THEORETICAL FRAMEWORK OF MATHEMATICS PROOF FOR S1 UNIVERSITY STUDENT

By

Kusnandi

Department of Mathematic Education FPMIPA UPI

ABSTRACT

Competence of proving insists student on analyzing and elaborating premises and conclusion, and also he should make connection between both. Kusnandi (2008) had developed learning model by abductive-deductive strategy to promoting reading and proving ability for the first year students in university. Based on theoretical research on structure of proof, the strategy with little extension also can be applied effectively on advanced subject such as analysis real and algebra abstract with the more operational problems. The extension strategy can be identified to be two kinds, namely knowledge of initial strategy and existential of key concept. The first one contain the knowledge of an indirect proof, a construction proof, and how to prove the conclusion that contain the quantifier "for every", statement "*p* or *q*", and the others. The initial strategy is very important as the first step in proving. Without this, it is very difficult to prove the conclusion. The second one often is met when we construct the existential an object. It is rather difficult to find ideas of the key concept. Only student who studies much in proving can appear the key concept.

A. Introduction

Developing mathematics education program that focus at increasing of ability mathematics think, is representing a compulsion to face various emulation and challenge in this globalization era. Ability to mathematics think is phases start at reproduce phase as the lowest step until analyses and connection think as higher-level step. Ability to prove on mathematics, claim the university student to elaborate and analyses that given fact, either through of premise, and the fact on conclusion. University student also claimed able to make a connection between both of the fact.

Curriculum of Mathematics Education Majors have presented various area of mathematics study which is spread at analyzes study group area, algebraic, computing and statistics, with the ability to demand the mathematics think that different each other. Development specifics study at algebra and analyses area lessons owning specification which relative is not differ. Discussion of that study, first defining an object. Properties (theorem) from object that defined, derived from definition based to order that had known previously and or through *lemma* that had known beforehand. Special things from object in theorem can be identified and yield a *corollary*.

Proof of a lemma, corollary, and theorem, which is presented in textbook is developed in deductively from premise to conclusion that oftentimes not easy to comprehending in comprehensively, particularly for beginner university student that learning to prove. University student difficulty in proving is representing a public symptom met, in either domestic university student (Sabri, 2003; Juandi, 2006; Arnawa, 2006) and also foreign university student (Moore, 1994; Tucker, 1999; dan Weber, 2002).

Kusnandi (2008) have developed a study of abduktive-deductive strategy (PSAD) to earn developing and growing ability read the proof and ability to prove learn the proof at beginner university student, with the framework like figure 1 follows.

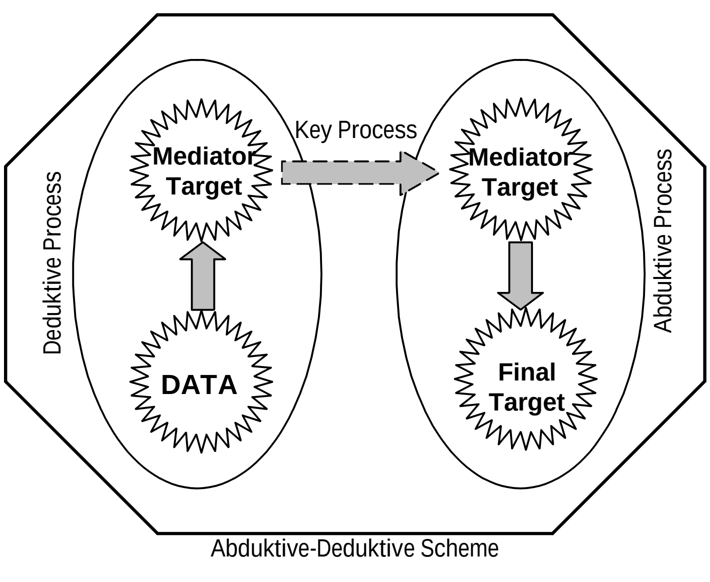


Figure 1 Abduktive-Deduktive Strategy Framework Model

This abduktive-deduktive strategy framework model is very effective for developing and growing ability prove on beginner student university to learn the proof, with the problem limited with implication form $p \Rightarrow q$ at lessons which its items not yet abstract as algebra group lessons or analysis. From this form, student uiversity is claimed to elaborate p data and final target q with the process like abduktive-deduktive scheme above.

For the lessons that more abstract, problem of proof not merely form of implication $p \Rightarrow q$, but also other form with the difficulty level can be more higher than previously. Form of proof problem such as problem show the existence of an object, proof with the contradiction, proof by contra positive, etc.

Based on above description, considered necessary to extend the effectiveness of strategy abductive-deduktive at the other proof problem in lessons, which is the topic of its study more abstract. So that its result expected compiled the framework of mathematics proof, process in general which can facilitate the mathematics lecturer to be implementation on the class.

B. Abduktif-Deduktif Proof Strategy

Toulmin (in Pedemonte, 2003) proposed a model describing the structure of the argumentation. In any argumentation the first step is expressed by standpoint (an assertion, an opinion). In Toulmin's terminology the standpoint is called the claim. Then, the claim that be expressed should be supported by the data. The Relation between the data and the claim is justified by a warrant. *Data – warrant – claim* is a base structure of argumentation. An auxiliary element such as backing is required when the authority of the warrant is not accepted straight away. The basic structure Toulmin's of the argumentation is used Krummheuer (in Hoyles & Kuhemann, 2003) for analyzing arguments. Schematic diagram is described by the following figure 2.

In mathematical proof, we shall consider the statements in every proving step as one of the argumentation form. As data are premises, and warrant are definitions or theorems. The schematic diagram will be used as tools to understand a proof and to construct mathematical proof informally. We use an

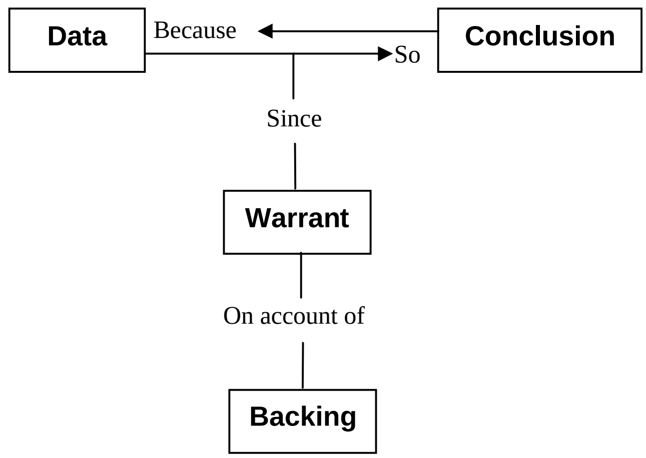


Figure 2 Skematic for analyzing arguments

inference rule deductively to get conclusion from the data when we construct the proof. So, the argumentation like that ia called a deductive argumentation.

However, we often face the problem that warrant which garante to get a conclusion from data has not been thought. One way to appear the idea toward a claim is by using an abduction. An abduction refers to an inference starting from an observed fact, and a given rule, led to a conclusion. An abductive step can be represented in the following way:

$$B$$
$$A \Rightarrow B$$
$$\therefore \text{ A is more probable}$$

where B is an observed fact, and $A \Rightarrow B$ ia a rule (as warrant). The argumentation like that is called abductive argumentation.

Argumentation which is done by combining both of the argumentation will be called abductive-deductive argumentation. So, the steps prove the statement form $A \Rightarrow B$ by using abductive-deductive can be represented in the following way

B	A
$C \Rightarrow B$	$A \Rightarrow C$
$\therefore$ Premis which is more probable is C	$\therefore$ C

where C is key concept that act as bridge between the fact A and conclusion B.

The argumentation models in the mathematical proof above are not process of writing proof model, but only be model to link to understand mathematical statement and how to construct the proof of the statement.

C. Applying The Strategy of Abductive-Deductive Proof at Algebra Structure and Analyses Area

Based on theoretical analysis toward proof structure of both study area, algebra structure and real analyze, visible that proof strategy by abductive-deductive can be applied on proving problem through deductive process, abductive process, or abductive-deductive process. Uppermost difference among the proving problem for the beginner university student with the proving problem at algebra structure and analysis area can be elaboration as follows:

1. Type of proving problem both in study area, especially at real analysis area, is more varying. This matter claims the initial strategy knowledge in proving it. That proving problem types among other things is:

   a. *Problem with conclusion "p or q" form.*

   Considering the conclusion that must shown is the truth is one from p or q, so the initial strategy to show is by taking example p not true, then show the truth from q. Truth from q can be processed through deductive process, abduktive or abduktive-deduktive. As illustration, please look proving problem at following example.

   “Let a and b be real numbers with $ab = 0$. Prove that $a = 0$ or $b = 0$.”

   Initial strategy: let $a \neq 0$, we have to show that $b = 0$. Furthermore is processing the proving deductively, abductively, or abduktive-deduktive with the fact owned:

PREMISE	CONCLUSION
P1: $ab = 0$ **P2:** $a \neq 0$	**C**: $b = 0$

In the statement of "*p or q*" is not close the possibility that both correctness. Therefore, the proving process is done by showing truth *p* and *q*.

b. *Problem with conclusion that load kuantor "for each" or "for all" expression.*

Initial strategy needed to prove a statement that containing this expression is so importance so that given premise could be used, organized, and aimed to expected conclusion. To see the initial strategy that required and how its link with given premise, please look at following example:

> *"Let S be a nonempty subset of real number R, and u = sup S. Show that for each* $\in\ > 0$ *there exists an* $s_\varepsilon \in S$ *such that* $u - \varepsilon < s_\varepsilon$*."*

Initial strategy to prove "for each $\in\ > 0$ there are $s_\varepsilon \in S$ so that $u - \varepsilon < s_\varepsilon$" Is take any $\in\ > 0$. In this case, we work only with one arbitrary number $\in\ > 0$. Thereby, we own the following fact:

PREMISE	CONCLUSION
P1: $u = sup\ S$ **P2:** $\in\ > 0$ is arbitrary	**C**: $\exists\ s_\varepsilon \in S \ni u - \varepsilon < s_\varepsilon$

Then we have to show existence $s_\varepsilon \in S$ that satisfy $u - \varepsilon < s_\varepsilon$. In relation with given premise, used $\in\ > 0$ on explanation $u = sup\ S$. In this case is $u - \in < u$ that based on suprimum properties will there exists $s_{\varepsilon} \in S$ such that $u - \varepsilon < s_{\varepsilon}$ (satisfy with expected conclusion).

Characteristic of owned by any $\in > 0$ on that conclusion will go into effect in general for each $\in > 0$. So that conclusion expected has been obtained.

c. *Problem with conclusion and premise that contain "for each" or "for all" expression.*

Initial strategy that conclusion owned, "*for each*" or "*for all*" expression on problem kind (b) can be used for this problem. Relations between problem kind (b) with premise that containing "*for each*" or "*for all*" expression need to be understand furthermore. If domain from quantifier expression at conclusion and premise is equal, then initial strategy of problem kind (b) can be valid at quantifier expression on premise, so there is characteristic on premise valid for initial strategy that owned also. As illustration, look at following problem example.

"*Let* (x_n) *be a convergent sequence real numbers. Prove that* (x_n) *is a Cauchy sequence*."

Based on definition, premise from the problem ($\lim (x_n) = x$) is

P: for each $\varepsilon > 0$ there is a natural number $K(\varepsilon)$ so that for all natural number

$$n \geq K(\varepsilon) \text{ satisfy } |x_n - x| < \varepsilon .$$

While condition which must be owned for arrive at conclusion is

C: for each $\varepsilon > 0$ there is a natural number $H(\varepsilon)$ so that for all natural number

$$n, m \geq H(\varepsilon) \text{ satisfy } |x_n - x_m| < \varepsilon .$$

Based on above conclusion (C), initial strategy that must be having is taking of any $\varepsilon > 0$. Then, we must find a natural number $H(\varepsilon)$ so that for all natural number

$$n, m \geq H(\varepsilon) \text{ satisfy } |x_n - x_m| < \varepsilon$$

Because domain from quantifier expression in premise (P) is equal, then the characteristic owned by that premise will go into effect $\varepsilon > 0$ that owned also. Therefore, existence of a natural number $K(\varepsilon)$ is guaranteed, so that for all natural number number

$$n \geq K(\varepsilon) \quad \text{satisfy} \quad |x_n - x| < \varepsilon .$$

The next process is linking between

$$n \geq K(\varepsilon) \quad \text{which satisfy} \quad |x_n - x| < \varepsilon$$

and

$$n, m \geq H(\varepsilon) \ \text{ which satisfy } \ |x_n - x_m| < \varepsilon$$

with $H(\varepsilon)$ which must be found.

d. *Problem with conclusion of showing uniqueness an object.*

Initial strategy to showing the uniqueness is take two objects that have characteristic on that conclusion. We must indicate that both objects are equal. As illustration look at the following example problem:

> *"Let M be a subset of G group. Prove that the smallest sub-group that containing M is a unique."*

Initial strategy showing the uniqueness is taking H_1 and H_2, each representing smallest sub-group that containing M. We will indicate that $H_1 = H_2$. Based on the smallest explanation, H_1 is the smallest sub-group that containing M, while we look H_2 is sub-group that containing M, then we have

$$H_1 \subseteq H_2$$

Now we look at smallest sub-group H_2 that contain M, with looking into H_1 as sub-group that contain M. Hence will be obtained.

$$H_2 \subseteq H_1$$

From $H_1 \subseteq H_2$ and $H_2 \subseteq H_1$ obtained

$$H_1 = H_2$$

So, we can conclude that the smallest sub-group that containg M is a unique.

2. In a few proof on the field of algebra structure and analyses, is very often emerge *key concept* that is sometime difficult to comprehended its idea appear. This key concept is very determining for attainment conclusion that expected. Therefore, university student experience on proving are so central in came out of this key concept. As an illustration, please give an attention to the problem proof in the following:

Cauchy's Inequality: *If* $n \in N$ *and* $a_1, \ldots, a_n$ *and* $b_1, \ldots, b_n$ *are real numbers, then*

$$(a_1b_1 + \ldots + a_nb_n)^2 \le (a_1^2 + \ldots + a_n^2)(b_1^2 + \ldots + b_n^2) \qquad (1)$$

To prove this inequality, can be define a function $F: R \to R$ *for* $t \in R$ *with*

$$F(t) = (a_1 - tb_1)^2 + \ldots + (a_n - tb_n)^2$$

or

$$F(t) = A - 2Bt + Ct^2 \ge 0$$

with

$$A = a_1^2 + \ldots + a_n^2, \quad B = a_1b_1 + \ldots + a_nb_n, \quad C = b_1^2 + \ldots + b_n^2$$

Since quadratic function $F(t)$ *is non-negative for all* $t \in R$*, then this function cannot have two distinct real rootsl. Therefore, its diskriminan*

$$\Delta = (-2B)^2 - 4AC = 4(B^2 - AC)$$

Must satisfy $\Delta \le 0$. *Consequently,we must have* $B^2 \le AC$ *that precisely* (1).

Key concept on proving Cauchy's inequality above is define the function $F: R \to R$ with

$$F(t) = (a_1 - tb_1)^2 + \ldots + (a_n - tb_n)^2$$

This F function is very determining on reaching the conclusion that expected. However, the idea for emerge or define the key concept is very difficult.

In the field of Algebra Structure study, the function definition as above is very often emerge at problem indicate an isomorphic like the following example:

Let G be a group and I(G) expressing group from all inner automorphism from G. Show that there is an isomorphism from I(G) to factor group G / Z where Z is center from G.

Key concept for proving this problem is definition function of

$$\varphi : G \to I(G)$$

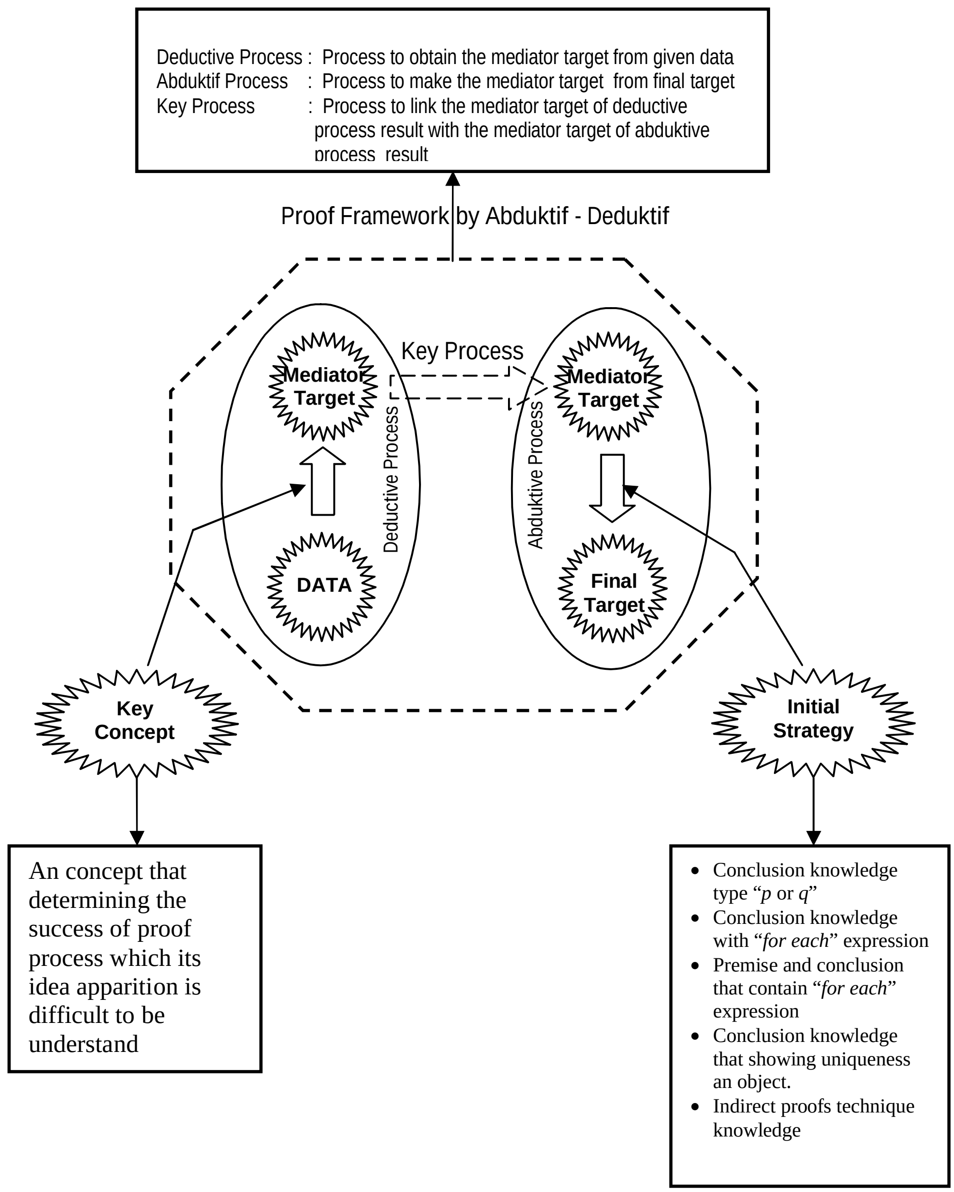
$$g \mapsto f_{g^{-1}} \quad \forall\, g \in G$$

Process the furthermore proof indicating that φ representing an homomorphism from G onto $I(G)$, and indicate that Ker (φ) = $Z(G)$

3. Initial strategy that corresponding with the technique proves at algebra structure and analyses area are more varying. Indirectly proof technique by contradiction and contra-positive technique, and the technique of existence construction about an object are often introduced in proof. Especially indirectly proof technique oftentimes earn more easily in proving a problem rather than a proof technique directly. Difference between indirectly proof and directly proof (proof strategy by abductive-deductive) is only located by the initial strategy existence on indirectly proof. While the process of proof furthermore is equal, that is developing the premise owned. Conclusion that expected from indirectly proof is about the existence of a contradiction for the contradiction technique or negation from premise beginning statement for the technique of contra-positive.

D. Theoretical Framework of Mathematics Proof

Based on theoretically study to proof structure of algebra structure and real analysis area, the theoretical framework of mathematics proof can presented at figure 2.



Figures 2

Theoretical Framework of Mathematics Proof for S1 Student

E. Suggestion and Conclusion

The conclusion from theoretically study based on mathematics proving of algebra structure and real analysis area lessons shall be as follows.

1. Proof process in algebra structure and analysis real area that have been expressed operationally (conclusion and premise able to be elaborate by a rule that have been guaranteed by truth) able to be constructed through applying abductive-deductive strategy.
2. Knowledge about initial strategy such as indirectly proof technique and conclusion elaboration technique on starting proof require owned by university student. So that problem can be formulated in the form of more operationally.
3. Key concept that determining the success of proving process, oftentimes emerge when proof construction in algebra structure and analysis area. Experience in proof construction so central in bearing idea from that key concept.

Framework yielded in this research compiled theoretically based on study of problem forms and structure proving on algebra structure and analysis real area. To see the validity and effectiveness of this framework, suggested to doing a continuation research with its implementation in the class.

F. Daftar Pustaka

Arnawa (2006). *Meningkatkan Kemampuan Pembuktian Mahasiswa dalam Aljabar Abstrak Melalui Pembelajaran Berdasarkan Teori APOS.* (Disertasi). Bandung: Universitas Pendidikan Indonesia.

Bhattascharya, P.B. & Jain, S.K. (1972). *First Course in Group Theory*. Wiley Eastern Private Limited-New Delhi.

Daepp, U. & Gorkin, P. (2003). *Reading, writing, and proving. A Closer Look at Mathematics*. Springer-Verlag.

Juandi, D. (2006). *Meningkatkan Daya Matematik Mahasiswa Calon Guru Matematika Melalui Pembelajaran Berbasis Masalah*. (Disertasi). Bandung: Universitas Pendidikan Indonesia

Kusnandi (2008). *Pembelajaran Matematika dengan Strategi Abduktif-Deduktif untuk Menumbuhkembangkan Kemampuan Membuktikan Pada mahasiswa* (Disertasi). Bandung: Universitas Pendidikan Indonesia.

Moore, R.C. (1994). Making the transition to Formal Proof. *Educational Studies in Mathematics,* 27: 249-266.

Raisinghania,M.D & Aggarwal, R.S. (1980). *Modern Agebra for Post-Graduate Students of Indian Universities*: S. Chand & Company LTD. Ram Nagar, New Delhi

Reiss, K. & Renkl, A. (2002). Lerning to prove: The idea of heuristic examples. *ZDM*, 34(1): 29-35.

Sabri (2003). *Prospective Secondary School Teachers' Conceptions of Mathematical Proof in Indonesia*. Thesis: Curtin University of Technology.

Solow, D. (1982). *How to read and proofs. An Introduction to mathematical thought process*. John WWiley & Sons

Tall, D. (1991). *Advanced Mathematical Thinking*. Kluwer Academic Publishers

Tucker, T.W. (1999). On the Role of Proof in Calculus Courses *Contemporary Issues in Mathematics Education MSRI*. Vol. 36.

Uhlig, F. (2003). The Role of Proof in Comprehending and teaching Elementary linear Algebra. *Educational Studies in Mathematics.*

Weber, K. (2002). Student Difficulty in Constructing Proofs: The need for Strategic Knowledge. *Educational Studies in Matematics* 48: 101-119.

Vellenman, D.J. (1996). *How to prove it. A Structured Approach*. Cambridge University Press.