

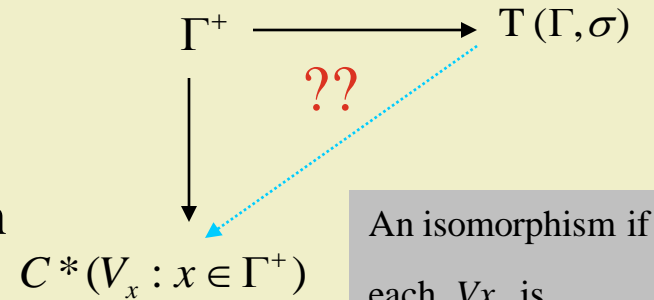
The Universal property of Twisted Toeplitz Algebras

Corollary II.15

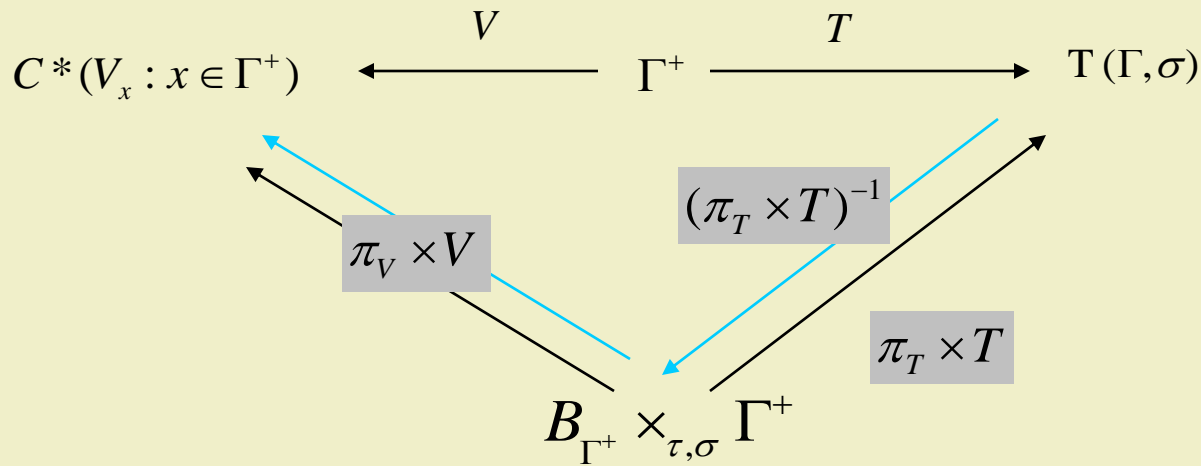
Γ : totally ordered abelian group, σ : cocycle on Γ .

$T(\Gamma, \sigma)$ is universal for isometric- σ representation of Γ^+ .

Proof :



An isomorphism if each V_x is nonunitary



The Universal property of Twisted Toeplitz Algebras

Corollary II.16

σ, ω are cocycles on Γ such that $[\sigma] = [\omega]$, then $T(\Gamma, \sigma) \cong T(\Gamma, \omega)$.

Proof :

$$\begin{array}{ccc} T(\Gamma, \sigma) & \xleftarrow{\quad} \Gamma^+ \xrightarrow{\quad} & T(\Gamma, \omega) \\ \uparrow \quad \downarrow & & \uparrow \\ B_{\Gamma^+} \times_{\tau, \sigma} \Gamma^+ & \xrightarrow{\quad} & B_{\Gamma^+} \times_{\tau, \omega} \Gamma^+ \end{array}$$

Invariant Ideals

Theorem II.20

A : C^* -alg, Γ : totally ordered abelian group, $\alpha: \Gamma^+ \rightarrow \text{Endo } A$ action,

I : extendibly α -invariant ideal of A ,

\therefore there is an exact dp. barisan eksak pendek

$$0 \rightarrow I \times_{\alpha|_I, \sigma} \Gamma^+ \rightarrow A \times_{\alpha, \sigma} \Gamma^+ \rightarrow A/I \times_{\bar{\alpha}, \sigma} \Gamma^+ \rightarrow 0.$$

Proposition II.21

Terdapat barisan eksak pendek

$$0 \rightarrow B_{\Gamma^+, \infty} \times_{\tau|_{B_{\Gamma^+, \infty}}, \sigma} \Gamma^+ \rightarrow B_{\Gamma^+} \times_{\tau, \sigma} \Gamma^+ \rightarrow (B_{\Gamma^+} / B_{\Gamma^+, \infty}) \times_{\tau, \sigma} \Gamma^+ \rightarrow 0.$$

Corollary II.22



Tdp. barisan eksak pendek

$$0 \rightarrow C(\Gamma, \sigma) \rightarrow T(\Gamma, \sigma) \rightarrow C^*(\Gamma, \sigma) \rightarrow 0.$$

Struktur dari $T(\Gamma, \text{inf } \sigma)$

Theorem III.1.

I : ideal urutan dr Γ , σ : kosikel pd Γ , $C(\Gamma, I, \text{inf } \sigma)$ ideal dr $T(\Gamma, \text{inf } \sigma)$ dibangun $\{1 - T_x T_x^* : x \in I^+\}$.

\therefore Tdp. barisan eksak pendek

$$0 \rightarrow C(\Gamma, I, \text{inf } \sigma) \rightarrow T(\Gamma, \text{inf } \sigma) \rightarrow \text{Ind}_{\perp}^{\hat{\Gamma}}(T(\Gamma/I, \sigma), \alpha^{\Gamma/I}) \rightarrow 0.$$

Sketsa Bukti

Menggunakan sifat universal alj. Toeplitz

$$V : \Gamma^+ \rightarrow C(\hat{\Gamma}, T(\Gamma/I, \sigma)), V_s(\gamma) := \gamma(s) T_{q(s)}^{\Gamma/I} \text{ rep. isometri-}\sigma$$

\Downarrow

tdp. hom. $\phi_V : T(\Gamma, \text{inf } \sigma) \rightarrow C(\hat{\Gamma}, T(\Gamma/I, \sigma)), \phi_V(T_s) = V_s.$

1. ϕ_V surjeksi pada $\text{Ind}_{\perp}^{\hat{\Gamma}}(T(\Gamma/I, \sigma))$: Lemma III.4

2. $\ker \phi_V = C(\Gamma, I, \text{inf } \sigma)$



Adji (2000): Produk silang, ideal invarian

$$0 \rightarrow C_{I^+} \times_{\tau} \Gamma^+ \rightarrow B_{\Gamma^+} \times_{\tau} \Gamma^+ \rightarrow (B_{\Gamma^+} / C_{I^+}) \times_{\tau} \Gamma^+ \rightarrow 0.$$