TEACHING MATHEMATICS BY USING ABDUCTIVE-DEDUCTIVE STRATEGY FOR IMPROVING STUDENTS' PROOF CONSTRUCTION ABILITY

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Abstract

Many students of all levels of education face serious difficulties with constructing mathematical proof. Some methods of proving such as generic proof, structured proof, WWHWT, and heuristic answer example had been developed. Nevertheles, they didn't overcome the students' difficulties on constructing the steps of logical proof. A quasi experiment was conducted to improve students' proof construction by using abductive-deductive strategy. The study involved 128 students of mathematics and mathematics education program, and four kinds of instruments those were a prior math ability test, proof reading test, a proof construction test, a self regulation scale. The study found that proof construction process was more difficult than proof reading for all students. Furthermore, students of mathematics education program performed better on proof reading ability than that of students.of mathematics program. However, there was no significant difference of proof reading ability between students.of both learning approaches. In relation to proof construction ability, students of abductive-deductive strategy class were leading than that of conventional teaching, but there was no significant difference of students' ability of both study program. The study also found that in both study programs and prior mathematics ability performed consistent role on developing proof reading and proof construction ability. The higher students' prior mathematics ability so the higher students' ability of proof reading and proof construction as well. Other study's findings was no significant difference on self regulated learning of students looking at it from study program, teaching approach and prior mathematics ability, and all of them were classified as fairly good. Beside that, study found not consistent interaction among students' prior math ability, study program, and teaching approach toward students' proof reading and construction ability, and self regulated learning.

Key word: abductive-deductive strategy**,** mathematical proof, premise, warrant, claim, mediate target, end target, conclusion, self regulated learning

A. Background

The opinion of mathematics education experts toward the necessity of introducing mathematical proof to be thought at high school levels was increased. Between 1970 and 1980 a number of mathematics teachers in America conducted intensive discussion about whether mathematics proof should be included or excluded in senior high school mathematics curriculum. The teachers said that actually mathematical proof had been developed in a topic which stress on formal aspects, but it was lack to focus on understanding mathematics (Hanna, 1983). The argument was developed so that National Council of Teachers of Mathematics (NCTM, 1989) stated that (1) it was not necessary that deductive proof to be taught at high school level because heuristics technique was more worthwhile for the students to develop they reasoning and justification than the deductive proof, and (2) the deductive proof was only for students who will pursue their study to university level.

Moore (1994) stated that the reason of freshman's difficulties on proving was because their proof experience of high school mathematics was limited only on constructing geometry proof. Whereas, the limitedness on proving ability would influence on learning other advanced mathematics such as real analysis, abstract algebra, and others. That condition would hamper the development of students' reasoning and others mathematical thinking abilities. Latter, NCTM (2000) recommended that mathematics proof as an essential mathematical process should be a part of the high school mathematics curriculum, and should be taught carefully so that the proof constructing ability of high school students will give positive effect on that ability in university level. Likewise, Sabri (2003) suggested that mathematical proof ability should be tought to preservice mathematics teacher students so that they were able to prepare themselves on teaching mathematical proof.

Those arguments above stimulate the researcher to conduct a study for overcoming the students' difficulties with constructing mathematical proof and for improving the development of their mathematical thinking level either for mathematics students or for preservice mathematics teacher students.

An alternative teaching approach that give opportunity to the students for developing their mathematical proof ability was abductive-deductive strategy. This strategy begun with presenting a problem situation, and then students were asked to elaborate the given information and facts. In this strategy, the problem should be able to help the students to understand all involved mathematical ideas and to look for the relation among them. Lecturer should motivate the students for conducting transactive reasoning such as to critictize, to explain, to clarify, to justify, and to elaborate the proposed opinion that inisiated by the lecturer or students. In order to each students involved actively in the transactive discussion, they should have a relevant prior mathematical ability, so that each new idea was able to be developed step by step and formed a comprehensive mathematical concept.

This proposed study was going to examine the effectivity of abductive-deductive strategy in improving proof reading ability and proof construction ability of mathematics students and mathematics education students. It was also expected that students' prior mathematics ability and self regulated learning (SRL) influenced toward attainment of students' ability on mathematical proof. reading and construction.

B. Theoritical Review

1. The Proof Reading and the Proof Construction Abilities

The objective of developing proof metodology was to improve students' ability on understanding mathematical proof, and proof constructing of mathematical statements. Some approaches had been developed, among them was concept of generic proof (Tall, 1991). Generic proof method of example level was an explaination of a concept in general based on a specific example or case. This condition was exactly different with proof in general that asked an abstraction in higher level. Then, Leron (Tall, 1991) proposed a structured proof which combined a formal and informal presentation of proof. The main objective of structured proof was not for convincing the truth of a statement, but for helping the reader to develop their understanding of the ideas beyond the proof, and the relationship among other mathemarics ideas.

Later, Reiss and Renkl (2002) offered concept of heuristic answer example that accompany with overview of an example that not only as a proof of the example but also the aspects of a proof in general. The steps of heuritic answer example as follow: (1) explore the problem situation, (2) formulate a conjecture, (3) gather information for investigating the conjecture, (4) proof the conjecture, and (5) examine the truth of the proof. All of those proof strategies didn' t explain explicitely how to show the main idea of the structure of a proof, either for understanding the proof or for constructing a proof. Furthermore, Uhlig (2003) developed an approach for understanding and constructing a proof in elementary linear algebra course. This course was considered as a transition course from informal proof to deductive proof Definition-Lemma-Proof-Theorem-Proof-Corollary. In order to prepare mental and emotional of students in facing a series of deduction. Uhlig proposed an approach of proving by using exploration intuitively toward the statement to be proved by asking as follow: (1)**W**hat happens if ? (2) **W**hy does it happen ? (3) **H**ow do different cases occur ?(4) **W**hat is true here ? By using those explorative questions it was hope that students' knowledge of the **T**heorems would be improved, and conceptual understanding as well.This approach was named as WWHWT

Toulmin (Pedemonte, 2003) developed a structure of argumentation. And then, Krummheuer (Hoyles & Kuhemann, 2003) analyzed argumentation by using Toulmin's argument form as in Figure 1.

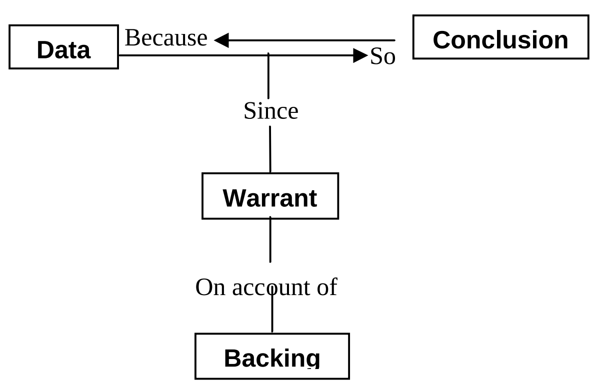


Figure 1 Schematic for analyzing argumentation Krummheuer (Hoyles & Kuhemann, 2003)

In relation to mathematical proof, statements in the proof was considered as a form of argumentation. In the argumentation of a mathematical proof, the data were premises and the warrant was definition or theorem. The schematic diagram could be used as a model for helping to read a proof of a mathematical statement, and by using modification, the schematic diagram could be used for constructing a mathematical proof.

Before student was able to construct mathematical proof, he should have sufficient proof reading ability. Sumarmo (2003) defined reading ability as ability to compile the core of information of a text A reader was said to understand a text, when he was able to express his mathematical idea in his own words orally or writenly. So, a reader not only to pronounce the text, but also to express the meaning of the text. Moesono (Sumarmo, 2003) identified four levels of reading ability those were: (1) literal reading, that was to get information for advanced understanding; (2) interpretatif reading that for drawing conclusion of a text in depth; (3) critical reading, which include to evaluate the core of a text, to compare ideas in text, and to draw conclusion from the result of the comparation; and the last (4) creative reading that was highest level of reading ability that to be able to compile a new idea, a new view, a new approach based on imagination toward the text.

For example, a student was able to read proof of a mathematical statement such as $p \Rightarrow q$ if he was able to identify the data (p) and the conclusion (q) of the statement, to form the connection among the data, and between data and conclusion by using a warrant; to guess a key concept that bridge between data and conclusion, to evaluate the rules of drawing conclusion from the given data or the attained data critically, and to be able to express idea and a mathematical process orally or writenly.

Krummheuer schematic diagram also helped to develop a model of informal mathematical proof strategy. In that schema conclusion either as a target-conclussion or as a mediate claim was utilized to draw conclusion deductively. That kind of argumentation was named as deductive argumentation. But, it was often difficult to find a guaranted warrant which produce a conclusion from unkonwn data. One way to find idea that direct to mediate claim was abductive strategy. Abduction was a drawing conclusion which began from an observed fact considered as a claim and a given rule that would bring into a required condition. The steps of those abduction were presented as follow:

$$\begin{array}{l} B \\ A \Rightarrow B \end{array}$$

The posible premis was A, and B was an observed fact (as a claim) and $A \Rightarrow B$ was a rule (as a warrant). Argumentation like this was named as abductive argumentation.

The argumentation.of proof by combining the two argumentation as in the proof of Theorm 1 was named as abductive-deductive argumentation. So, the steps of proving $A \Rightarrow B$ statement by using abductive-deductive strategy could be presented as follow.

$$\begin{array}{l} B \\ C \Rightarrow B \end{array} \qquad\qquad \begin{array}{l} A \\ A \Rightarrow C \end{array}$$

Then the posible premise was C in which C was a key concept that bridge of fact A and conclusion B.

The both argumentation as tools for setting the main idea of a proof of a statement. This atrategy reduced the formallity of the proof without decreasing the reasoning aspects of the proof. By this strategy, it was hoped that the students would understand easily the structure of the proof

In proving mathematics process, mathematical expression either from the lecturer or students would motivate to happen a transactive and facilitative discussion among lecturer and students. In this transactive discussion, students were demanded to use their transactive reasoning. Berkowitz (Blanton et al, 2003) defined transactive reasoning as abilities of criticizing, explaining, clarifyng, and elaborating an idea. While facilitative statement was lecturer's restatement or clarification of students' statement (Blanton et al, 2003)

In abductice-deductive strategy there were many transactive discussion and only limited facilitative lecturer's clarification. This learning situation provided to persist an interaction among lecturer and students for solving general problems or proof problems. That analysis supported opinion that abductive-deductive strategy as a good alternative strategy for developing students' proof constructing ability.

Beside proof reading ability, others prerequisite ability of proof construction were prior mathematics ability.That hypotesis was in line with the opinion that mathematics as a systematics knowledge, so before ones understoond a mathematical concept he or she should comprehend the prerequisite topics first. Furthermore, as proof construction was one of complex and high order thinking, so it was estimated that for constructing a proof ones should have high motivation such as self regulated learning (SRL). In fact SRL was an indirect object of learning process for attaining high order ability such to construct proof. Sumarmo (2004) defined SRL as planning and monitoring processes that involved cognitive and affective processes in solving academic task. The SRL was not mental or spesific academic ability but more than self directed for solving a task. There were three main characteristics of SRL, those were: 1) to plan for learning, 2) to select strategy and to excecute learning design, and 3) to monitor and to evaluating the learning result, and to compare it to the standard quality.

3. Related Studies

Some studies on proof ability of university students were reported by Alibert & Thomas (Tall,1991), Raman (2003), Tucker (1999), Weber (2002), and Moore (1994) Alibert & Thomas (Tall,1991) studied about the difficulties of students on understanding proof. They identified two kinds of students' difficulties those were: 1) "How do we include the main ideas through which we understand why the result is true at the same time as the necessary details to make it rigorous ?", and 2) How can we manage to make students see proof as a necessary step in the scientific process, alongside activities such as research, the formulation of conjectures etc. and not just as formal necessity required by the teacher, or as an answer given by the teacher in response to a question which the student may not have asked ?".

Rather different with Alibert & Thomas, Moore (1994) found seven source of students' difficulties on constructing a proof, those were: 1) Students were not able to explain a definition in their own words; 2) Students had limited understanding toward a concept; 3) Students' overview toward concepts was not sufficient; 4) Students were not able and interested on composing some examples by themselves; 5) Students did not know how to use definition in constructing a proof entirely; 6) Students were not able to understand and to use mathematical language and symbols; 7) Students did not know how to begin a proof .

Different analysis with the both studies above, Tucker (1999) suggested that first year students whom learnt Calculus should have been introduced on how to understand a proof, although not for constructing proof. The reason of Tucker was that proof were going to help students to understand concept and to belief the presented results. For constructing proof, Tucker suggested that the lecturer should sellect some topics which students had possibility to overcome a proof. In other study, Weber (2002) succeed to identify students' difficulties on constructing a proof. The fact was that students knew and were able to apply the given facts, nevertheles they were not able to construct a valid proof. It was becaused students had less strategic knowledge which need for selecting inferences derived from given facts so that arrive on the necessary end target.The strategic knowledge constituted of knowledge of proof technique, knowledge of the importance of and the usefull of a theorem, and knowledge of when a sintaxis strategic should been used. The sintaxis strategic could be brought about by loosening the given premises by using definition, and or manipulating relevant symbols to the given information.

In his study, Raman (2003) found a theoritical frame work of characterization idea which turned up by the students while they brought about the proof. Raman had identified three important ideas, those were 1) *Heuristic Idea*, such as empirical data or visualization of figure just only for understading the result of proof.; 2) *Procedural Idea,* which used in proving based on logical and formal manipulation to arrive at formal proof. This idea was proposed just for convincing and not for understanding that the statements was true; 3) *Key Idea,* which appeared in formal proof based on relevant rules. This idea not only for convincing the truth of the statement but also for indicating of understanding toward each step of proving.

Four studies (Arnawa, 2006, Herman, 2005, Juandi, 2006, Suryadi, 2005) were conducted not to investigate students' proof ability but to improve secondary and tertiery students' ability of higher order mathematical thinking by using various teaching approaches. Two studies, Suryadi (2005) by using direct-indirect approach and Herman by using problem based learning they succeed to improve yunior students' higher order mathematical thinking. The more detailed findings were reported by Suryadi that the students learnt by using indirect and modified indirect-direct approaches attained higher quality than that of conventional students. Similar to the findings of Suryadi, Herman reported that of problem based learning either of opened problem or stuctured problem both of them were more effective to develop students' higher order mathematical thinking.than that of conventional approach as well. Those indirect and combination direct-indirect approaches (Suryadi, 2005) and problem based learning (Herman, 2005) for improving high oder mathematical thinking of high school students, could be modified for improving university students' ability of proof reading and proof construction as well..

Similar findings of studies in university level were reported by Arnawa (2006) with teaching approach based on the action, process, object, and schema (APOS) theory and Juandi (2006) with problem based learning. The spesific characteristic of the APOS theory were that: (1) students' knowledge was constructed by students through the phases of APOS's mental construction; (2) using computer; (3) students learned in small group discussion; 4) using learning cycles laboratorium activity, class discussion, and exercises (ADE). Arnawa found that proof ability of students taught by using APOS theory approach was higher than that of conventional students. Similar findings also reported by Juandi with problem based on sudents' mathematical power as well.

C. Metodology

The purpose of this study was to examine the effectivity of abductive-deductive strategy (X) on developing students' proof reading ability (O_1) and proof constructing ability (O_2). The study was a quasi experiment with pretest-postest control group design as follow.

$$\begin{array}{llll} O_1 & X & O_1 & O_2 \\ O_1 & & O_1 & O_2 \end{array}$$

This experiment was carried out in number theory course at a state university in Bandung. This course had an unique characteristic that was involved many proof problems but they were not too abstract as in analyses or abstract algebra. So, number theory course was sufficient for implementing the abductive-deductive strategy which would bridge proof process from the less formal to the more formal proof. The number subject of this study was 128 students. They were selected from students of four classes of number theory course, two classes of mathematics program and the other two classes of mathematics education program.

The study involved two kinds of test, proof reading test and proof construction test, and two scales, students' opinion scale and SRL scale. Before the experiment was carried out, the both tests were tried out first, so that they fulfilled the sufficient characteristic of good instruments. The proof reading ability test and the proof constructing abilty test were written in essay form and they consisted successively of five (5) items and four (4) items. In order to minimize subjectivity in scoring process, the researcher provided a rubric as scoring guidance. The sufficient characteristic of the two scales were estimated by their content validity and judged by two experts. The opinion scale and SRL scale were writen in Likert model in four option answers: SA (strongly agree), A (agree), DA (disagree), and SD (strongly disagree), and they consisted successively of 20 and 30 statements. The hypotheses were tested by using two way anova analysis and t-test.

For ilustrating the instruments, in the following we presented some samples of the item tests and item scale statements.

1. Samples of Proof Reading Test Item

1) Read the following argument carefully.

Suppose a and b were whole numbers that $\gcd(a, b) = 1$ then

$$\gcd(2a + b, a + 2b) = 1 \text{ or } 3\text{”}$$

(note: gcd is the greatest common divisor)

The proof of the statement as follow.

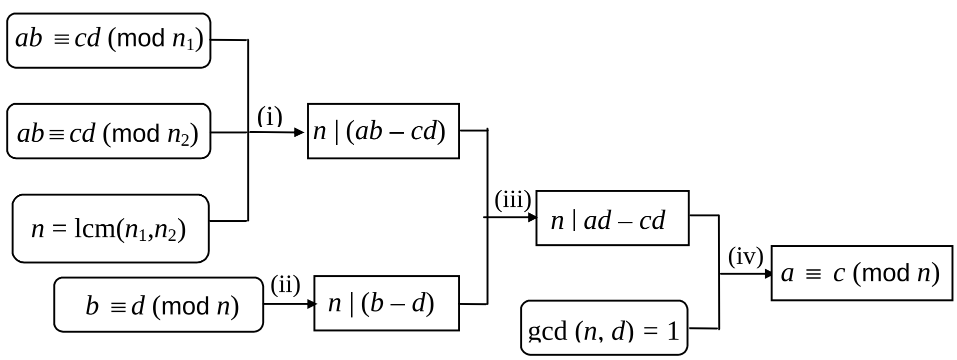
Suppose gcd($2a + b$, $a + 2b$) = d, so based on the definition of gcd, $d \mid (2a + b)$ and $d \mid (a + 2b)$. This expression result $d \mid 3a$ and $d \mid 3b$. And then based on alternative definition of it is found that $d \mid \gcd(3a, 3b)$ or $d \mid 3\gcd(a, b)$. But $\gcd(a, b) = 1$, so $d \mid 3$. Since $d > 0$ then the values of d are 1 or 3. So, $\gcd(2a + b, a + 2b) =$ 1 or 3.

By using similar argument, answer this problem.

If a and b are natural numbers that $\gcd(a, b) = 1$, determine the value of
$\gcd(2a + 3b, 3a + 2b)$

2) Observe this statement carefully.
Suppose a, b, c, d, n_1 and n_2 were whole numbers. If $ab \equiv cd \pmod{n_1}$, $ab \equiv cd \pmod{n_2}$, and $b \equiv d \pmod{n}$ then $a \equiv c \pmod{n}$ in which $n = \text{lcm}(n_1, n_2)$ with n and d are relatively prime
(note: lcm is the least common multiple)

The structure of the proof argumentation that statement is follow.



Where:
$\rightarrow$ denotes an implication, ▭ denotes a premise and ▭ denotes a conclusion (either end target or mediate target)
Explain each step of the proof above, from the begining (i) up to implication (iv).

2. Samples of Proof Construction Test Item

1) Observe this statement carefully.
Suppose a, b, c, d, n_1 and n_2 were whole numbers If $ab \equiv cd \pmod{n_1}$ and $b \equiv d \pmod{n_2}$ then $a \equiv c \pmod{n}$ in which $n = \gcd(n_1, n_2)$ with n and b are relatively prime". (note: gcd is the greatest common divisor)
   i) Write all premises of the statement above and its implication.
   ii) Write the conclusion of the statement and then by using definition and or theorem that you know for determining a condition in oder to find the conclusion.

2) Suppose a, b, m_1 and m_2 were whole numbers with $a \equiv b \pmod{m_1}$ dan $a \equiv b \pmod{m_2}$.

i) If $m = \text{lcm}(m_1, m_2)$, exhibit that $a \equiv b \pmod{m}$.

ii) If $\gcd(m_1, m_2) = 1$, exhibit that $a \equiv b \pmod{m_1 m_2}$.

(note: lcm is the least common multiple)

3. Samples of Opinion and SRL Scale Statements

1) Samples of opinion scale statements

No.	**Statements**	SA	A	DA	SD
1.	I like to participate in small group discussion				
2.	To learn proof construction is better individually than grouply				
3.	I like to solve task of mathematical proof				
4.	During a mathematics lesson, I prefer to listen and to make a note than to ask or to propose my opinion.				
5.	During learning in small proup, I learn much from my friends.				
6.	I can solve proof problem easily.				
7.	When teacher ask something to students, I wait better than I try to answer it.				
8.	The teacher's proof strategy motivate me to understand the concepts better.				
9.	I realize that task of proof is not memorise task.				
10.	I am brave to ask to my lecturer when I face difficulty				

2) Samples of SRL scale statements

No.	**Statements**	SA	A	DA	SD
1.	I learn only when I have tasks must be collected				
2.	I do exercise and task as want to my self				
3.	I don't know what have to prepare for my examination				
4.	I am satisfy when I got a C grade on my test				
5	I like to look for an illustration or an example for understanding a concept.				

D. Findings of The Study and Discussion

1. Students' PRA and PCA based on Study Program and Teaching Approaches

The attained proof reading ability (PRA) and proof construction ability (PCA) of students based on study program and teaching approaches as presented in Table 1.

Table 1. Mean and Standard Deviation of PRA and PCA Based on Kinds of Program and Teaching Approaches

Study Program	Teaching Approaches			
	Conventional		Abductive-deductive Strategy	
	PRA	PCA	PRA	PCA
Mathematics education	75.41 (12.80)	54.81 (15.74)	77.13 (16.22)	69.31 (15.97)
Mathematics	64.09 (13.15)	54.44 (13.36)	64.44 (12.52)	70.34 (12.30)
Total	69.75 (14.08)	54.62 (14.48)	70.78 (15.73)	69.83 (14.15)

Note: 1) PRA = proof reading ability 3) …… = Mean score 5) Ideal scor was 100
2) PCA = proof constructing ability 4) (…..) = Standard Deviation

The significancy of the difference of mean scores of PRA and PCA of students of mathematics education and mathematics programs and teaching approaches were tested by using two way anova analysis. The findings were presented in Table 2

Table 2 Two Way Anova Analysis PRA and PCA Based on Study Program and Teaching Approaches

Factor	PRA			PCA		
	F	**P**	**Ho**	**F**	**P**	**Ho**
Study Program	24,360	0,000	Rejected	0,017	0,898	Accepted
Teaching Appr	0,180	0,672	Accepted	35,536	0,000	Rejected
Interaction	0,080	0,778	Accepted	0,076	0,783	Accepted

Ho : No difference PRA or PCA among students' group based on study program and teaching approaches

Based on Table 2 it was concluded that of PRA, Ho was rejected for study program. It meant that there was significant difference between students' PRA of mathematics education program and students' PRA of mathematics program. In other words, study program had significant influence toward the attainness of students' PRA. In this case, students' PRA of mathematics education program was classified as good (75.41 and 77.13) and it was better than students' PRA of mathematics program that classified as mediocore (64.09 and 64.44). However, Ho was accepted for teaching approaches, it was meant there was no significant difference between students' PRA of conventional teaching and abductive-deductive strategy, and both of them were classified as fairly good.(69.75 and 70.78). This finding supported that teaching approaches had no significant influence toward the attainness of students' PRA.

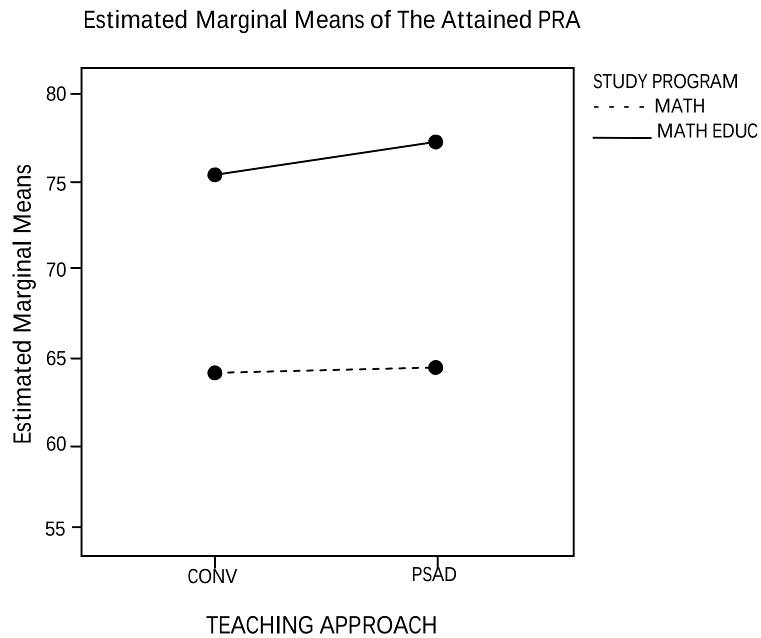


Figure 2 Interaction between teaching approaches and study program toward students' PRA

Moreover, it was found that there was no interaction between study program and teaching approaches toward the attainness of students' PRA. Interaction diagram between those variables toward students' PRA was presented in Figure 2. In this case, mathematics education program performed bigger role than that of teaching approaches on attaining students' PRA It might be because on daily teaching learning process, students of mathematics education were strived not only to understand a written text but also to explain the text to other students. In other words, those students used to communicate their ideas to each other. That ability precisely were measured in the proof reading test.

However, for proof construction ability (PCA), Ho was accepted for study programs, that was meant there was no significant difference between students' PCA, of prospective mathematics teacher program and students' PCA of mathematics program, and both of them were classified as fairly good. (69.31 and 70.34) and (75.41 and 77.13) In relation to the influence of teaching approraches toward students' PCA, Ho was rejected, it was meant that there was significant difference between students' PCA, of conventional teaching and abductive-deductive strategy. It was found that students' PCA, of abductive-deductive strategy was classifeid as fairly good (69.83) and it was better than students' PCA of conventional teaching that classied as mediocore (54.62) .Those findings could be explained as follow. Based on the curriculum, the both programs had the same mathematics courses as prerequisite courses before students took the number theory course (as an object of this study). So, all of the subject of this study had similar experience on mathematical proof, and this experience gave similar influence toward attaining proof ability on the later courses such as number theory course. So, it was understadable that students' proof ability was not influenced by study programs but was more influenced by the abductive-deductive strategy that designed specifically for developing students'PCA.This analysis supported opinion that on attaining students' PCA, the abductive-deductive strategy performed bigger role than the kinds of study program. Beside that, it was also found that there was no significant interaction between study program and teaching approaches toward the attainness of students' PCA,. Interaction diagram between those variables toward students' PCA, was presented in Figure 3.

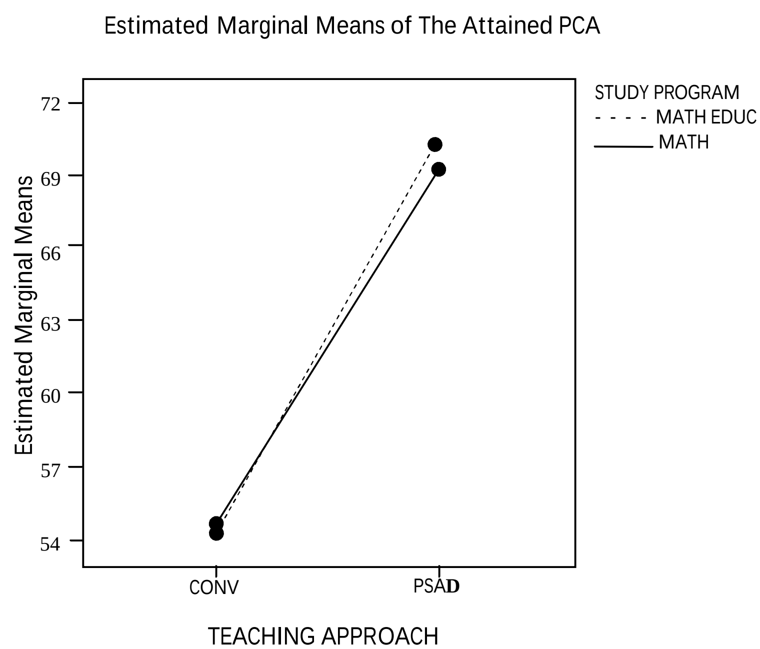


Figure 3   Interaction between teaching approaches and study program toward students' PCA,

2. Students' PRA and PCA based on Study Program and Prior Math Ability

The attainess of students' PRA and PCA based on study program and prior math ability were presented in Table 3. It were found that students' PRA and PCA of both study programs of high prior math ability was higher than that of students of medium prior math ability, and both of were higher than that of students of low prior math ability. It meant that prior math ability had great influence toward the attainess of PRA and PCA. This finding get along with the characteristic of mathematics as a systematic and structured study which understanding of prior mathematics proccess supported to the attainess of understanding of advanced mathematics process

Tabel 3. Mean score and standard deviation of PRA and PCA based on study program and prior math ability

Prior math ability	Study program			
	Mathematics Education		Mathematics	
	PRA	PCA	PRA	PCA
Low	63,06 (14,28)	50,38 (12,89)	56,94 (13,17)	56,62 (13,28)
Medium	76,59 (11,70)	63,72 (17,78)	64,09 (09,96)	60,53 (14,70)
High	88,81 (06,66)	70,44 (14,74)	71,94 (13,53)	71,88 (13,76)
Total	76,27 (14,11)	62,06 (17,34)	64,27 (12,74)	62,39 (15,05)

Note 1) ….. = mean score 3) Ideal score was 100
2) (….) = standard deviation

To test the difference of the mean of students' PRA based on study program and prior math ability by using two way analysis was presented in Table 4.

Table 4. Two way analysis of PRA based on study program and prior math ability

Factor	PRA		
	F	P	Ho
Prior math abiity	24,721	0.000	Rejected
Study programi	30,015	0.000	Rejected
Interaction	1,750	0.178	Accepted

Ho : no difference PRA between among students group of study program and level of prior math ability.

Based on Table 4, it was also found that there were not interaction between study program and prior math ability toward the attainness of students' PRA. Interaction diagram was presented in Figure 4.

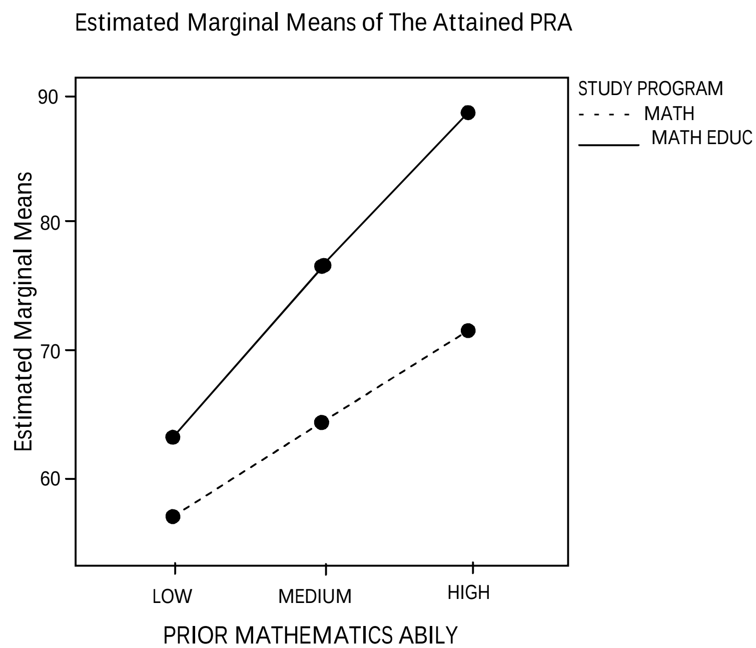


Figure 4 Interaction between prior math ability and study program toward PRA

To test the difference of the mean of students' PCA based on study program and prior math ability by using Tamhane test was presented in Table 5.

Tabel 5 Tamhane test of the difference of students' PCA among study program and prior mathaability

Prior math ability	Prior math ability	Tamhane-test			
		Mean Difference	Sig.	95% Confidence Interval of the Difference	
				Lower	Upper
Medium	Low	8,63	.021	1,05	16,20
High	Low	17,66	.000	9,28	26,04
High	Medium	9,03	.019	1,18	16,88

Ho : no difference PCA among levels of prior math abnility

Table 5 indicated that there was significant difference of students' PCA among levels of students' prior math ability. Furthermore, the difference of students' PCA between mathematics students and mathematics education students was tested by using t-test. The result was presented in Table 6.

Table 6. t-test of the difference of students' PCA between study program and level of prior math ability.

Level of prior mathe ability	t-test for Equality of Means				
	t	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
				Lower	Upper
Low	-1,35	0,187	-6,250	-15,699	3,199
Medium	0,781	0,438	3,188	-4,966	11,341
High	-,285	0,778	-1,438	-11,735	8,860

Ho : no difference of students' PCA ability between the two study programs and among level of prior math ability.

Table 6 pointed that there was no difference of students' PCA between mathematics program and mathematics education program. Furthermore, analysis of interaction between prior math ability and study program variables toward students' PCA was presented in Figure 5. The Figure 5 indicated that there was interaction between prior math ability and study program toward students'PCA

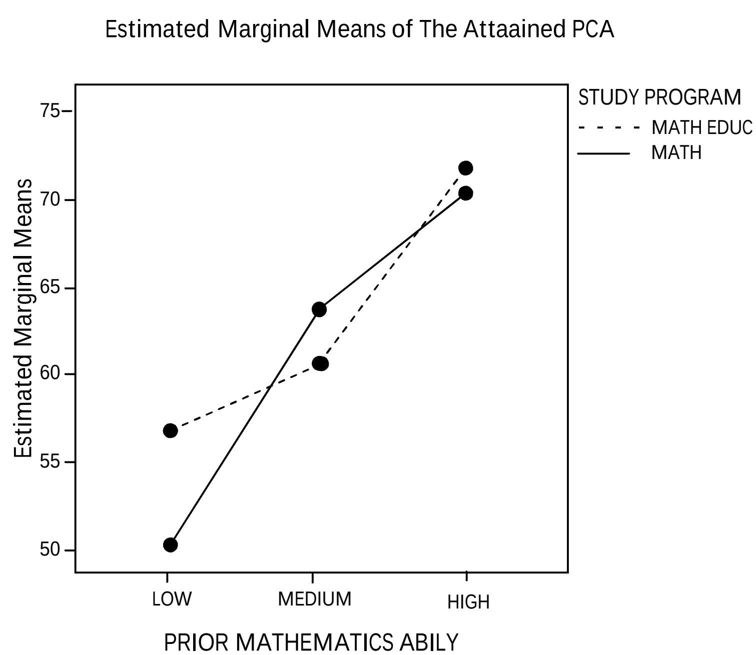


Figure 5 Interaction between study program and level of prior math ability toward PCA.

4. Students' Self Regulated Learning based on Study Program and Teaching Approaches

The study found students' self regulated learning based on study program and teaching approaches as reflected on Table 7.

Table 7. Mean score and standard deviation of students' self regulated Learning based on study program and teaching approaches

Study Program	Teaching Aproaches			
	Ccnvensional		Abduktive-Deduktive Strategy	
	Mean SRL	SD SRL	Mean SRL	SD SRL
Math. Education	69,47	6,42	70,72	4,00
Mathematics	70,56	6,59	69,72	7,23
Total	70,02	6,48	70,22	5,82

Note: SRL = self regulated learning (ideal score : 100)

Based on Table 7, there was no mean difference of SRL between students of conventional teaching and of abductive-deductive strategy and both of them were classified as fairly good (70,02 and 70,22 of 100). Furthemore it was found no mean difference of SRL between students of mathematics education program and mathematics program (69,47 and 70,56 of conventional teaching; and 70,72 and 69,72 of Abduktive-Deduktive Strategy) all of them were classified as fairly good. Students' SRL did not influenced by study program and teaching approaches. Those analysis was tested by using two path anova as presented in Table 8.

Table 8. Two path anova of SRL based study program and teaching approaches

Factor	Students' self regulated learning		
	F	P	Ho
Study program	0,035	0,853	Accepted
Teaching approaches	0,002	0,966	Accepted
Interaction	0,917	0,340	Accepted

Ho : no mean difference of students' SRL based on study program and on teaching approaches

Furthermore it was aslo found no inteaction between study program and teaching approaches toward students' SRL Interaction diagram was presented in Figure 6.

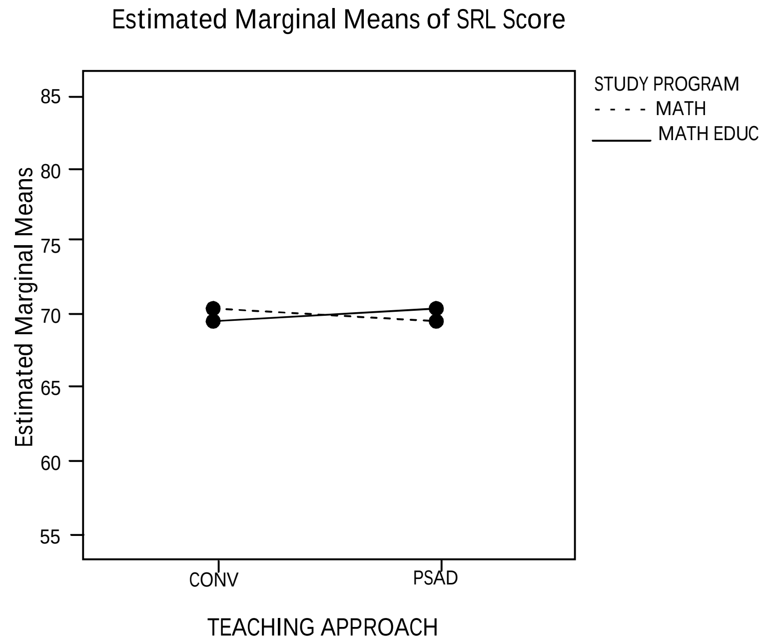


Figure 8 Inteaction between study program and teaching approaches toward students' SRL

Table 9. Mean score and standard deviation of students'SRL based on prior math ability and teaching approaches

Prior Mathematics Ablity	Teaching approaches			
	Conventional		Abductive-Deductive strategy	
	Mean SRL	SD SRL	Mean SRL	SD SRL
Low	67,69	6,954	66,88	6,531
Medium	70,22	7,024	70,69	4,714
High	71,94	4,024	72,63	5,920
Total	70,02	6,479	70,22	5,819

Note;: ideal score = 100

Analysis of students' SRL based on students' prior math ability was presented in Table 9. The findings showed that there was significant difference of students' SRL based on level of student prior mathematics ability. The higher the students prior mathematics ability then the higher students' SRL. It was meant that students' prior mathematics ability influenced the attainments of students' SRL. Nevertheles, there was no significant difference of students SRL based on conventional teaching and Abductive-Deductive Strategy and there was no interaction between study program and prior mathematics ability toward SRL, and between teaching approaches and prior mathematics ability. The result of the analysis was presented in Table 10.

Table 10. Two path analysis of SRL based on prior mathematics ability and teaching approaches

Factor	Self Regulated learning		
	F	P	Ho
Prior mathematics ability	5,809	0,004	Rejected
Teaching approaches	0,011	0,918	Accepted
Interaction	0,158	0,854	Accepted

Ho : no difference of students' SRL between Prior mathematics ability and Teaching approaches

Interaction diagram between prior mathematics ability and teaching approaches towrd students' SRL presented in Figure 9.

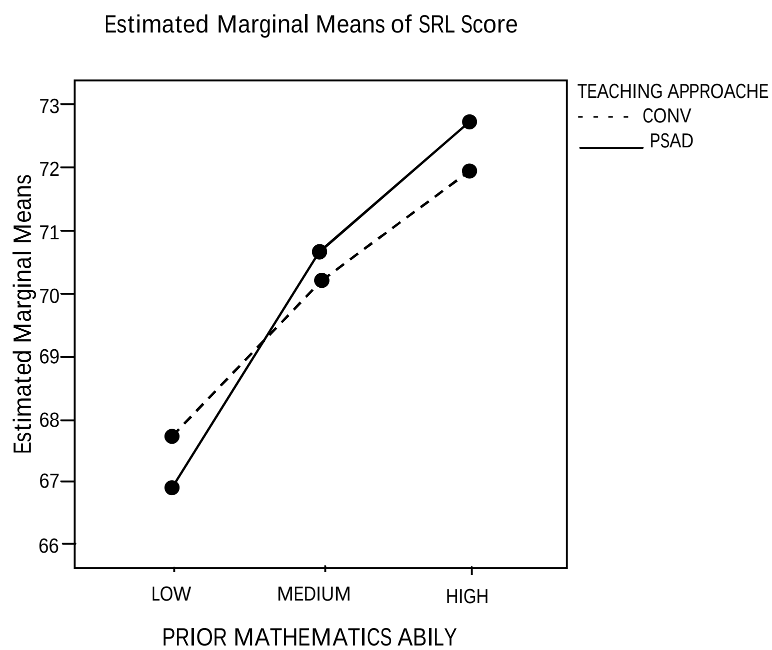


Figure 9   Interaction between prior mathematics ability and teaching approaches towrd students' SRL

E. CONCLUSSION AND IMPLICATION

1. Conclussion

Based on the findings and it analysis, the study drew some conclusion as follow.

There was no difference quality of students' proof reading ability (PRA) of conventional teaching and of abductive-deductive strategy, and both of them were classified as fairly good. However in relation to study program, on both teaching approaches, students' PRA of mathematics education program were classified as good and it were higher than students's PRA of mathematics program that classified as mediocore. The findings of this study supported that on attaining students's PRA, mathematics education program had bigger role than mathematics program and the both teaching approaches.

While on proof construction ability (PCA,), and on both study programs, students of abduktive-deductive strategy attained fairly good quality and those were higher than that of students of conventional teaching. Nevertheles in relation to conventional teaching, there was no difference quality of students' PCA, between mathematics education program and mathematics program, and both of them were classified as mediocore. Likewise, in relation to abductive-deductive strategy there was no difference quality of students' PCA, between prospective mathematics teacher program and mathematics program, and both of them were classified as fairly good. Contradiction with the conclusion about students' PRA, the study concluded that on attaining students' PCA, the abductive-deductive strategy performed bigger role than conventional teaching and the both of study programs.

In relation to the level of students' prior math ability it was concluded that the higher prior math abiliity of students then the higher students' PRA, students' PCA and students' SRL on both teaching approaches as well. Toward the attainment of students' PRA, students' PCA, and SRL the influence of prior math ability was the biggest than that of study program. and teaching approaches.

In relation to interaction among variables, the study concluded that there were not interaction among study programs and teaching approaches toward attaining students' PRA and students' PCA. Likewise, there were not interaction among study programs and students' prior math ability toward attaining students' PRA and students' PCA.as well.

2. Implication

Based on the analysis of this study, the abductive-deductive strategy performed more effective on developing students' proof reading ability (PRA) and students' proof constructing ability (PCA) either in prospective mathematics teacher program or in mathematics program.. The abductive-deductive strategy constituted as a schematic model of students' learning activities on process of forming new mental mathematical object when they carried out proof of a mathematical statement.

In the framework of Action Process Object Schema (APOS) theory, the abductive-deductive strategy could be ilustrated as in Figure 4. The problem of proof could be simplified operasionally to become a problem of how to show the truth of the demanded end target based on a set of given information that involved in the data. The data and end target constituted two mental objects that posed to the students. In general there were two action could be carried out directly when we posed with proof problem. Fisrtly, examine each given information of the data, and then compile them so that we got medite targets, and from those targets were analyzed again for getting the next medite targets, and so forth so we got the end target. Those medite targets constituted other mental object that might have been possessed by the students in advance. .

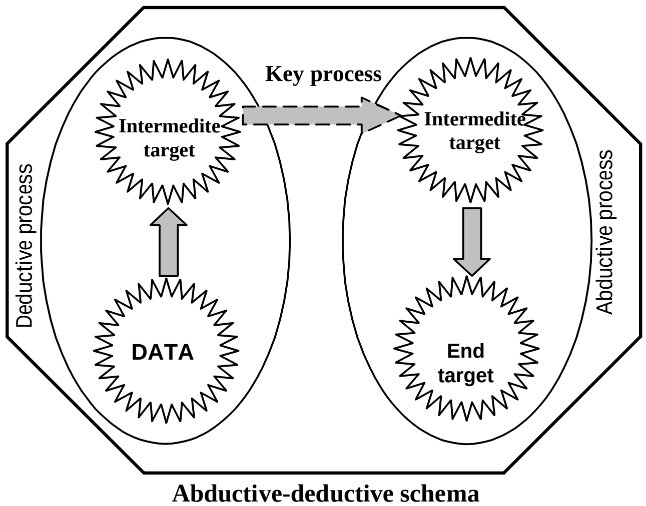


Figure 10 Model of abductive-deductive strategy on framework of APOS theory-

The process of getting medite target from given data as above constituted deductive process in PSAD. While the second action was to analyze the demanded end target and looked for a medite target so that by using definition and theorem we got the end target. The process of conditioning medite target constituted the abductive process in PSAD. The other step process in PSAD was carried out mental action that bridged.medite target of deductive process with intermediate target of abductive process. Becaused of this process determined the success of mathematical proof, so the process was named as key process.

In teaching-learning situation, lecturer's intervention should have motivated students to carried out mental actions so that they were able to carry out the three kinds of process. Transactive and facilitative statements or scaffolding technique could be implemented so that transactive discussion went on to form new mental objects especially related to the development of proof ability.

Schema of proof construction process that built by abductive and deductive process would increase continually as long as the level of complexity of the relation between the mental objects of presented proof problem. The development of the schema would motivate the students' actual development and potential development to the higher level. In general the relation of schema development, actual development and potential development was named as Zone of Proximal Development (ZPD) that was illustrated in Figure 11.

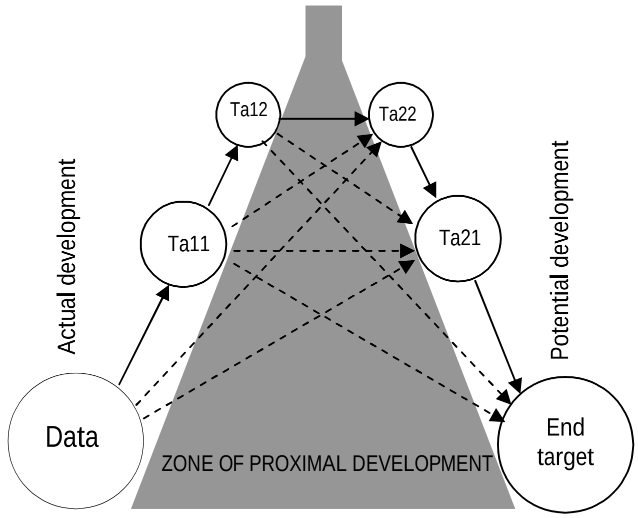


Figure 11 Model of development of ZPD by forming schema in PSAD

Proof problems that presented in abductive-decuctive strategy would motivate to form a schema which developed from given data and demanded end target. Students' actual development could be increased by deductive process and optimalized their knowledge of the given data for constructing the mental objects Ta11, Ta12, . . . , Ta1*n*..Whereas the students' potential development would increase when they formulated the mental objects Ta21, Ta22, . . . , Ta2*n* by interaction with other higher ability students. Then the new schema would be formed when they carried out mental action at key process level which it bridged mental objects Ta1*i* with one of mental objects Ta21, Ta22, . . . , Ta2*n*. The more complex relation among the mental objects Ta1*i* with mental objects Ta2*j* then the formed proof schema became more complex. This condition trigered the development of students' ZPD to the higher level.

In relation to the students' difficulty with proof constructing that identified by Moore (1994), the development of ZPD by forming schema in PSAD would decrease the students' difficulty with understanding of a concept and on beginning to construct proof. Definition of a concept was identified by representing a statement in $p \leftrightarrow q$. form. This form of definition was used to prove $r \rightarrow s$ statement by formulating p from relevant premise r. Illustration of a concept, either from a definition or from a theory was presented in the form of abductive-deductive argumentation. While intuition of concept understanding would develop in line with understanding toward the structure of abductive-deductive argumentation of the concept. Thus, the difficulty on beginning a proof could be overcome by to identify the data with its implication, and the end target with formulation of the posible intermediate target for reaching at the end target.

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