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# Designing symbol sense tasks: the case of quadratic equations

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**Abstract.** Symbol sense refers to an ability to give meaning and to see important structures to symbols, expressions and formulas. Designing algebra tasks for assessing student symbol sense ability is therefore important. This study aims to design types of tasks based on the theory of symbol sense for the topic of quadratic and related equations. To do this, first, we conducted a literature study on the theory of symbol sense. Second, we designed tasks based on characteristics of symbol sense and provided possible strategies to solve the tasks. Third, we validated the designed tasks to seven mathematics and mathematics education experts. The results included: (1) Three types of symbol sense tasks and predictions of strategies to solve the tasks; (2) Experts comments and suggestions for improving the quality of the tasks. We conclude that the designed tasks meet the criteria of the symbol sense characteristics and can further be used for research after some minor revisions.

## 1. Introduction

The notion of symbol sense, firstly introduced by Arcavi [1], includes an intuitive feel for when to use symbols and when to disregard them in the process of solving a problem. Symbol sense is considered as an ability that shows proficiency in algebra [2], and shows a relational rather than only an instrumental understanding [3].

The idea of symbol sense has been used in several previous studies for, for instance, investigating student algebraic expertise in a digital environment [4], assessing algebraic proficiency [3], and understanding student difficulties on the concept of parameter [5]. In case of Indonesia, to a limited extent, the symbol sense framework has been used for understanding students' algebraic thinking in solving substitution problems [6]. This symbol sense idea, however, is not yet further investigated for instance for assessing Indonesian student algebraic skills and proficiency.

Considering the above into account, we conducted a small-scale case study on developing algebraic symbol sense problems. This study therefore aims to validate theoretically designed algebraic symbol sense tasks for assessing high school algebra students. To achieve this aim, the notion of symbol sense is used as a main theoretical lens.

Symbol sense, which is an analogy of number sense [1], can be defined as an ability to give meaning and to see important structures to symbols, expressions and formulas [5, 7]. According to Arcavi [7] characteristics of symbol sense include, for instance, the skill to use symbols in recognizing relationship, in displaying generalization and proofs; the skill to manipulate and to read through symbolic expressions; and the skill to check for the symbol meanings in the implementation of a procedure, the solution of a problem, or during the inspection of a result.



For the purpose of this study, we distinguish between procedural and symbol sense strategies in solving algebra problems. By procedural strategy we mean a strategy that is used by someone in solving algebra problems using a standard procedure without considering the efficiency of the procedure [8]. By symbol sense strategy we mean a strategy that is used by someone in solving algebra problems using characteristics of symbol sense, as suggested by Arcavi [1, 7], to achieve an efficiency and elegance of solving problems.

## 2. Method

As a part of a larger study, this case study aims to design types of tasks based on the theory of symbol sense. To carry this out, first, we conducted literature study on the theory of symbol sense. Second, we designed different types of tasks based on the main ideas of symbol sense as described in [1, 7] and provided possible strategies to solve the tasks. The designed tasks are on the topic of algebra, and in particular for the case of quadratic and related equations. Third, we validated the designed tasks theoretically to seven experts, including three mathematics experts and four mathematics education experts. In this validation process, the experts were requested to give comments and suggestions on the designed tasks. The comments included, for instance, whether the tasks are appropriate with the symbol sense characteristics, and whether the tasks are suitable for high school students in terms of quality and difficulties.

## 3. Result and Discussion

In this section, we subsequently describe in an integrative manner about types of designed tasks, prediction of strategies to solve the tasks, and experts' comments and suggestions on the designed tasks. Based on symbol sense characteristics [1, 7] we designed three types of algebra tasks on quadratic and related equations that invite the use of symbol sense strategies as shown in Table 1.

**Table 1.** Symbol sense tasks on quadratic and related equations

Type of tasks	Symbol sense characteristics	Example of tasks
1	Check symbol meanings before or during the implementation of a procedure, the solution of a problem, or during the inspection of a result	$38 - (1 - 2x)^2 = 13$
2	Manipulate and read through symbolic expressions	$\frac{(2x - 1)(x + 2)}{6x^2 + 9x - 6} = 2$
3	Recognize symbolic relationship, display and do symbolic generalization and proofs	$(2x - 3)(x + 2) = (x + 1)(x + 2)$

We predicted that a procedural strategy might emerge for solving tasks of type 1, for example for solving the equation  $38 - (1 - 2x)^2 = 13$ , if someone is tempted to expand  $(1 - 2x)^2$  into  $(1 - 4x + 4x^2)$  and then solve the quadratic equation using a standard procedure—such as using a quadratic formula. We also predicted that the symbol sense strategy might emerge if someone at the first step sees the equation meaningfully as a simple arithmetic problem  $38 - \dots = 13$ , and as such conclude  $(1 - 2x)^2 = 25$ , which is then relatively easy to deduce that  $1 - 2x = 5$  or  $1 - 2x = -5$ .

All the seven experts commented that the task of type 1 is appropriate to the idea of symbol sense and has good quality for assessing student algebraic proficiency. Two of the experts predicted that there might be students who would be trapped with this task, for instance, by only concluding from  $(1 - 2x)^2 = 25$  to  $\sqrt{(1 - 2x)^2} = \sqrt{25}$ , and to obtain  $1 - 2x = 5$ .

For the tasks of type 2, for example for the equation  $\frac{(2x-1)(x+2)}{6x^2+9x-6} = 2$ , we predicted that a procedural cross multiplication strategy might emerge. This strategy, however, leads to incorrect solutions, i.e., from the equation to obtain  $(2x - 1)(x + 2) = 2(6x^2 + 9x - 6) \Rightarrow 10x^2 + 15x - 10 = 0 \Rightarrow x = -2$  or  $x = \frac{1}{2}$ . In line with our prediction, this strategy is also predicted to emerge by two experts. A symbol sense

strategy—which is also predicted by the experts—might emerge if someone reads through the equation and finds that the left hand side of the equation is  $\frac{1}{3}$  (because  $\frac{(2x-1)(x+2)}{6x^2+9x-6} = \frac{2x^2+3x-2}{3(2x^2+3x-2)} = \frac{1}{3}$ ) but the right hand side is 2. By using this symbol sense strategy, someone will deduce that the equation has no solution. In addition to comment that the task is appropriate, has good quality, and shows that an equation does not necessarily have solution, one of the experts suggests to add one similar equation, such as,  $\frac{(2x-1)(x+2)}{6x^2+9x-6} = \frac{1}{3}$  for further investigation.

For the equation  $(2x - 3)(x + 2) = (x + 1)(x + 2)$ —as an example of the tasks of type 3, we predicted that there might emerge two different procedural strategies. First, the two terms on each side of the equation are multiplied to obtain  $2x^2 + x - 6 = x^2 + 3x + 2 \Leftrightarrow x^2 - 2x - 8 = 0$ . Then, a standard procedure of solving a quadratic equation, such as using factorization method, is used. Second, since each side has the same term  $(x + 2)$ , even if this might lead to incorrect solutions, cancelation method might emerge. This second prediction is also predicted by all seven experts involved in this study. One of the experts suggested that to improve the quality of the task, the direction should be added, such as, by adding information whether  $x$  is real number or not to avoid an assumption that  $(x + 2) \neq 0$ .

We also predicted the emergence of a symbol sense strategy to deal with the equation  $(2x - 3)(x + 2) = (x + 1)(x + 2)$ . The strategy is employed by factorizing the same terms to obtain  $(x + 2)[(2x - 3) - (x + 1)] = 0$ , and then to have  $(x + 2)(x - 4) = 0$ . This prediction is also in line with the experts' predictions, in which this strategy might emerge if someone is able to see a symbolic relationship in the equation and able to apply the distributive property for factorizing the same terms.

As a reflection to the result above, we consider that use of procedural strategies in the solution processes shows instrumental understanding [3], and the use of symbol sense strategies shows relational understanding [3] or shows better quality of algebraic proficiency [2, 9]. Also, the use of symbol sense strategies shows flexible, holistic, and comprehensive understanding on symbolic expressions in general, and on solving equations in particular [10].

#### 4. Conclusion

From the description in the previous section, we draw the following three conclusions. First, for equation solving on quadratic and related equations, we can design three types of tasks according to symbol sense characteristics. Other characteristics of symbol sense that are not related to equation solving [1, 7] are not included in our design, but are prolific to be investigated for relevant topics, such as, developing mathematical models from word problems. Second, our predictions about possible strategies to solve the designed tasks, including procedural and symbol sense strategies, are in line with the seven experts' comments and suggestions. Finally, according to the experts, even if our designed tasks need minor revisions, they are considered to be appropriate with the notion of symbol sense, and are having good quality in terms of design and difficulty for assessing student algebraic proficiency.

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