Investigating Primary School Mathematics Teachers' Deductive Reasoning Ability through Varignon's Theorem

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Investigating Primary School Mathematics Teachers' Deductive Reasoning Ability through Varignon's Theorem

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Abstract. The responsibility to promote the growth of deductive reasoning ability of school students through learning mathematics is in the hand of mathematics teachers and particularly primary school mathematics teachers. However, how we can make sure whether teachers are able to do so. To investigate this issue, we conducted a three-step of an exploratory survey study. First, we designed tasks from the Varignon's theorem. Second, we administered an individual written test involving twenty master students of primary education program, in which they are prospective of and primary school mathematics teachers. Finally, we address the results in the light of Van Hiele theory. The results showed that participated students lack of deductive reasoning ability in the context of geometry. For further research, we wonder whether the designed tasks are also applicable to assess student deductive reasoning ability if the students have acquired appropriate teaching.

1. Introduction

In mathematics, geometry is considered a rich topic to promote student deductive reasoning ability ([1], [2]). This indispensable topic is therefore included in school mathematics curriculum over the world [3], including in Indonesia, from elementary to secondary school [4]. The success of promoting the growth of the deductive reasoning through learning mathematics is of course in the responsible of mathematics teachers and particularly primary school mathematics teachers as the spearhead of the mathematics curriculum implementer (e.g., [5]) in the primary school level. The question is how we can make sure that (prospective and) mathematics teachers in the primary school level have ability in fostering student deductive reasoning.

To deal with this issue, we have conducted an explorative survey study to investigate the ability of prospective and primary school mathematics teachers in applying their geometry knowledge on various quadrilaterals and their properties to solve Varignon problem. The Varignon problem concerns tasks that are designed according to Varignon's theorem [6]. The theorem states that *the figure formed when the midpoints of the sides of a quadrilateral are joined in order is a parallelogram, and its area is half that of the quadrilateral*. This theorem is named after the French mathematician, Pierre Varignon (1654-1722), proved it [7].

The theoretical framework to use for the purpose of the study is the Van Hiele theory on the development of geometric thought. According to this theory, student geometric thought can be classified into five levels: visualization, analysis, abstraction, deduction, and rigor ([8], [9], [10], [11]). The ability to identify geometric shapes according to general appearance without attention to properties of the shapes concerns the visualization level. For example, the student recognizes the form of a square and considers it as a different shape from a rectangle. In the analysis level, the student can recognize properties of geometric shapes as discrete entities, such as that a rectangle has two pairs of sides having the same length and each angle is a right angle. In the abstraction level, the student can relate between properties of geometric figures. For example, a square is recognized as being a rectangle as it has all properties of the rectangle. In the deduction level, the student is able to reason deductively, i.e., s/he is able to read, to understand and to do proofs in the context of geometry. For example, the student is able to prove that the opposite angles of a parallelogram have the same sizes.

Finally, in the rigor level, the student understands axiomatic systems in geometry. For example, the student is able to compare between Euclidean and non-Euclidean geometry without using concrete models. Important to note about this theory is that the levels show a progression of student geometric reasoning ([8], [9]); also the levels are invariant, sequential, hierarchical, and the progress depends largely on the instruction, not age ([8], [12]).

2. Experimental Method

To investigate primary and prospective school mathematics teachers' ability in deductive reasoning, we carried out a three-step of an exploratory survey study involving 20 master students of primary mathematics education program. Of the twenty, seven of the students have experienced as primary school mathematics teachers and the rest are having relevant educational background to be mathematics teachers in primary school level. In the first step, we designed two tasks, adapted from Oliver's work based on Varignon's theorem [13], to assess student deductive reasoning ability. We consider the tasks can be solved by geometry knowledge—such as knowledge on various quadrilaterals and their properties—that should have been mastered by primary school mathematics teachers. The tasks, including tasks A and B, are presented in Figure 1. From the perspective of the Van Hiele theory, the tasks require students to do mathematical proofs and as such assessing their deductive reasoning ability. Second, we administered an individual written test, lasted for 60 minutes, and called for students to provide reasons for their solutions. Finally, we analyzed student written work in the light of the Van Hiele theory.

A. What figure is formed if the consecutive midpoints of the sides of a quadrilateral are joined? What if the original quadrilateral were: a square, a rectangle, a kite, a rhombus, an isosceles trapezoid, a parallelogram, an arbitrary quadrilateral, respectively? Give appropriate reasons why you think your answer is true.

B. Find the proportion of the area between each of the original quadrilaterals and the new figures, and provide reasons for your answer.

Figure 1. Designed tasks based on Varignon's theorem (adapted from [13]).

3. Result and Discussion

Table 1 summarizes results of master student written work on Tasks A and B. The columns #Correct and #Reason mean the number of correct answer and the number of correct answer with reasoning, respectively. We found that all reasons given by students are in terms of mentioning properties of a type of quadrilateral that formed from joining the four midpoints of the original quadrilateral, but are not in terms of formal proving and argumentation. This surprising result showed that the involved master students lacked of deductive reasoning ability.

Table 1. Summary of student written work on Tasks A and B

Type of Quadrilateral	Task A		Task B	
	#Correct (%)	#Reason (%)	#Correct (%)	#Reason (%)
Square	12 (60)	8 (40)	5 (25)	4 (20)
Rectangle	12 (60)	11 (55)	8 (40)	5 (25)
Kite	19 (95)	12 (60)	3 (15)	3 (15)
Rhombus	6 (30)	4 (20)	4 (20)	4 (20)
Isosceles Trapezoid	12 (60)	11 (55)	5 (25)	2 (10)
Parallelogram	12 (60)	8 (40)	4 (20)	4 (20)
Arbitrary Quadrilateral	7 (35)	5 (25)	7 (35)	0 (0)

Notes. N: 20 master students

For the Task A, we found that the case of a kite as the original quadrilateral be the most frequent of correct answer and providing reasons, and the rhombus being the least frequent. For the Task B, the result showed that most of students (40%) are able to answer correctly for the case of a rectangle, but surprisingly they seemed to find it difficult for the case of a kite. For the case of an arbitrary quadrilateral, which is the general case for the Varignon's theorem, we found that even if 35% students are able to answer correctly for Tasks A and B, reasons provided by students are not convincing and seemed as a result of guessing. To illustrate the results shown in Table 1, below we address for the case of the rectangle and the arbitrary quadrilateral.

Figure 2 presents solutions for the designed tasks for the case of the rectangle. The Figure 2 (left part) shows reasons to conclude that $EFGH$ is a rhombus, and the right part shows the proof that the proportion of the area of $EFGH$ and the area of $ABCD$ is $1:2$. Figure 3 presents student written work for the case of the rectangle. Even if there is no formal argumentation, the left part of Figure 3 shows a correct answer and reasoning both for Tasks A and B; while the right part of Figure 3 shows an incorrect argumentation for Task B—rather than providing general case, the student provides special measures of a rectangle. In the light of Van Hiele theory, this result provides evidence that the participated students lacked of deductive reasoning ability ([8], [9]).
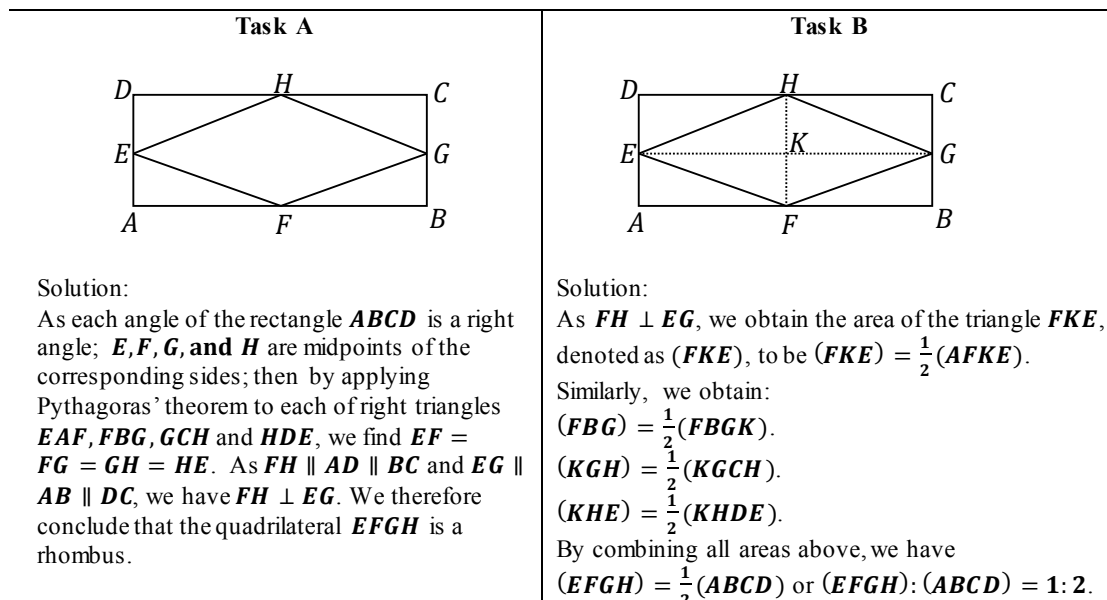
Task A	Task B
Solution: As each angle of the rectangle $ABCD$ is a right angle; $E, F, G,$ and H are midpoints of the corresponding sides; then by applying Pythagoras' theorem to each of right triangles EAF, FBG, GCH and HDE, we find $EF = FG = GH = HE$. As $FH \parallel AD \parallel BC$ and $EG \parallel AB \parallel DC$, we have $FH \perp EG$. We therefore conclude that the quadrilateral $EFGH$ is a rhombus.	Solution: As $FH \perp EG$, we obtain the area of the triangle FKE, denoted as (FKE), to be $(FKE) = \frac{1}{2}(AFKE)$. Similarly, we obtain: $(FBG) = \frac{1}{2}(FBGK)$. $(KGH) = \frac{1}{2}(KGCH)$. $(KHE) = \frac{1}{2}(KHDE)$. By combining all areas above, we have $(EFGH) = \frac{1}{2}(ABCD)$ or $(EFGH):(ABCD) = 1:2$.

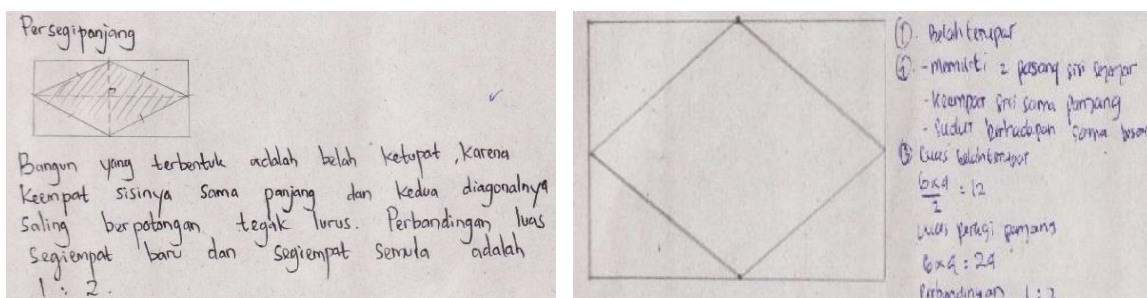**Figure 2**. Designed tasks and their solutions for the case of a rectangle

Figure 3. Representative examples of student written work for the case of a rectangle

Figure 4 presents the proof of Varignon's theorem, i.e., for the case of an arbitrary quadrilateral, which is adapted from ([6], [7], [13]). Figure 5 presents representative examples of student written work which shows an inability of students to do a formal proof, using deductive reasoning, for the case of an arbitrary quadrilateral. This means that the participated students are still in the abstraction level, and not in the deduction level ([8], [9]).
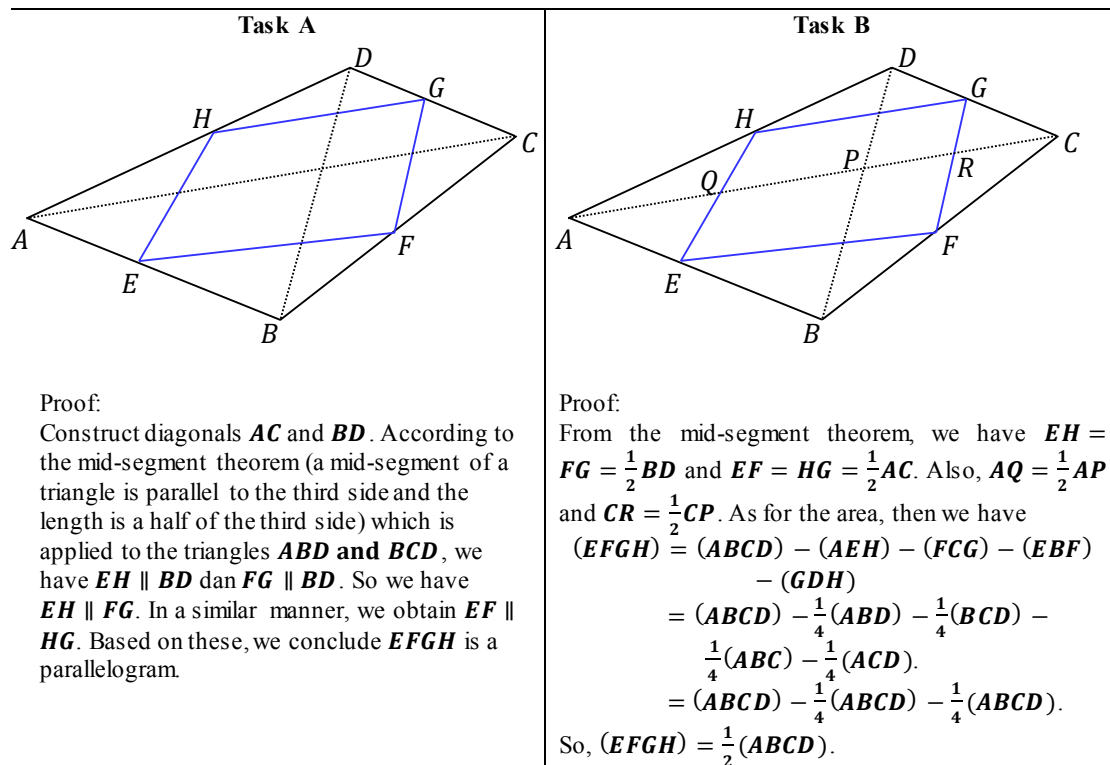
Task A	Task B
Proof: Construct diagonals AC and BD. According to the mid-segment theorem (a mid-segment of a triangle is parallel to the third side and the length is a half of the third side) which is applied to the triangles ABD and BCD, we have $EH \parallel BD$ dan $FG \parallel BD$. So we have $EH \parallel FG$. In a similar manner, we obtain $EF \parallel HG$. Based on these, we conclude $EFGH$ is a parallelogram.	Proof: From the mid-segment theorem, we have $EH = FG = \frac{1}{2}BD$ and $EF = HG = \frac{1}{2}AC$. Also, $AQ = \frac{1}{2}AP$ and $CR = \frac{1}{2}CP$. As for the area, then we have $$(EFGH) = (ABCD) - (AEH) - (FCG) - (EBF) - (GDH)$$ $$= (ABCD) - \frac{1}{4}(ABD) - \frac{1}{4}(BCD) - \frac{1}{4}(ABC) - \frac{1}{4}(ACD).$$ $$= (ABCD) - \frac{1}{4}(ABCD) - \frac{1}{4}(ABCD).$$ So, $(EFGH) = \frac{1}{2}(ABCD)$.

Figure 4. The proof of Varignon's theorem (adapted from [6], [7], [13]).
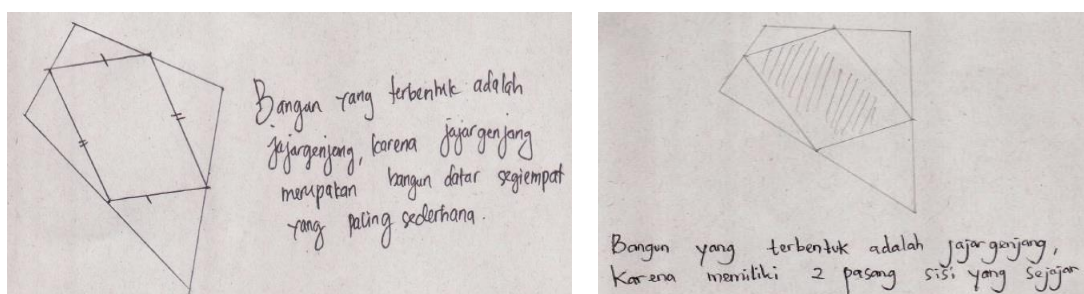

Figure 5. Representative examples of student written work for the case of an arbitrary quadrilateral

From the results above, we consider three points to discuss. First, the result showing the lack of master students' deductive reasoning ability implies that the teaching of mathematics courses for primary school mathematics teachers should be improved and be more emphasized on using deductive reasoning through, for instance, doing mathematical proofs in geometry topics. Also, the mathematics courses should address contents not only for primary school level, but also for fostering deductive reasoning of the prospective mathematics teachers themselves. Second, as dynamic geometry software

is currently available for free, such as GeoGebra, it would probably be more interesting and easier for students if the investigation of the Varignon Problem is by using this software. By using this in the teaching process, the students will be encouraged to explore the problem in a dynamic way. Also, the theorem probably can firstly be investigated informally for other two-dimensional shape cases before doing a formal proof. Finally, the proof of Varignon's theorem in Figure 5 holds for the case of a convex quadrilateral, and as such for further investigation we need to provide proofs that hold for convex and crossed quadrilaterals.

4. Conclusion

From results described in the previous section, we draw the following conclusions. The result showing the lack of deductive reasoning ability of the (prospective and) primary school mathematics teachers, as investigated through the Varignon problem, encourage us as university teachers to improve the content of mathematics courses and to emphasize the use of deductive reasoning through doing mathematical proving and problem solving. The use of tasks designed from Varignon's theorem seems effective to investigate students' deductive reasoning ability in the context of geometry. The tasks directly call for students to use their geometry knowledge on properties of the quadrilateral and triangle—which are two main contents of primary school mathematics. For further research, we wonder whether the designed tasks are also applicable to assess student deductive reasoning ability if the students have acquired appropriate teaching in geometry.

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