STRUCTURE THEORY OF TWISTED TOEPLITZ ALGEBRAS AND IDEAL STRUCTURE OF THE (UNTWISTED) TOEPLITZ ALGEBRAS OF ORDERED GROUPS

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Abstract

Suppose Γ is a totally ordered abelian group and σ is a cocycle on Γ. In this dissertation we discuss a theory of twisted Toeplitz algebras $\mathcal{T}(\Gamma, \sigma)$ of Γ. We generalise previous works on the untwisted version $\mathcal{T}(\Gamma)$ by employing the twisting concept of a cocycle σ. Theory of the twisted Toeplitz algebras is not new, but previous work just covers subgroups of real numbers. In line with previous work on $\mathcal{T}(\Gamma, \sigma)$, this algebra is universal for isometric σ-representation of Γ^+. Important results on the untwisted version have their generalisation in our twisted version. The method of crossed product by semigroups of endomorphisms of previous works are applicable in these generalisations, so we also have developed theory of twisted crossed products by semigroups of endomorphisms.

Another focus of our dissertation is the ideal structure of the untwisted Toeplitz algebras $\mathcal{T}(\Gamma)$. This work is an extension of previous work which proved that the primitive ideal space is parametrised by disjoint union X of the duals of order ideals in Γ. It was assumed that the chain of order ideals in Γ contains a copy of $\{-\infty\} \cup \mathbb{Z} \cup \{\infty\}$. The most important result in the previous work is, that X can be topologised so that this topology is precisely the hull-kernel topology (the standard topology for the primitive ideal space). We give the necessary and sufficient condition for the previous result; the topology in X is the hull-kernel topology if and only if the chain of order ideals is well ordered.

Abstrak

Misalkan Γ sebuah grup abelian terurut total dan σ sebuah kosikel pada Γ. Disertasi ini membahas sebuah teori tentang aljabar Toeplitz terpelintir $\mathcal{T}(\Gamma, \sigma)$ dari Γ. Di sini dibahas perumuman dari aljabar Toeplitz tak terpelintir $\mathcal{T}(\Gamma)$ dengan menerapkan konsep pelintiran dari kosikel σ. Kajian tentang aljabar Toeplitz terpelintir bukanlah sesuatu yang baru, tetapi penelitian sebelumnya hanya mencakup subgrup dari bilangan real. Sejalan dengan penelitian sebelumnya, aljabar ini adalah universal terhadap representasi isometrik-σ dari Γ^+. hasil-hasil penting pada versi takterpelintir mempunyai perumuman terhadap versi terpelintirnya. Metoda produk silang atas semigrup dari endomorfisma pada penelitian sebelumnya dapat digunakan dalam perumuman ini, dengan demikian dalam disertasi ini dikembangkan pula teori dari produk silang atas semigrup dari endomorfisma versi terpelintir.

Kajian yang lain dalam disertasi ini adalah struktur ideal dari aljabar Toeplitz $\mathcal{T}(\Gamma)$. Penelitian ini merupakan pengembangan dari penelitian terdahulu yang membuktikan bahwa ruang ideal primitif dari aljabar Toeplitz dapat diparametrisasi oleh gabungan lepas X dari grup-grup dual dari ideal urutan di Γ. Pada penelitian terdahulu diasumsikan bahwa rantai dari ideal ururan dari Γ memuat salinan dari $\{-\infty\} \cup \mathbb{Z} \cup \{\infty\}$. Hasil yang terpenting dari penelitian terdahulu adalah, bahwa X dapat diberikan topologi sedemikian sehingga topologi ini benar-benar sesuai dengan topologi hull-kernel (topologi baku untuk ruang ideal primitif). Dalam disertasi ini diberikan syarat perlu dan cukup untuk hasil terdahulu, yaitu bahwa topology dalam X menyatakan topologi hull-kernel jika dan hanya jika rantai $\Sigma(\Gamma)$ adalah terurut rapi.

Guidelines to Use the Dissertation

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Contents

Chapter I

Preliminary

I.1 Motivation

Since Adji, Laca, Nilsen and Raeburn initiate the study of Toeplitz algebras from point of view of semigroup crossed products [4] , attempts to explore the potency of its theory have motivated significantly to the development of semigroup crossed products and to the generalisation of Toeplitz algebras. Adji [3] contributes theorems on the structure of semigroup crossed products, Laca and Raeburn [16] study the semigroup crossed products and the Toeplitz algebras of non-abelian groups, [6, 17, 19] are some collection of papers in employing the theory of semigroup crossed products to Hecke C^*-algebras arising in number theory.

In this thesis we study the twisted Toeplitz algebra (the algebra introduced by Ji in [13]) by employing the twisted semigroup crossed products. We extend the technique in [4] to the twisted version, and we want to continue to study the ideal structure of twisted Toeplitz algebra.

I.2 Problem Formulation

Research investigations in this project is mainly concerned with the following problems:

Suppose $\sigma : \Gamma \times \Gamma \to \mathbb{T}$ is a cocycle on a totally ordered abelian group Γ

1. Given an isometric σ-representation V of Γ^+. How does C^*-algebra $C^*(V_x, \sigma)$ generated by isometric σ-representation depend on V?

2. How does the structure theorem of Adji [3, Theorem 3.1] extend to the twisted Toeplitz algebra $\mathcal{T}(\Gamma, \sigma)$?

3. Given an order ideal I in Γ, and denoting by $\mathcal{C}(\Gamma, I, \sigma)$ the ideal of $\mathcal{T}((\Gamma, \sigma)$ generated by $\{1 - T_x T_x^* : x \in I^+\}$. How does this relate to twisted commutator ideal $\mathcal{C}(I, \operatorname{res}\sigma)$ of $\mathcal{T}(\Gamma, \operatorname{res}\sigma)$ generated by

$$\{T_y T_x^* - \operatorname{res}\sigma(y, -x)\overline{\operatorname{res}\sigma(-x, y)} T_x^* T_y : x, y \in \Gamma^+\}?$$

4. How does the topology introduced by Adji and Raeburn in [5] represent the hull-kernel topology?

I.3 Objectives

Referring to our problems listed in Problem Formulation, our objectives are to obtain general theorems which solve the problems. We want to have results of [4] for twisted semigroup crossed products and twisted Toeplitz algebra of totally ordered abelian groups. Then an analogue theorem to Theorem 3.1 of Adji [3] for twisted Toeplitz algebra is one of the goals. Our main goal is to obtain the ideal structure of the twisted Toeplitz algebra $\mathcal{T}(\Gamma, \sigma)$.

I.4 Outline

This thesis consists of five chapters. The motivation of study, formulation of the problems, objectives of study, and literature reviews are discussed in Chapter I. Results of the study are discussed in Chapter II through Chapter IV. Chapter II consists of construction and properties of twisted crossed products. The twisted Toeplitz algebras as well as their invariant ideals are discussed in this chapter. The results in this chapter mostly are generalisation of previous results, to a more general group or either to the presence of the cocycle, so they are not quite new. Chapter III consists of structure ideals of twisted Toeplitz algebras. Our main result is a twisted version of Theorem of Adji. On the second half of this chapter we give a structure ideal theory of twisted Toeplitz algebras. We give some new theorems in the structure ideal theory, as well as a theorem to implement a Morita equivalence between two algebras. Chapter IV discusses the primitive ideal space of Toeplitz algebras which is generalisation of theory of [5]. In this chapter we specialise to figure out the behaviour of primitive ideal space of Toeplitz algebra. Our main result is the necessary and sufficient condition of Adji-Raeburn's theorem. All results in this chapter are quite new. Finally Chapter V presents the conclusions of the research.

Our original results in this dissertation are indicated by $\diamondsuit$, these are all the propositions, lemmas, theorems and corollaries in Chapter IV, and some in Chapter II and III. Our results coming up with references in brackets are generalisation of previous results to our twisted setting. Unless otherwise stated, these results mostly proved by adapting previous method.

I.5 Literature Review

I.5.1 Structure Theory of Toeplitz Algebra

Douglas in [9] studied the C^*-algebra generated by semigroup of isometries $\{V_x\}_{x\in\Gamma^+}$ where Γ^+ is the positive cone of a subgroup Γ of $\mathbb{R}$. He proved that when each V_x is non-unitary, $\{V_x\}_{x\in\Gamma^+}$ generate a canonically isomorphic algebras, in the sense that if $\{W_x\}_{x\in\Gamma^+}$ is another semigroup of non-unitary isometries, then the C^*-algebra $C^*(\{W_x\})$ generated by $\{W_x\}_{x\in\Gamma^+}$ is isomorphic to $C^*(\{V_x\})$. This canonical algebra is called the Toeplitz algebra $\mathcal{T}(\Gamma)$ of group Γ. Further he stated that the commutator ideal $\mathcal{C}(\Gamma)$ of $\mathcal{T}(\Gamma)$ is simple, and there is a short exact sequence

$$0 \longrightarrow \mathcal{C}(\Gamma) \longrightarrow \mathcal{T}(\Gamma) \longrightarrow C(\hat{\Gamma}) \cong C^*(\Gamma) \longrightarrow 0. \tag{I.1}$$

These results are extensions of earlier wellknown theorem of Coburn [8] for the case $\Gamma = \mathbb{Z}$.

For a more general case, involving analysis on crossed product by automorphism, Murphy in [21] generalised the previous results for any totally ordered abelian group Γ. The Toeplitz algebra of a totally ordered abelian group Γ is the C* subalgebra $\mathcal{T}(\Gamma)$ of $B(H^2(\Gamma^+))$ generated by Toeplitz operators T_f of continuous symbol f in $C(\hat{\Gamma})$. In Theorem 2.9 Murphy showed that $\mathcal{T}(\Gamma)$ is universal for non-unitary isometric representation of Γ^+. For every $x \in \Gamma^+$, the evaluation maps $\epsilon_x : \gamma \mapsto \gamma(x)$ spans $C(\hat{\Gamma})$, hence the Stone-Weierstrass theorem implies that $\mathcal{T}(\Gamma)$ is generated by T_{ϵ_x} for $x \in \Gamma^+$. In Theorem 3.14 Murphy showed that $\mathcal{T}(\Gamma)$ is then generated by $T : \Gamma^+ \longrightarrow B(H^2(\Gamma^+))$ where $T_x := T_{\epsilon_x}$, because T is a nonunitary isometric representation of Γ^+. Extending the previous results, also he proved the short exact sequence (I.1) for totally ordered abelian group Γ.

An alternative approach to the Murphy's theorem is given in [4]. Given a totally ordered abelian group Γ, and let V be an isometric representation of the positive cone Γ^+, Adji, Laca, Nilsen and Raeburn in [4] gave a short, selfcontained proof of Murphy's theorem. Consider the subalgebra B_{Γ^+} of $\ell^\infty(\Gamma)$ spanned by $\{1_x : x \in \Gamma^+\}$ where

$$1_x(y) = \begin{cases} 1 & \text{if } y \geq x \\ 0 & \text{else.} \end{cases}$$

They developed and used the theory of semigroup crosed product by endomorphism to characterise that $C^*(\{V_x : x \in \Gamma^+\})$ is a crossed product which is universal for isometric representation of Γ^+. For every $x \in \Gamma^+$, the automorphism $\tau_x \in \operatorname{Aut} \ell^\infty(\Gamma)$ defined by translation $\tau_x(f)(y) = f(y-x)$ leaves

B_{Γ^+} invariant because $\tau_x(1_y) = 1_{x+y}$. The restriction of τ on Γ^+ is then an action of Γ^+ by endomorphisms of B_{Γ^+}. In Proposition 2.2 they proved that the crossed product $B_{\Gamma^+} \times_\tau \Gamma^+$ has the universal property which characterises the semigroup C^*-algebra $C^*(\Gamma^+)$. Their main theorem proved that $B_{\Gamma^+} \times_\tau \Gamma^+ \cong C^*(\{V_x : x \in \Gamma^+\})$ iff each V_x is nonunitary. This theorem is a remarkable innovation, they tied up two C^*-algebras which has different universal property by an isomorphism. As a consequence, Theorem 2.9 of [21] can be derived as a corollary of this theorem [4, Corollary 2.5]. Another consequence, if each V_x is nonunitary, through the Stone-Weierstrass theorem they proved that $\mathcal{T}(\Gamma)$ is the crossed product $B_{\Gamma^+} \times_\tau \Gamma^+$.

Suppose I is an order ideal of a totally ordered abelian group Γ. In a more recent paper, Adji in [3] considered the relationship between the Toeplitz algebra $\mathcal{T}(\Gamma)$ and $\mathcal{T}(\Gamma/I)$. The universal property of Toeplitz algebra $\mathcal{T}(\Gamma)$ assures that there is a canonical surjection $Q_I : \mathcal{T}(\Gamma) \longrightarrow \mathcal{T}(\Gamma/I)$ which takes generators T_x to generators $T^{\Gamma/I}_{q(x)}$. If α be the dual dual action of $\hat{\Gamma}$ on $\mathcal{T}(\Gamma)$ characterised by $\alpha_\gamma(T_x) = \gamma(x)T_x$, in her main theorem Adji stated that there is a short exact sequence

$$0 \longrightarrow \mathcal{C}_I \longrightarrow \mathcal{T}(\Gamma) \xrightarrow{\Theta} \operatorname{Ind}_{I^\perp}^{\hat{\Gamma}} \mathcal{T}(\Gamma/I) \longrightarrow 0 \tag{I.2}$$

in which $\mathcal{C}_I$ is the ideal in $\mathcal{T}(\Gamma)$ generated by $\{T_uT_u^* - T_vT_v^* : v - u \in I^+\}$, and Θ is defined by $\Theta(a)(\gamma) := Q_I(\alpha_\gamma^{-1}(a))$.

Another direction of generalising the Toeplitz algebra $\mathcal{T}(\Gamma)$ is involving the cocycle σ on Γ as studied by Ji in [13] for the case Γ is a dense subgroup of $\mathbb{R}$. Given a cocycle σ on $\mathbb{R}$, he considered the isometries $\{V_x : x \in \Gamma^+\}$ on $\ell^2(\Gamma^+)$ defined by

$$V_x f(y) = \begin{cases} \sigma(-y, x) f(y-x) & \text{if } y \geq x \\ 0 & \text{else,} \end{cases}$$

and stated that $\{V_x : x \in \Gamma^+\}$ generates a canonical algebra; the twisted

Toeplitz algebra $\mathcal{T}(\Gamma, \sigma)$. His short exact sequence

$$0 \longrightarrow \mathcal{C}_\Gamma^\sigma \longrightarrow \mathcal{T}(\Gamma, \sigma) \longrightarrow C^*(\Gamma, \sigma) \longrightarrow 0 \tag{I.3}$$

involves a twisted commutator ideal $\mathcal{C}_\Gamma^\sigma$ and the twisted group C^*-algebra $C^*(\Gamma, \sigma)$.

We want to generalise the results of [4] and the apply to the twisted Toeplitz algebra $\mathcal{T}(\Gamma, \sigma)$.

Here we also prove a twisted version of Adji's short exact sequence (I.2). For this purpose we specialise on the cocycle which is inflated from quotient group Γ/I for an order ideal I in Γ. We use a different technique to Adji, our method is more direct and shorter than of Adji. We use Lemma **??** instead of the characterisation of induced C^*-algebras due to Echterhoff.

I.5.2 The Ideal Structure of Toeplitz Algebra

As we discussed earlier that in the case Γ is a subgroup of $\mathbb{R}$, Douglas proved that the commutator ideal $\mathcal{C}(\Gamma)$ of $\mathcal{T}(\Gamma)$ is simple, moreover Ji and Xia in [14] proved that $\mathcal{C}(\Gamma)$ is the only nonzero simple ideal in $\mathcal{T}(\Gamma)$.

In a more general case Murphy in [23] analysed the ideal structure of $\mathcal{T}(\Gamma)$ for finitely generated abelian group Γ of order n, therefore Γ is isomorphic to a lexicographic direct sum of subgroups of $\mathbb{R}$ [23, Theorem 2.1]. He stated explicitly the necessary and sufficient condition of Γ in order $\mathcal{T}(\Gamma)$ to be type I is that Γ must be isomorphic to $\mathbb{Z}^n$.

In the most recent paper, Adji and Raeburn in [5] investigate the ideal structure of Toeplitz algebra $\mathcal{T}(\Gamma)$ of a totally ordered abelian group Γ. They considered the problem of describing the ideal structure of $\mathcal{T}(\Gamma)$ for nonfinitely generated totally ordered group, hence is a generalisation of earlier Murphy's work. The crucial ingredient in their analysis is the set $\Sigma(\Gamma)$ of ordered ideals, which is itself a totally ordered set under inclusion. They showed that each irreducible representation of $\mathcal{T}(\Gamma)$ factors through an irreducible representation of $\mathcal{T}(\Gamma/I)$ for some order ideal $I \in \Sigma(\Gamma)$, and therefore they found that primitive ideals $\operatorname{Prim}\mathcal{T}(\Gamma)$ of $\mathcal{T}(\Gamma)$ can be parametrised by the disjoint union $X := \bigsqcup\{\hat{I} : I \in \Sigma(\Gamma)\}$ [5, Theorem 3.1]. Furthermore for the case $\Sigma(\Gamma)$ is isomorphic to a subset on $\mathbb{N} \cup \{\infty\}$ they proved in Proposition 4.7 that the topology in $\operatorname{Prim}\mathcal{T}(\Gamma)$ can be described through the topology on X they defined in Definition 3.1.

Motivated by Adji and Raeburn's results, this thesis gives extensions of previous results. We prove the previous results for a more general situation. Here we assume that $\Sigma(\Gamma)$ is well-ordered instead of order isomorphic to a subset of $\mathbb{N} \cup \{\infty\}$ as was used in [5].

Chapter II

Twisted Toeplitz Algebra $\mathcal{T}(\Gamma, \sigma)$

Chapter III

Twisted Toeplitz Algebra $\mathcal{T}(\Gamma, \inf \sigma)$ of Inflated Cocycle

Chapter IV

The Corresponding Topology for the Hull-Kernel Topology in the Primitive Ideal Space of Toeplitz Algebras

Chapter V

Conclusions and Suggestions for Further Research

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Curriculum Vitae

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