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Learning Algebra on Screen and on Paper: The Effect of Using a Digital Tool on Students' Understanding

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Abstract. The use of digital tools in algebra education is expected to not only contribute to master skill, but also to acquire conceptual understanding. The question is how digital tools affect students' thinking and understanding. This paper presents an analysis of data of one group of three grade seventh students (12-13 year-old) on the use of a digital tool for algebra, the Cover-up applet for solving equations in particular. This case study was part of a larger teaching experiment on initial algebra enriched with digital technology which aimed to improve students' conceptual understanding and skills in solving equations in one variable. The qualitative analysis of a video observation, digital and written work showed that the use of the applet affects student thinking in terms of strategies used by students while dealing with the equations. We conclude that the effects of the use of the digital tool can be traced from student problem solving strategies on paper-and-pencil environment which are similar to strategies while working with the digital tool. In future research, we recommend to use specific theoretical lenses, such as the theory of instrumental genesis and the onto-semiotic approach, to reveal more explicit relationships between students' conceptual understanding and the use of a digital tool.

INTRODUCTION

To pursue either advanced study or profesional work, secondary school students should master algebra, a main topic in secondary school mathematics [1]. However, mastering algebra is not easy for most of students over the world, and for Indonesian students in particular, as indicated in the 2011 Trends in International Mathematics and Science Study (TIMSS) [2]. This TIMSS study reveals that Indonesian student have low performances in algebra, ranked in 38th position out of 42 countries [2]. Concerning this, the question on how to improve Indonesian student performance in algebra domain becomes an important issue to be addressed. We propose that considering the role of Information and Communcation Technology (ICT) in mathematics education is an alternative way to address this issue.

In today society, digital tools for learning mathematics are used more frequently in the teaching and learning processes (e.g., [3]). For example, the use of ICT affects positively on mathematics achievement [4] and on students' mathematical attitude [5]. Also, ICT use contributes to student conceptual understanding and skills in algebra [6], including symbol sense and procedural skills [7], and understanding the notion of the concept of function [8].

Taking the above into consideration, we conducted an ICT-rich teaching experiment to improve student performance in algebra. In this article, we report on a part of the results of this experiment, and in particular we investigate the effect of the use of a digital tool, i.e., the Cover-up applet, to student conceptual understanding.

THEORETICAL FRAMEWORK AND RESEARCH QUESTION

Difficulties in Initial Algebra Learning

The difficulties emerged in the learning and teaching of algebra can be used to analyze student conceptual understanding in algebra. In [9], Jupri, Drijvers and Van den Heuvel-Panhuizen identified five main categories of difficulties in algebra learning. First, the difficulties in using arithmetical operations and properties include difficulties in adding like terms (e.g., [10]; [11]); in using commutative, associative, inverse, and distributive properties; and in using priority rules of arithmetical operations (e.g., [12]; [13]). Second, the difficulties in

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understanding the notion of variable concern understanding it as an unknown, a generalized number, a placeholder, or a varying quantity ([10]; [12]). Third, the difficulties in understanding algebraic expressions consist of various obstacles, such as the expected answer obstacle, the lack of closure obstacle, the parsing obstacle, and the gestalt view obstacle ([14]; [15]). Fourth, the difficulties in understanding the different meanings of the equal sign: as a sign that urges to do a calculation, and as a sign of equivalence ([10]; [16]). The fifth, the difficulties in mathematization embrace the difficulties in transforming the problem situation into the world of mathematics, and reorganizing the symbols within the world of mathematics ([17]; [18]).

Didactical Functions of Technology in Algebra Education

Drijvers, Boon and Van Reeuwijk [19] distinguish three didactical functions of technology in the algebra education. First, as a tool for doing mathematics, technology plays as an assistant to carry out algebraic routine procedures, such as solving equations. In this case, the user of technology does not necessarily understand how the outcomes are produced. Second, as an environment for practicing skills, technology provides feedback to students' responses [20], a variation of problems [7], and a compatibility of the technological and the paper-and-pencil environment [7]. Third, as an environment for developing algebraic concepts, technology is aimed to guide the development of algebraic thinking (e.g. [21]). The use of digital tools in this study is mainly on the second and third functions: conceptual understanding as a basis for a mastery of algebraic skills, and procedural skills are expected to corroborate the conceptual understanding.

Research Question

To address the issue described in the Introduction, we formulate the following research question: *What is the effect of the use of a digital tool to students' conceptual understanding in algebra?*

The domain of algebra in this study concerns equations in one variable, which is intended for grade VII students (12-13 year-old) in Indonesian curriculum. The digital tool is the Digital Mathematics Environment (DME), i.e., the Cover-up applet, which is developed at the Freudenthal Institute, Utrecht University, The Netherlands.

METHOD

The case study reported in this paper was a part of a larger experimental study [22], and was based on one lesson, the algebra learning with the Cover-up applet on equations in one variable. The learning organization consisted of the activities with the applet embedded within the DME. The DME is a web-based environment which provides: interactive digital tools for learning mathematics; a design of open online tasks; immediate feedback for the tasks; access at any time and place as long as technological infrastructure is met; and a storage for student work ([23]; [24]).

The lesson was lasted for 80 minutes, and consisted of three parts. The first part concerns a paper-and-pencil activity. This activity was carried out to introduce the concept of equation through posing problems and class discussion. The second part include a whole class demonstration on how to work with the applet, and a group-based digital activity which was done by students under the teacher guidance. Third, an individual paper-and-pencil test was carried out after the two previous parts.

The data reported in this paper included video registrations of one group of three grade VII students, its corresponding student digital work, and written work. To investigate the effect of the use of the Cover-up applet to student conceptual understanding and skill in dealing with equations, an integrative qualitative analysis on these data, with the help of Atlas.ti software, was carried out.

RESULTS AND DISCUSSION

This section presents the results of analysis of one group, consists of three grade seventh students, from the Cover-up applet activity. The three students are named in this article Farhan, Toyib and Wahib. We start the description with an analysis of a typical task addressed in the digital group work and a video observation while solving the task. Next, we discuss the effect of the digital activity on student written paper-and-pencil work.

Figure 1(a) shows a student digital group work on a typical task of the Cover-up lesson, i.e., solving the equation $\frac{18}{5a-2} = 6$. If we only consider this digital work, the students seemed to have no problem for solving the equation. A video observation while the students were working on this task, however, shows that the process of obtaining the correct solution was not easy. Figure 1(b) provides an excerpt of this observation. Based on the conversation during the group work, we predict that the three students seemed to acquire different understanding: Farhan and Wahib seemed to have a better understanding than Toyib—who was not so active and only followed his two colleagues while working on the task.

(a)

Observer : Which part should you cover first for solving $\frac{18}{5a-2} = 6$?

Farhan : $5a$ [This is still incorrect and the three students do not understand yet. The observer then guides the students to get $5a - 2 = 3$].

Observer: Now, which part should you cover for solving $5a - 2 = 3$?

Farhan : $a - 2$. Eh, no, no… It should be $5a$ to be covered! [He covers $5a$, then $5a = \cdots$ appears in the next line. Toyib and Wahib have no response to this.]

Observer: Okay, good!

Wahib : So...[the value of $5a$] is 5.

Observer: Now, which part should you cover from $5a = 5$?

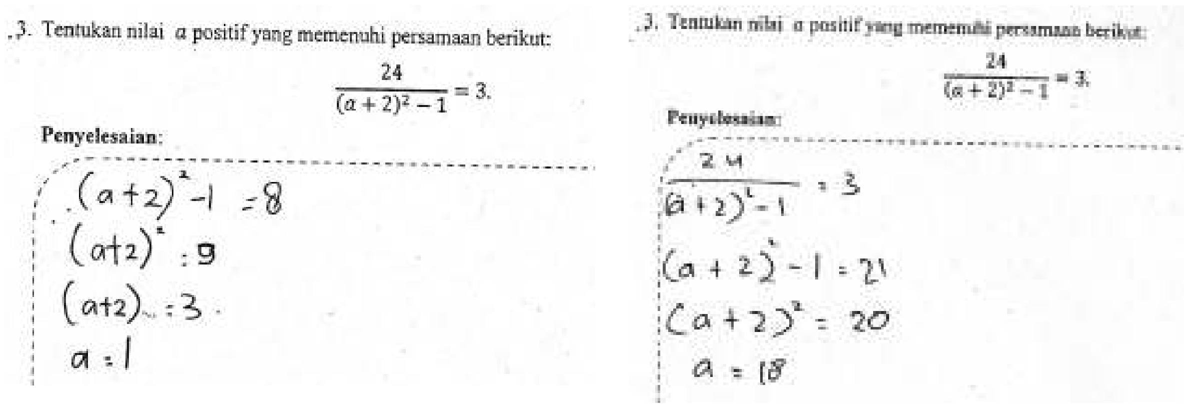
Wahib : a.

(b)

FIGURE 1. An example of student digital work and its corresponding observation excerpt

Different acquired conceptual understanding is manifest in students' individual written work after the Cover-up applet activity, when solving the equation $\frac{24}{(a+2)^2-1} = 3$, as shown in Figure 2. Figure 2(a) shows Farhan's correct solution, which is quite the same as Wahib's work. Farhan showed that he is able to select appropriate expressions as well as able to assign correct numerical values to them when applying the cover-up strategy for solving the equation. The strategy that he used on paper-and-pencil environment is similar to the strategy while working on the screen with the applet. This shows a transfer of notation and strategy from a digital environment to a paper-and-pencil environment ([7]; [3]; [22]).

Figure 2(b) shows Toyib's incorrect solution. Even if Toyib was able to select appropriate expressions, he assigned incorrect values to them while applying the cover-up strategy. For example, he assigned 21 to $(a+2)^2 - 1$, rather than 8. The value of 21 seems to come from $24 - 3 = 21$. As a consequence, this incorrect step affected next steps. If we do not consider the previous step, Toyib also made mistakes while solving the equation $(a+2)^2 - 1 = 21$. First, rather than assigning $(a+2)^2 = 22$, Toyib assigned 20. This mistake is classified into the first category of difficulty, i.e., an arithmetical difficulty and applying additive inverse property in particular ([9]; [13]). Second, rather than taking a square root for the equation $(a+2)^2 = 20$, Toyib ignored the square and subtract both sides with 2 to get $a = 18$. This analysis shows that even if Toyib was able to select appropriate parts of the equation while applying the cover-up strategy, he did not really understand the idea of this strategy.



(a) (b)

FIGURE 2. Written work by Farhan (a) and Toyib (b) after the Cover-up lesson

CONCLUSION

Based on the results described in the previous section, we draw the following conclusion. The effect of the use of digital technology, the Cover-up applet, includes strategies used by students on paper-and-pencil environment which are similar to strategies while working with the applet. That is students used the same cover-up strategy for solving equations in one variable in both environments: paper-and-pencil and digital environments. In our view, this phenomenon reflects a direct effect of the applet use. Also, observable mistakes in the written work which correspond to student difficulties during the digital group work depict this subtle effect and to certain extent show student conceptual understanding.

We however acknowledge that the results are still limited to the effects of the use of the digital tool to student written and digital work in terms of difficulties in algebra and the strategies that students use. We do not yet address the interplay between the use of the applet and student conceptual understanding: how the applet affects students thinking, and in turn how students' thinking influences the use of the applet? Therefore, to reveal more explicit interrelationship between student conceptual understanding and the use of a digital tool, we recommend applying specific theoretical lenses, such as the theory of instrumental genesis and the onto-semiotic approach (e.g., [25]). In this way, we expect to obtain a better comprehension of the effects of the digital tool use on student conceptual understanding and skills.

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