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# Designing an algebra learning sequence: the case of operations on algebraic expressions

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# Designing an algebra learning sequence: the case of operations on algebraic expressions

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**Abstract**. This study aims to design a learning sequence for algebra learning on the topic of operations on algebraic expressions—which is taught for junior high school students. To do so, we used design research method, particularly the preliminary design phase. First, we compiled student difficulties in the topic of algebraic expressions from previous relevant studies. This compilation is used as a consideration for designing a learning sequence. Second, we designed the learning sequence based on the theory of realistic mathematics education. This theory is used because it offers meaningful mathematics for students. Finally, we conducted a focus group discussion to revise the design in order to produce a ready-use learning sequence for the teaching experiment. The result of this study included a sequence of algebra learning material on the topic of operations on algebraic expressions. In sum, we consider that the designed learning sequence for this algebra topic is more meaningful than the conventional learning sequence for students.

## 1. Introduction

Operations on algebraic expressions is one of main algebra topics in school mathematics that should be mastered by junior high school students over the world, including Indonesia [1], either for further study or for professional life [2]. Previous relevant studies in Indonesian context, however, have shown that this algebra topic is difficult for most of students [3-4]. The difficulties that students encounter include parsing obstacle, lack of closure obstacle, and the expected answer obstacle [3]. The question is how we overcome these student difficulties, particularly for the topic of operations on algebraic expressions.

To deal with this question, we carried out the present study which aims to design an algebra learning sequence according to the theory of realistic mathematics education (RME). This RME theory is used because it has been successful in the Netherlands and has been adapted in the United States for presenting mathematics in a meaningful manner [5]. We propose to use three didactical principles of the RME theory in the design of the learning sequence, including the reality principle, the level principle, and the intertwinement principle [6].

According to the reality principle, mathematics learning should start from meaningful problems, that is, rich contextual problems that need to be mathematized [6-7]. In the case of algebra topics, mathematization refers to an activity of translating contextual problems into symbolic algebra problems and vice versa, and of reorganizing as well as reconstructing the symbolic problems within the world of mathematics [6-8].

The level principle emphasizes that in the process of learning mathematics students pass various levels of understanding: from context-related solutions to acquiring insight into relationships between concepts and strategies [6]. In the context of algebra learning, this principle provides a bridge between informal and formal world of mathematics in the form of mathematical models.

In the view of the intertwinement principle, mathematical content domains such as number, algebra and geometry are considered as integrated rather than as isolated mathematical topics [6, 7]. Therefore, for the context of algebra learning, students are offered rich algebra problems in which they can use various mathematical concepts: not only can use the algebra domain itself, but also can use geometry knowledge to understand and solve algebra problems.

## 2. Methods

To design an algebra learning sequence on the topic of operations on algebraic expressions, we used design research method. This method includes three phases: developing a preliminary design, conducting a teaching experiment, and carrying out a retrospective analysis [9-10]. In this article, we report on the results of the preliminary design phase which was carried out in three steps.

First, we collected student difficulties for the topic of operations on algebraic expressions from previous relevant studies. We identified three main difficulties in this topic, i.e., parsing obstacle, lack of closure obstacle, and the expected answer obstacle [3-4]. By the parsing obstacle we mean student inability to disentangle the order of operating in the context of algebraic expressions and often having conflict with the order of natural language. For example, in dealing with $25-8x$, a student incorrectly considers it as $17x$. The lack of closure obstacle is a discomfort from attempting to handle an algebraic expression which represents a process that cannot be calculated. For example, a student incorrectly considers $x+2018$ as $2018x$. The expected answer obstacle is an incorrect expectation of having a numerical answer for an algebraic expression. For instance, $12+13x$ is incorrectly considered as $12+13x=25$. This collection was then used as a consideration in designing a learning sequence.

Second, as an attempt to overcome the aforementioned difficulties, we designed an algebra learning sequence on the topic of operations on algebraic expressions. This topic, in Indonesian curriculum, is taught for grade VII students [1]. The design was based on the theory of realistic mathematics education, in particular the reality principle, the level principle, and the intertwinement principles [6]. The sequence includes three main consecutive activities: introduction to algebraic expressions, addition and subtraction on algebraic expressions, and multiplication on algebraic expressions. The main context for these three activities is the area and perimeter of rectangles. In this way, we expected that students can do operations on algebraic expressions in a meaningful manner. The use of this geometry context for learning algebra is mainly based on the intertwinement principle, and the level principle. The reality principle appears in the form of problems that can be imagined by students—as they have already learned the topic of area and perimeter in primary grades.

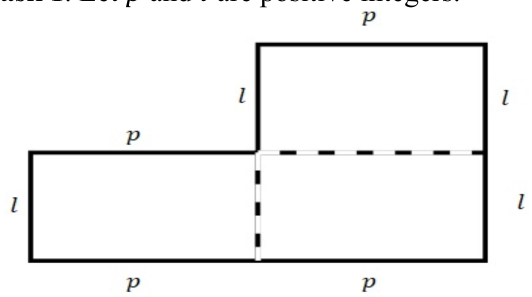
Finally, the third, we conducted a focus group discussion among research group members to revise the design produced in the previous step. This discussion was carried out to produce a ready-use learning sequence for the teaching experiment.

## 3. Results and Discussion

This section presents and discusses the algebra learning sequence for the topic of operations on algebraic expressions. The sequence of the learning includes three activities. First, for the activity of the introduction to operation on algebraic expressions, the learning process starts by reminding students with the idea of perimeter and area of a rectangle, that is, the perimeter of a rectangle with the length $p$ and width $l$ is $p+l+p+l=2p+2l$; and its area is $pl$. Next, students should solve two main tasks presented in Table 1. The learning goal for this activity is to introduce students that the same variables can be added. By using the context of perimeter and area of a rectangle, which is considered to be meaningful for students, the students are expected to grasp the idea of operations on algebraic expressions. For Task 1(a), for example, students can formulate that the perimeter is $p+p+l+l+p+l+p+l$ and then can add the same variables to get $4l+4p$. This also holds similarly for calculating the area of the figure, i.e., $pl+pl+pl=3pl$. If students are successful in solving this task, they are expected to apply their knowledge for solving Task 2 in a similar way.

**Table 1.** Main tasks used in the activity of introduction to operations on algebraic expressions.

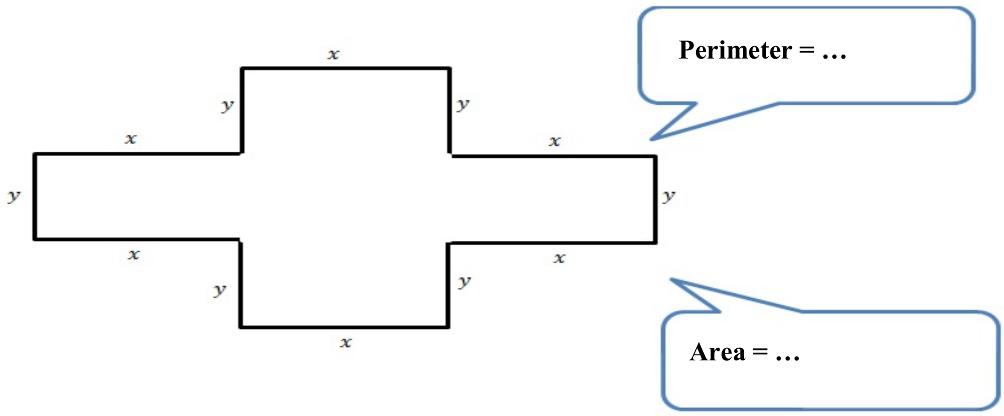
**Task 1**. Let $p$ and $l$ are positive integers.




(a) Find the perimeter and the area of the region in terms of $p$ and $l$.

(b) If the perimeter of the region is 44 cm, find possible values for $p$ and $l$. Then, find its area.

**Task 2.** Find the perimeter and area for the following figure in terms of $x$ and $y$.

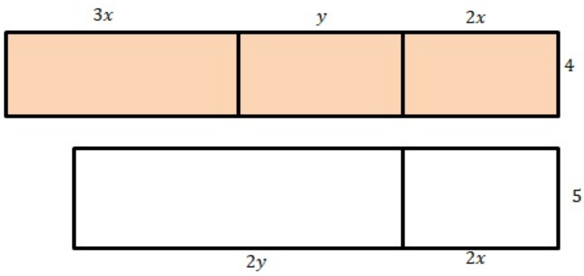



Second, for the activity of addition and subtraction on algebraic expressions, the learning process starts with a problem of finding an area and perimeter of a region, in which the length or the width of the region is not in terms of a single variable. Main tasks used in this activity are presented in Table 2. The learning goal for this activity is to guide students acquiring skills and understanding in addition and subtraction of algebraic expressions.

For Task 3, for example, students are expected to be able to formulate the area of the shaded region into $12x + 4y + 8x$, and then simplify it into $20x + 4y$. Similarly, for the unshaded region students are expected to get $10x + 10y$. Finally, students can do addition and subtraction of algebraic expressions respectively for $A + B = (20x + 4y) + (10x + 10y) = 30x + 14y$ and for $A - B = (20x + 4y) - (10x + 10y) = 10x - 6y$. If students have already been familiar with this, students are expected to be able to solve Task 4. In this way, we expect students would not encounter parsing obstacle, lack of closure obstacle, or the expected answer obstacle [3-4].

**Table 2.** Main tasks used in the activity of addition and subtraction on algebraic expressions.

**Task 3.** Consider the following figure.

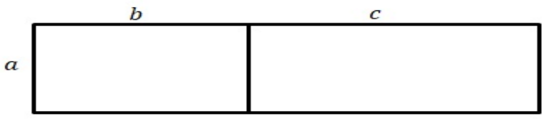



(a) Find the area of the shaded region in terms of $x$ and $y$.
(b) Find the area of the unshaded region in terms of $x$ and $y$.
(c) If $A$ is the area of the shaded region and $B$ is the area of unshaded region, find $A + B$ and $A - B$.

**Task 4.** If $P = x^2 + 2xy + y^2$, $Q = 2x^2 + 3xy + 5y^2$ and $R = 4x^2 - 4xy + y^2$, then find: $P + Q$, $P + R$, $Q + R$ and $Q - P$.

**Table 3.** Main tasks used in the activity of multiplication on algebraic expressions.

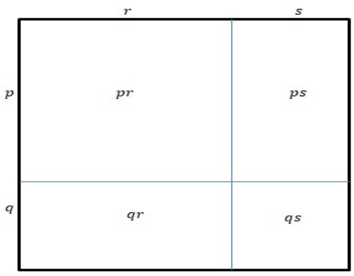
Consider the following figure. Its area is $a \times (b + c) = ab + ac$.




**Task 5.** Based on the process of finding the area of the figure, find:
(a) $3(x + y) = \cdots$
(b) $4(2x + 3y) = \cdots$
(c) $2x(x + 5y) = \cdots$
(d) $3x(4x + 3y) = \cdots$

Consider that

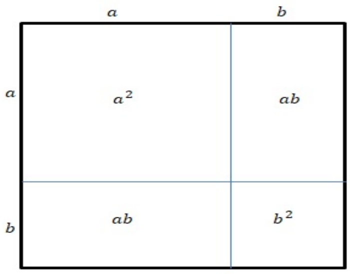
$$(p + q).(r + s) = p.r + p.s + q.r + q.s$$
$$= pr + ps + qr + qs$$




**Task 6**. Based on the process of multiplication of $(p + q).(r + s)$ as shown, find:
(a) $(m + n)(x + y) = \cdots$
(b) $(2p + q)(r + 2s) = \cdots$
(c) $(x + 1)(y + 2) = \cdots$
(d) $(x + 2)(y - 2) = \cdots$
(e) $(3x + 2)(2y - 3) = \cdots$

Consider that $(a + b)^2 = (a + b)(a + b)$.

$$(a + b).(a + b) = a.a + a.b + b.a + b.b$$
$$= a^2 + ab + ba + b^2$$
$$= a^2 + 2ab + b^2$$




**Task 7**. Based on the process of multiplication $(a + b)^2 = (a + b)(a + b)$ as shown, find:
(a) $(p + q)^2 = \cdots$
(b) $(2a + 3b)^2 = \cdots$
(c) $(a + 3)^2 = \cdots$
(d) $(2a + 1)^2 = \cdots$
(e) $(a - 1)^2 = \cdots$

The learning goal for the third activity, i.e., multiplication on algebraic expressions, is to guide students to understand the process of multiplication on algebraic expressions using the context of area of a rectangle. The learning process starts with a simple task of finding the area of combination of two rectangles shown in Task 5 in Table 3. The activity in Task 6 deals with multiplication on algebraic expressions of the form $(p + q).(r + s)$, and Task 7 deals with an algebraic expression of the form $(a + b)^2$. Based on this activity, students are expected to be able to solve multiplication on algebraic expressions.

Three main components, including the learning goal, the learning activity, and the expectation of student learning process, that guide the description of the learning sequence above, are called a *hypothetical learning trajectory* (HLT) [11]. With this HLT idea in mind, we can do a design process of a learning sequence in an appropriate manner.

## 4. Conclusion

Based on the description in the previous section, we draw the following conclusion. The learning sequence for the topic of operations on algebraic expressions starts from introducing simple addition of algebraic expressions to addition, subtraction, and multiplication on algebraic expressions. This learning sequence uses the context of perimeter and area of a region—as a combination of rectangles. According to the RME theory, this context intertwines the domain of algebra and geometry in a meaningful manner for students. Also, the reality and level principles that underpin the presentation of tasks play an important role to support student understanding. In this way, student difficulties in algebraic expressions, such as lack of closure and parsing obstacles, are expected to be reduced. For further research, we wonder whether the designed learning sequence can be implemented successfully and whether the difficulties can be reduced are interesting to investigate.

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