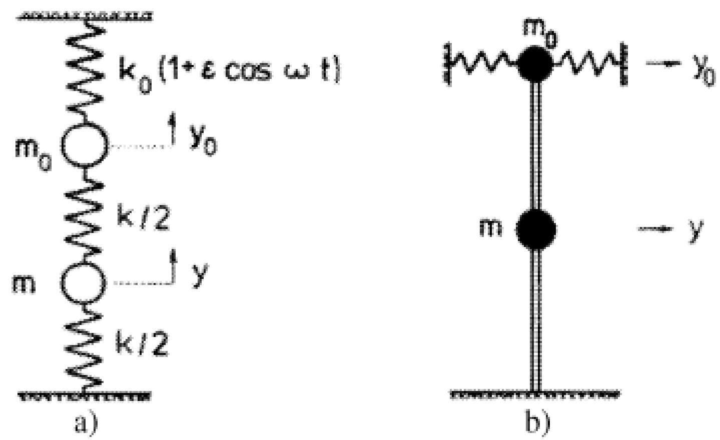
$$m\,\ddot{y} + b\,\dot{y} + k\,y = \gamma\,u\ ,$$

$$\ddot{u} + \alpha_0\,\dot{u} + \omega_s^2\,[u - \delta\ \mathrm{sgn}(\dot{y})] = 0\ ,$$

Two-degrees of freedom

$$m\,\ddot{y} + [b - \beta_0\,U^2\,(1 + \gamma\,y^2)]\,\dot{y} + k\,y - \frac{1}{2}\,k\,y_0 = 0\ ,$$

$$m_0\,\ddot{y}_0 + b_0\,\dot{y}_0 + k_0\,(1 + \varepsilon\,\cos\omega t)\,y_0 - \frac{1}{2}\,k\,(y - y_0) = 0\ .$$



Sumber: A. Tondl.. To The problem of Self-Excited Vibration Suppresion. *Engineering Mechanics, Vol. 15, no. 4, p. 297-307,* 2008.

Hasil Studi numerik dari

two-degrees of freedom system tereksitasi secara parametrik

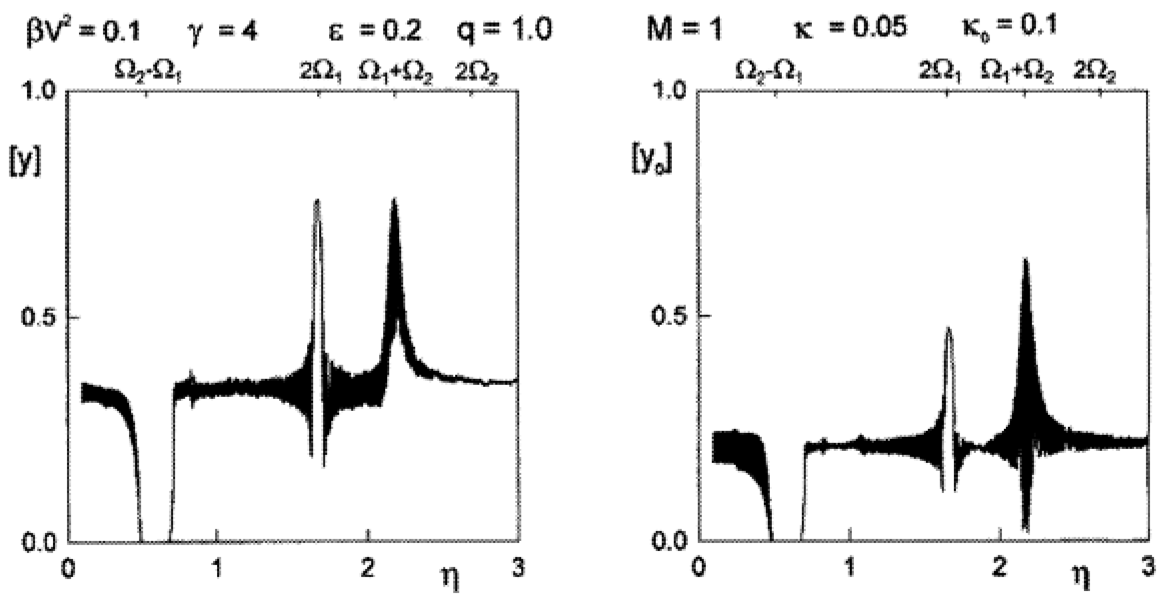


Fig.5: Extreme of y, y_0 in dependence on parametric excitation frequency ω

Hasil Studi Analitik: Dinamik pada *two-degrees of freedom system* tereksitasi secara parametrik (memuat persamaan Mathieu)

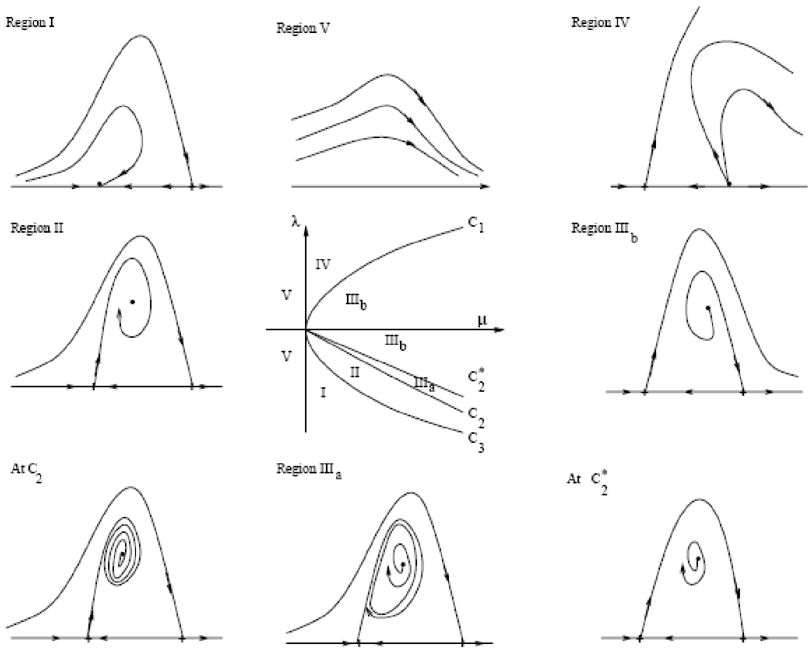


Figure 7: The phase-portraits of equation (71) in the (x, y)-plane for a specific value (μ, λ) in each region.

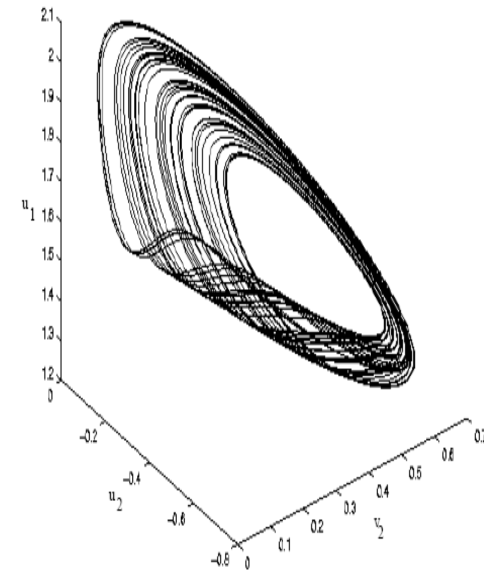


Figure 6: The strange attractor of the averaged system (68). The phase-portraits in the (u_2, v_2, u_1)-space for $c_2 < 0$ at the value $\sigma_2 = 5.3$. The Kaplan-Yorke dimension for $\sigma_2 = 5.3$ is 2.29.

Sumber: A. S.Fatimah and M. Ruijgrok. Bifurcations in an autoparametric system in 1:1 internal resonance with parametric excitation. *International J. Non-linear Mechanics*, 37(2), 297-308, 2002.

Hasil Studi Analitik: Dinamik pada *two-degrees of freedom system (memuat persamaan Rayleigh)*

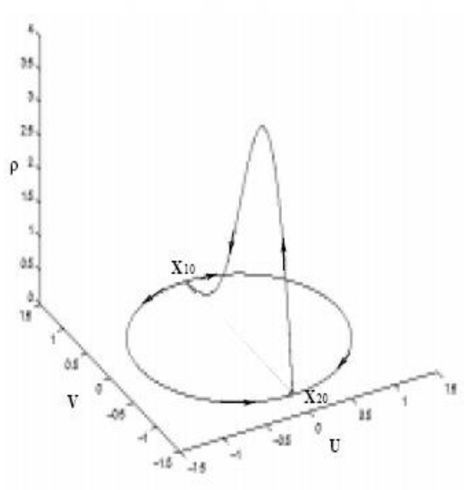
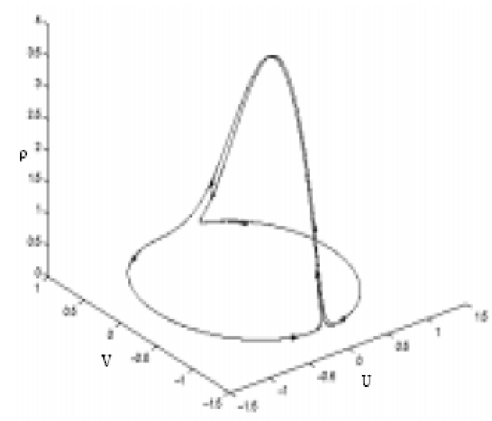


Figure 11: The robust heteroclinic cycle connecting the saddle points $\mathbf{x}_{10}$ and $\mathbf{x}_{20}$.

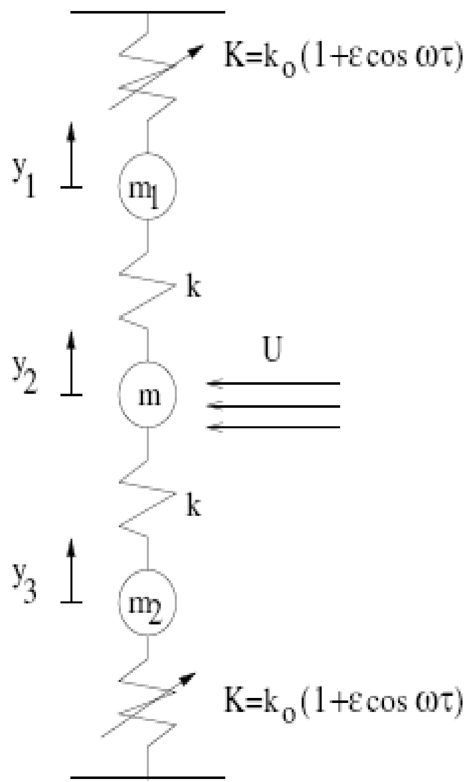


(iii)

Figure 12: **Forced symmetry breaking.** (i) a long-periodic orbit for $\sigma = 0.01$, (ii) a long-periodic orbit for $\sigma = -0.01$, (iii) the combination of (i) and (ii).

Sumber: Abadi. On self-excited autoparametric systems. Nonlinear Dynamics, 24, 147-166, 2002.

Model *tiga-massa*



- Massa pusat m dan massa eksternal m1 dan m2.
- Konstanta pegas k
- Kecepatan gaya yg mengenai m =U, serupa persamaan Rayleigh
- Peredam berperiodik dg koefisisien K(t)
- K(t)=0 tidak ada pengaruh eksitasi parametrik

Persamaan Rayleigh:

$$\ddot{x} + x - \varepsilon(1 - \dot{x}^2)\dot{x} = 0$$

Persamaan Mathieu:

$$\ddot{x} + (1 + \varepsilon \cos \varpi t)x = 0$$

Sistem tiga-massa

$$m_1\ddot{y}_1 + b_0\dot{y}_1 + k_0(1+\varepsilon cos\omega t)y_1 + k(y_1 - y_2) = 0$$
$$m\ddot{y}_2 - bU^2(1 - c\dot{y}_2)\dot{y}_2 + 2ky_2 - k(y_1 + y_3) = 0 \qquad (1)$$
$$m_2\ddot{y}_3 + b_0\dot{y}_3 + k_0(1+\varepsilon cos\omega t)y_3 + k(y_3 - y_2) = 0$$

- Pada model di atas untuk kasus $m_1 = m_2 = m_0$ kemudian transformasikan dengan $\tau = \omega_0 t$ dimana $\omega_0 = \sqrt{2k/m}$ diperoleh

$$y_1'' + \varepsilon\hat{\kappa}y_1' + q^2(1+\varepsilon cos\eta\tau)y_1 + \tfrac{1}{2}M(y_1 - y_2) = 0$$
$$y_2'' - \varepsilon\hat{\beta}V^2(1 - \gamma y_2'^2)y_2' + y_2 - \tfrac{1}{2}(y_1 + y_3) = 0 \qquad (2)$$
$$y_3'' + \varepsilon\hat{\kappa}y_3' + q^2(1+\varepsilon cos\eta\tau)y_3 + \tfrac{1}{2}M(y_3 - y_2) = 0$$

Dengan $q^2 = k_0/m_0\omega_0^2$, $M = m/m_0$, $\eta = \omega/\omega_0$ $\kappa = \varepsilon\hat{\kappa} = b_0/m_0\omega_0$

$\gamma = c\omega_0^2$, $V = U/U_0$

Analisis dari Sistem Tiga-massa yang Tereksitasi Secara Parametrik

- Transformasi sistem (2) dengan transformasi linear:

$$y_1 = x_1 + x_2 + x_3$$
$$y_2 = a_1 x_1 + a_2 x_2$$
$$y_1 = x_1 + x_2 - x_3$$

$$a_{1,2} = \frac{1}{M}\left(q^2 + \tfrac{1}{2}M - 1\right) \pm \sqrt{\left(q^2 + \tfrac{1}{2}M - 1\right)^2 + 2M}$$

- Diperoleh sistem yang memuat persamaan Mathieu berikut:

$$x_1'' + \Omega_1^2 x_1 + \varepsilon f_1(\mu_1, \cos\eta\tau, x_1, x_1', x_2, x_2', x_3, x_3') = 0$$
$$x_2'' + \Omega_2^2 x_2 + \varepsilon f_2(\mu_2, \cos\eta\tau, x_1, x_1', x_2, x_2', x_3, x_3') = 0$$
$$x_3'' + \Omega_3^2 x_3 + \varepsilon f_3(\cos\eta\tau, x_3, x_3') = 0$$

$$f_1 = \theta_{11}x_1' + \theta_{21}x_2' + (Q_{11}x_1 + Q_{12}x_2)cos\eta\tau + c_1{x_1'}^3 + c_2{x_1'}^2x_2' + c_3x_1'{x_2'}^2 + c_4{x_2'}^3$$

$$f_2 = \theta_{21}x_1' + \theta_{22}x_2' + (Q_{21}x_1 + Q_{22}x_2)cos\eta\tau - c_1{x_1'}^3 - c_2{x_1'}^2x_2' - c_3x_1'{x_2'}^2 - c_4{x_2'}^3$$

$$f_3 = \hat{\kappa}x_3' + q^2cos\eta\tau x_3$$

- dengan

$$\Omega_{1,2}^2 = \frac{1}{2}\left(q^2+\frac{1}{2}M+1\right) \pm \sqrt{\frac{1}{4}\left(q^2+\frac{1}{2}M+1\right)^2 - q^2}$$

$$\Omega_3^2 = q^2+\frac{1}{2}M$$

$$\theta_{11} = -\frac{1}{\alpha_1-\alpha_2}\left(\alpha_2\hat{\kappa} + \alpha_1\hat{\beta}V^2\right) \qquad \theta_{12} = -\frac{\alpha_2}{\alpha_1-\alpha_2}\left(\hat{\kappa} + \hat{\beta}V^2\right) \qquad \theta_{21} = -\frac{\alpha_1}{\alpha_1-\alpha_2}\left(\hat{\kappa} + \hat{\beta}V^2\right)$$

$$\theta_{22} = \frac{1}{\alpha_1-\alpha_2}\left(\alpha_1\hat{\kappa} + \alpha_2\hat{\beta}V^2\right) \qquad Q_{11} = Q_{12} = -\frac{\alpha_2}{\alpha_1-\alpha_2}q^2 \qquad Q_{21} = Q_{22} = \frac{\alpha_2}{\alpha_1-\alpha_2}q^2$$

$$c_1 = \frac{{\alpha_1}^3}{\alpha_1-\alpha_2}\gamma\hat{\beta}V^2 \qquad c_2 = \frac{3{\alpha_1}^2\alpha_2}{\alpha_1-\alpha_2}\gamma\hat{\beta}V^2 \qquad c_3 = \frac{3\alpha_1{\alpha_2}^2}{\alpha_1-\alpha_2}\gamma\hat{\beta}V^2 \qquad c_4 = \frac{{\alpha_2}^3}{\alpha_1-\alpha_2}\gamma\hat{\beta}V^2$$

Kestabilan dari Solusi Trivial

- Keberadaan solusi trivial ditentukan oleh interval dengan kondisi berikut:

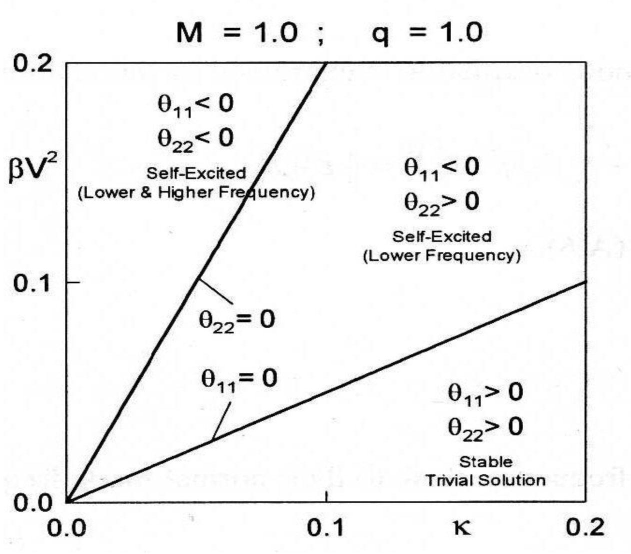
$$\theta_{11} + \theta_{22} > 0 \qquad \text{atau} \qquad \hat{\kappa} - \hat{\beta} V^2 > 0$$

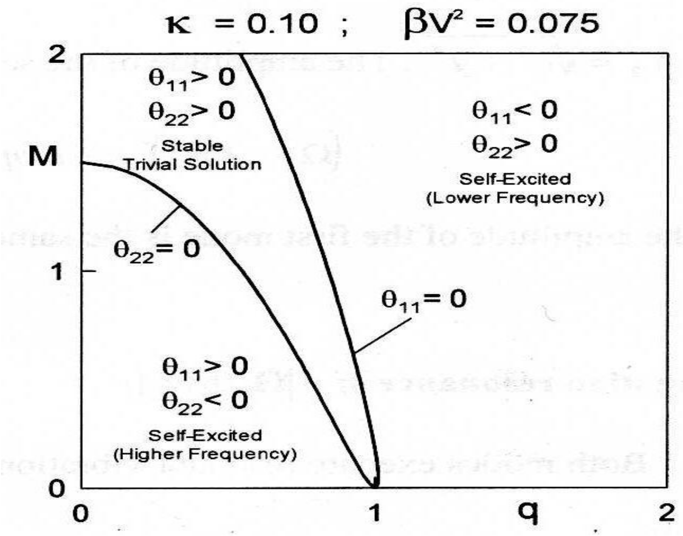
- Batas kestabilan ditentukan oleh:

$$\eta_0 - \delta < \eta < \eta_0 + \delta$$

Dengan: $\eta_0 = \Omega_2 - \Omega_1$ dan $\delta = \varepsilon \dfrac{\theta_{11} + \theta_{22}}{\sqrt{|\theta_{11}\theta_{22}|}} \sqrt{-\dfrac{Q_{12}Q_{21}}{16\Omega_1\Omega_2} - \theta_{11}\theta_{22}}$

Grafik keberadaan solusi





$M = 0.1 \quad q = 1$

$\hat{\kappa} = 0 \qquad \hat{\beta} V^2 = 0.075$

Solusi Non-trivial

- Pada sistem dengan parameter berperiodik, resonansi parametrik terjadi di sekitar nilai-nilai:
- $$\eta = \frac{2\Omega_j}{N} \qquad j = 1,2. \quad N = 1,2,3,\ldots$$
  $$\eta = \frac{|\Omega_j + \Omega_k|}{N} \qquad j,k = 1,2. j \neq k$$
  $$x_1(\tau) = R_1 cos\Omega_1 \tau$$
  $$x_2(\tau) = R_2 cos\Omega_2 \tau$$
  $$R_1 = \sqrt{-\frac{4\theta_{11}}{3c_1\Omega_1^2}} \qquad R_2 = \sqrt{\frac{4\theta_{22}}{3c_4\Omega_1^2}}$$
- Untuk kasus $c_1 > 0$ dan $c_2 < 0$, solusi non trivial ada untuk
- $\theta_{11} > 0$ dan $\theta_{22} < 0$

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