Predictions of Students' Thinking for The Learning of System of Linear Equations in Two Variables

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Abstract. This research aims to predict grade VIII students' thinking for the learning of system of linear equations in two variables. To achieve this, first, we surveyed algebra education literature and Indonesian school mathematics textbooks for collecting information about a learning sequence for the topic of system of linear equations in two variables. Second, based on the first step, we designed a sequence of learning for this topic according to the theory of realistic mathematics education and made predictions about student thinking process particularly on solving relevant problems in this topic. The results consisted of a sequence of algebra tasks and various predictions of student thinking for learning algebra. In conclusion, we consider that predictions of student thinking process can be used by the teacher for anticipating the actual learning process either for helping students who encountering difficulties or for teaching actions to invite students' problem solving skills.

Keywords: Algebra education, system of linear equations in two variables, hypothetical learning trajectory, realistic mathematics education.

1 Introduction

In Indonesia system of linear equations in two variables is one of algebra topics included in school mathematics for grade VIII junior high school level [1]. The learning objective for this topic is that students are expected to be able to understand and explain the topic and to solve relevant problems. The Indonesian curriculum suggests that student conceptual understanding for this topic is developed through the use of meaningful problems during the learning process [1]. However, mathematics textbooks used popularly in Indonesia, such as [2-4], usually present contextual problems as application of the concept rather than as problems that can develop student conceptual understanding. The question is how a learning sequence for the topic of system of linear equations is designed so it can develop student conceptual understanding in a meaningful way.

To address this question, we carried out the present study which aims to design an algebra learning sequence for the topic of system of linear equations in two variables according to the theory of realistic mathematics education (RME). In this paper we present results of predictions of students‘ thinking process as a part of the design process. The RME theory, which is coming from the Netherlands, is used because it presents mathematics in a realistic and meaningful

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manner [5]. For the purpose of the design of a learning sequence, we use three didactical principles of the RME theory, including the reality principle, the level principle, and the intertwinement principle [5].

From the perspective of the reality principle, mathematics is conveyed from meaningful problems to a more symbolic problems, that is, problems that need to be mathematized [5-7]. By mathematization we mean an activity of transforming contextual or words problems into symbolic mathematics problems and of reorganizing the symbolic problems within the world of mathematics [5-7].

According to the level principle, students pass various levels of understanding in the process of learning mathematics: from context-related solutions to acquiring insight into relationships between concepts and strategies [6]. In the case of learning algebra, using this principle, the learning design should provide a link between informal and formal world of mathematics in the form of mathematical models.

In the light of the intertwinement principle, various mathematical content domains such as number, algebra and geometry are considered to be integrated rather than as separated mathematical topics [5-7]. In this manner, students are offered rich algebra tasks that relate various mathematical concepts: not only the algebra domain itself, but also other domains, including arithmetic and geometry.

2 Methods

We used design research method to design an algebra learning sequence on the topic of system of linear equations in two variables [8]. This research method includes three phases, namely the phase of a preliminary design, carrying out a teaching experiment, and conducting a retrospective analysis [8-9]. In this article, we report on part of the results of the preliminary design phase particularly on predictions of students‘ thinking for the learning of system of equations.

To do this, first, we surveyed algebra education literature [10-11] and Indonesian school mathematics textbooks for grade VIII students [2-4] for collecting information about a learning sequence for the topic of system of linear equations in two variables. Second, we designed a sequence of learning for this topic according to the theory of realistic mathematics education and made predictions about student thinking process particularly on solving relevant problems in this topic. Prediction of student thinking is one of main components of hypothetical learning trajectory [8-9] that helps researchers in the design process and retrospective analysis, and helps the teacher in the teaching experiment.

Table 1 presents three main tasks used in the design of the learning sequence. Task 1 is typical task for the first lesson which aims to introduce linear equations in two variables through realistic situation. Task 2 is typical for the lesson two which aims to introduce the idea of system of linear equations in two variables and to explore solution strategies of the system. Task 3 is typical task for the lesson three which aims to understand substitution and elimination methods for solving system of equations. These three typical tasks comply with the three RME principles—i.e., the reality, the level, and the intertwinement principles—and are mainly adapted from [11]. In this study, this sequence of tasks formed a design of learning sequence for the topic of system of linear equations in two variables.

Table 1. Three typical tasks used in an algebra learning design

Lesson 1	Tasks
1	Given two groups of cards A and B, where each group contains cards numbered from 1 to 12. If you take one card from each group, then in which cases will the sum of "Two times A" plus "B" equal seventeen?
2	Given two groups of cards A and B, where each group contains cards numbered from 1 to 12. If you shuffle each group of cards and you take one card from each group, when do the following two conditions occur: (a) The sum of three A plus B equals 21; (b) The sum of two A minus B equals 9.
3	Solve the following system of equation: $\begin{cases} x + 3y = 14 \\ y = x + 2 \end{cases}$.

3 Results and discussion

This section addresses our predictions of students' thinking on solving tasks presented in Table 1. The predictions include possible correct and incorrect solution strategies. Also, we provide some suggestions on how the designed tasks can be presented in order for achieving the objective of the learning process.

3.1 Predictions of students' thinking on Task 1

A typical task for the lesson one (as presented in Table 1) to introduce linear equations in two variables through realistic situation is the following:

Given two groups of cards A and B, where each group contains cards numbered from 1 to 12. If you take one card from each group, then in which cases will the sum of "Two times A" plus "B" equal seventeen?

We predicted that there are at least two possible correct solution strategies. First, after students understand the task, they probably will use a systematic table strategy as shown in Figure 1. In this way, students will obtain correct solutions of $A: 3, 4, 5, 6, 7, 8$ and its corresponding values of $B: 11, 9, 7, 5, 3, 1$.

A	1	2	3	4	5	6	7	8	~~9~~	~~10~~	~~11~~	~~12~~
2A	2	4	6	8	10	12	14	16	~~18~~	~~20~~	~~22~~	~~24~~
B	~~15~~	~~13~~	11	9	7	5	3	1				

Fig. 1. The use of table strategy for solving typical task for the lesson one

Second possible correct solution strategy is the use of what so-called substitution strategy. In this case, for instance, a student transforms the contextual problem into an equation with two variables, such as $2A + B = 17$ or $B = 17 - 2A$. Next, by substituting each possible value of A, the student will obtain values of B. For example, by substituting $A = 1$, the student will obtain $B = 15$ (which is not the solution); by substituting $A = 3$, then the student will get $B = 11$; and so on. The student will finally end up with complete and correct solutions.

Possible incorrect solutions might occur if, for example, a student uses a trial-and-error strategy, namely by substituting values of A or B in non-systematic way. The student might find incomplete solutions or might made incorrect calculations.

By having various predictions of student thinking process, the teacher can anticipate possible mistakes that might occur from students. For the purpose of the learning process, we suggest that this task can be posed by the teacher to individual students or to groups of four-to-five students. They are then asked to solve the task. Next, the teacher guides the classroom discussion to discuss all possible solutions and strategies.

3.2 Predictions of students‘ thinking on Task 2

For the lesson two, a typical task as presented in Table 1 for introducing the idea of system of linear equations in two variables and for exploration of solution of the system is the following:

Given two groups of cards A and B, where each group contains cards numbered from 1 to 12. If you shuffle each group of cards and you take one card from each group, when do the following two conditions occur:

(a) The sum of three A plus B equals 21;
(b) The sum of two A minus B equals 9.

We predicted again that there are at least two possible different strategies to solve this task. First, students might use an informal strategy by using tables as shown in Figure 2. Using this way, the students will find that $A = 6$ and $B = 3$ satisfy both conditions, and as such they are the solutions of the task.

A	1	2	3	4	5	6	7	8	9	10	11	12
B	~~18~~	~~15~~	~~13~~	9	6	3	~~0~~	~~-3~~	~~-6~~			
3A+B	21	21	21	21	21	21	21	21	21			

A	1	2	3	4	5	6	7	8	9	10	11	12
B	~~-7~~	~~-5~~	~~-3~~	~~-1~~	1	3	5	7	9	11	~~13~~	
2A-B	9	9	9	9	9	9	9	9	9	9		

Fig.2. The use of an informal table strategy for solving typical task of the lesson two

Second, students might be able to transform the task into two equations to make a system of equations (1) as the following:

$$\begin{cases} 3A + B = 21 \\ 2A - B = 9 \end{cases} \quad (1)$$

By adding the first to the second equations, for instance, a student will obtain $5A = 30$ or $A = 6$. Next, by substituting $A = 6$ into one of the equations, the student will obtain $B = 3$. If this second prediction occurs, the teacher can introduce to students that this solution strategy combines both elimination and substitution methods.

Incorrect solutions might happen if students are not able to transform the task into a correct of system of equation. Also, even if students are able to transform the task into a system of equation correctly, students might make mistakes during, for instance, substitution or elimination processes.

For the purpose of the learning process, for instance in a classrom discussion, we suggest that the teacher guides step-by-step in translating the word task into a system of equation. Also, the teacher can remind students to check solutions of the system by substituting the solutions to each equation in the system of equations.

3.3 Predictions of students‘ thinking on Task 3

A typical task for the lesson three—as presented in Tabel 1—to understand substitution and elimination methods for solving system of equations is the following:

Solve the following system of equations:

$$\begin{cases} x + 3y = 14 \\ y = x + 2 \end{cases} \quad (2)$$

By having an experience from the lesson two, we predicted that students will first use the substitution method to solve the system of equation (2). By substituting $y = x + 2$ into $x + 3y = 14$, a student will get $x + 3(x + 2) = 14 \Leftrightarrow 4x = 8$ and to deduce $x = 2$. So, by substituting $x = 2$ into $y = x + 2$, the student will obtain $y = 4$. For students who are able to see the second equation as $x - y = -2$, they might probably use the elimination method.

Another possible strategy that might emerge from students is by the use of area strategy [11]. This area strategy, for solving the system of equations (2), can be seen in Figure 3. From the perspective of the RME theory [5], the use of area strategy shows an intertwinement between algebra and geometry domains. The intertwinement between mathematical domains might also happen if students use a graphical method to solve the system of equations.

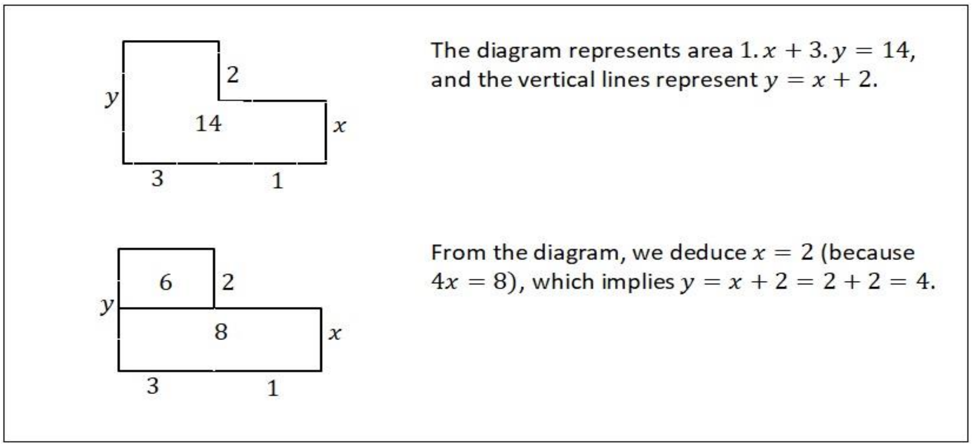

Fig. 3. The use of area strategy for solving a system of equations

Mistakes might occur if students make incorrect calculations, for example, during the use of elimination or substitution methods. For the purpose of implementing the design in a teaching process, the teacher can encourage students to use different strategies: not only use elimination and substituion methods, but also use graphical and area methods. Mistakes that might occur can be prevented by, for example, encouraging students to do final check by substituting solutions to the original system of equations.

4. Conclusions

We conclude that predictions of student thinking process can be considered by the teacher for preparing a lesson plan and can be used for anticipating the actual learning process including for helping students who encountering difficulties and for teaching actions to invite students' problem solving skills. From the perspective of design research, predictions of student thinking is a part of hypothetical learning trajectory in the prelimnary design phase. From the perspective of the realistic mathematics education theory and the level principle in particular, recognizing various different solution strategies is important for the teacher to help students pass a more abstract mathematics in a more meaningful manner.

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