GROUPS OF LINEAR OPERATORS AND APPLICATION TO THE SCHRÖDINGER EQUATION

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Abstrak

In this paper, the semigroup is understood as a class of bounded linear operators in Banach space X that satisfy semigroup properties, namely, being closed under multiplication (composition) and having an identity element. The term semigroup refers to properties and involves a non-negative parameter t interpreted as time parameter. Meanwhile, if both properties are satisfied for negative one the class is called group. Therefore, the semigroup describes an irreversible process while the group reversible one.

Specifically, we will study C_0-semigroup (group), namely, a semigroup whose operators are continuous for $t \geq 0$ ($t \in \mathbb{R}$ for group). Related to this kind of semigroup, it will be explained the Hille-Yosida theorem. This theorem suggests the equivalence between the operator that act as generator and the semigroup generated by the generator.

Theoretically, every semigroup can be extended to a group. In particular, when the elements of semigroup have invers. But in some cases, it is impossible to extend a semigroup to group because the system represented by semigroup does not permit it, such as the semigroup for diffusion equation.

The evolution problem are usually represented by equation so called *abstract Cauchy problem*. The unique solution to this equation involves the semigroup (group) of linear operators. This says the close connection between semigroup (group) and differential equation. In particular, the equation that represents a *well-posed* system (has unique solution and depend continuously on initial data), has unique solution involving the C_0-semigroup.

One main theorem in group of linear operators connects the unique solution to the abstract Schrödinger equation with a unitary group. This solution describes the dynamics of the system. An important example in this case is oscillator harmonic in the quantum system represented by Schrödinger equation. We will see how to represent this equation in the abstract form, so we know the unique solution and the dynamics of system.