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# Analysis of Junior High School Students' Attempt to Solve a Linear Inequality Problem

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Abstract. Linear inequality is one of fundamental subjects within junior high school mathematics curricula. Several studies have been conducted to asses students' perform on linear inequality. However, it can hardly be found that linear inequality problems are in the form of "ax + b < dx + e" with " $a, d \neq 0$ ", and " $a \neq d$ " as it can be seen on the textbook used by Indonesian students and several studies. This condition leads to the research questions concerning students' attempt on solving a simple linear inequality problem in this form. In order to do so, the written test was administered to 58 students from two schools in Bandung followed by interviews. The other sources of the data are from teachers' interview and mathematics books used by students. After that, the constant comparative method was used to analyse the data. The result shows that the majority approached the question by doing algebraic operations. Interestingly, most of them did it incorrectly. In contrast, algebraic operations were correctly used by some of them. Moreover, the others performed expected-numbers solution, rewriting the question, translating the inequality into words, and blank answer. Furthermore, we found that there is no one who was conscious of the existence of all-numbers solution. It was found that this condition is reasonably due to how little the learning components concern about why a procedure of solving a linear inequality solution.

## **INTRODUCTION**

Learning linear algebraic inequality is fundamental. One of the reasons is that it is the first inequality which students deal with. Hence, a good understanding on this inequality will help students to acknowledge further inequality. In the accordance that this is an important material, it is not considered as a main topic within Indonesian curricula [1]. To exemplify, inequality has become subordinate subject relating to equation. With this circumstance, how will it be likely in the students' perspective?

Several studies were conducted on this inequality. In Indonesia, it is clearly revealed that students faced difficulties when trying to solve linear inequality tasks [2]. Moreover, they failed to do operations involving algebraic form, such as simplifying "4x + 7" to "11x". This result shows that they treat it as same as dealing with linear equation i.e. using algebraic manipulations. Although this method is acceptable, it can lead to a misconception that is to solve an inequality means to solve appropriate equation [3]. It is not surprising since the problems in both of the inequality and the equation, apart from equal symbol, are similar.

Most questions in linear algebraic inequality tasks are in the form of "ax + b < c" and "ax + b < dx + e" with " $a, d \neq 0$ ", and " $a \neq d$ ." It can be seen from textbooks used by students such as two books entitled '*Matematika:* Konsep dan Aplikasinya untuk Kelas VII SMP dan MTs', and '*Penunjang Belajar Matematika untuk SMP/MTs Kelas VII*" [4,5]. In addition, this type was used in another research [2]. Students tend to approach the problems in this type algorithmically and analogically by using the similar technique used for equation [6,7]. But, what outcome that will be found if a question in another type is provided to the students?

The 4th International Conference on Research, Implementation, and Education of Mathematics and Science (4th ICRIEMS) AIP Conf. Proc. 1868, 050033-1–050033-12; doi: 10.1063/1.4995160 Published by AIP Publishing. 978-0-7354-1548-5/\$30.00 This condition leads to a question: How do junior high school students answer a simple question such as "2x + 1 > 2x - 2"? Although it is simple, this kind of question has a potential chance to reveal how students actually think about solving linear inequality problems [3]. This study aims to answer that question in case of Indonesian students. In addition, apart from this, we also focus on another similar linear inequality problem in the form of "ax + b < dx + e" with " $a, d \neq 0$ ", and " $a \neq d$ ," like "3x + 1 > 2x - 1". By this way, analysis can be done in such a way so the result and review in both of the two inequalities can support one another.

## THEORETICAL BACKGROUND

#### **Linear Inequality**

Most of books used by the students stated that linear algebraic inequality (or simply linear inequality) is an open sentence which contains linear algebraic form i.e. ax + b,  $a \neq 0$  and inequality symbol i.e.  $\langle , \rangle, \leq , \geq$ , such as 2x + 1 < 3 and  $3x + 4 \ge 2x - 7$ . In this definition, open sentence means a sentence that cannot be decided yet whether it is true or not. Considering this definition of open sentence, it seems hard to decide whether  $x^2 + 1 > 0$ ,  $\log x < 0$ , |x| < 0, and x + 1 > x - 1 are inequality or not. Nevertheless, we found a better definition of open sentence which states that it is left to you what the unknown is [8]. Regarding this, a sentence like the above will be considered as an inequality.

It can be seen in most of the books that all numbers replacing the unknown which makes the sentence true are called solution. This definition leads to three kind of linear inequality's solutions: The first is an empty set  $\emptyset$ ; The second is particular subset of real number in the form  $\{x \in \mathbb{R} | x > (\text{or } <) a\}$  for a particular number a; The third is a whole real number  $\mathbb{R}$ . To exemplify  $\emptyset$  is solution for  $2(x-2) \ge 1 - (3-2x)$ ,  $\{x \in \mathbb{R} | x > 1\}$  is solution for 2x + 1 > 3, and  $\mathbb{R}$  is solution for x + 1 > x - 1. Then, how we get such solution from the problems?

According to [3], "to solve" means "to find all acceptable values of the variable that will make the sentence true". Nevertheless, in what way we do that? Henry O. Pollak said that some operations like adding and multiplying with non-zero number can be done as long as the truth set does not change [8]. On the other hand, some researcher suggested to use functional approach or graphical approach for solving the inequality [7,9,10]. But, what techniques mostly used by the students when solving inequality problems?

There are various approaches used by students when facing inequality problems. Apparently, there are, at least, three kinds of techniques used by students: the first is immediate solution without algebraic manipulations; the second is using a rule; the third is using the number line [7]. Besides, it was found that there are two most-used techniques which are the algorithmic (algebraic manipulations) approach and the verbal examination of the given expressions [11]. However, while some students successfully used these techniques, others did not.

Some researchers have addressed studies to conceal how students solve a simple linear algebraic inequality which has null set and all real numbers solution. One of the results is a study of university students' way of solving 5-3(2-x) > 4-3(1-x) [12]. It is revealed that there are three category: the first is those who got 0 > 2 or -1 > 1 and added that the inequality is not satisfied by any real number; the second is those who got 0 > 2 or -1 > 1, but did not add anything further; the third is those who incorrectly did operations. Another study observed university students way of solving 9(x + 1) > 9(x - 2) [3]. There are four categories from this research: the first is those who answer no solution for this problem; the second is those who write down 27 > 0 or 9 > -18 but not stated anything further; the third is those who only write down 9x + 9 > 9x - 18; the fourth is those who answer that the solution for this problem is the whole real number. From these results, it can be assumed that some students face difficulties when using the techniques.

## **Difficulties in Linear Inequality**

The difficulties possibly come from previous learning or linear inequality learning itself. According to [2], there are several difficulties confronted by the students concerning linear equation and inequality as follows:

- 1. A student makes mistake when carrying out addition, subtractions, multiplication, or division of numbers.
- 2. A student makes mistake when carrying out addition, subtractions, multiplication, or division of algebraic expressions.
- 3. A student does not follow the rules of order of arithmetical operations in numerical expressions.
- 4. A student misapplies a commutative property in calculating a division in numerical expressions.
- 5. A student misuses a distributive property of a multiplication over an addition in algebraic expressions.

- 6. A student does not use an additive or multiplicative inverse in solving an equation.
- 7. A student interprets a symbol has only a single rather than more than one value.
- 8. A student substitutes a literal symbol in an equation with a particular value and the result is incorrect.
- 9. A student experiences a conflict between the order in natural language and in algebraic language.
- 10. A student expects to have a numerical answer for an algebraic expression.
- 11. A student adds or subtracts algebraic terms and numbers to get an algebraic term within an algebraic expression.
- 12. A student ignores local salience in an algebraic expression, such as the inequality sign <, the variable x, positive or a negative sign of an algebraic term.
- 13. A student ignores pattern salience in algebraic expression, such as an algebraic expression with two terms and within a bracket.
- 14. A student does not understand the meaning of the equal sign as algebraic equivalence, such as the student makes a notational error as a result of a combination of operations.
- 15. A student does not understand the meaning of the equal sign as algebraic equivalence, such as the student ignores the equal sign and applies an incorrect simplification on algebraic expression.
- 16. A student mistranslates words or phrases into mathematical notations.
- 17. A student fails to formulate an equation or an inequality from the given word problem.
- 18. A student encounters a difficulty in interpreting a mathematical concept and pattern, in substituting information into a formula and in using a formula.
- 19. A student encounters difficulties in combining, in integrating, or in using information either given in the task or given as a result of calculation in solving symbolic algebra problems.
- 20. A student uses an arithmetical method to solve symbolic algebra problems.
- 21. A student encounters a difficulty in manipulating symbol when solving symbolic algebra problems.
- 22. A student misapplies equation solving when simplifying algebraic expressions.

In addition, the difficulties can be seen in the further learning. As in this case, difficulties in linear inequality are seen on quadratic inequality. More specifically, Indonesian students presented these kinds of answer which are a blank answer and solving corresponding equation [13]. It means that students let the answer sheet to be not written and solved the inequality by solving an equation which is resulted by merely replacing the inequality sign with equal sign.

## **Research Question**

Taking all above into account, as mentioned in the introduction that this study concerns on how junior high school students answer 9x + 1 > 9x - 2 problem, this study proposes the research questions as follows:

- 1. What are the students' typical answers on solving 9x + 1 > 9x 2 problem and related inequalities?
- 2. What type of difficulties that students face when solving the problem?
- 3. What are the students' explanations behind their answer?
- 4. What are possible sources of these difficulties found from teachers and the textbooks perspective?

## METHODOLOGY

To answer the research question, we offered an individual written test followed by student interviews based on the student's work. In addition, we interviewed the teachers and analysed the text book, allowing us to have a slight look into the learning process.

## Sample

Participants in this study are 58 Indonesian junior high school students. Particularly, they were in grade VIII, as they enrolled linear inequality subject [1]. These students are from two different school, public school and private school. The aim of this choice is to represent both kinds of schools.

#### **Data Collection**

The data were collected from two schools in 2016. First of all, written test containing the problem was held. The students were asked to solve linear inequality task in 60 minutes that contains the "9x + 1 > 9x - 2" problem. During the test, the students were told that this test does not affect their final assessment, so they will feel free to express their answer based on what they think. Few days after the test, the students were interviewed.

Secondly, semi-structured interview was addressed to explore the reason behind the student's work. The students were encouraged to show their reasoning without any interventions. There were several questions such as "why did you make this step?", and "what did you mean by this step?" during the interview.

Thirdly, we interviewed two mathematics teachers teaching mathematics subject from the schools using semistructured interview. The goal of this interview is to have an insight about their perspective about linear algebraic inequality learning. In addition, to broaden the insight the text book was analysed.

#### Task

This research study used a paper-pencil task containing "9x + 1 > 9x - 2" problem. The design of this task was motivated by two tasks which are 9(x + 1) > 9(x - 2) and 5 - 3(2 - x) > 4 - 3(1 - x) [3,12]. Considering that many Indonesian students have difficulty to use distributive law, this study did not use the bracket used in both previous task [2]. Moreover, the aim of this absence is that to look what possibly students do after their arrive at 1 > -2 or 0 > -3. Though, it may vary on how they think about linear inequality.

This study addressed the problem for revealing the students' understanding on the meaning of "to solve", and the existence of all-real-numbers solution of an inequality. When they think that "to solve" an inequality means to get the value of the variable as in an equation problem i.e. by algebraic manipulations, it leads to puzzlement as they realize that the variable vanishes. In addition, it seems hard for them to be aware of the original inequality as they get 1 > -2 or 0 > -3. However, to foreseen about what students may answer, a pre-research test is held, as well as for validation purpose.

In addition, we also provide this problem: 5x + 1 > 3x - 2, which is usually found in the text books. By this way, we can compare the results of both the two inequalities and try to figure out what we can learn from it.

#### **Possible Responses**

Based on the validation test combining with the findings from other researches the prediction of students' respond was made as seen on Table 1 [2,3,12,13].

Question	Possible Answer
Find all real number which	Blank answer
satisfy this inequality	Doing operations incorrectly
9x + 1 > 9x - 2	Doing operation correctly and get $1 > -2$ or $0 > -3$ , but not doing anything further
	Getting $1 > -2$ or $0 > -3$ and stating that the inequality has all-real-number solutions
	Getting $1 > -2$ or $0 > -3$ and stating that the inequality has no solutions
	Immediately stating that the inequality has all-real-number solution
	Immediately stating that the inequality has no solution

TABLE 1. The Prediction of Students' Respond

#### **Data Analysis**

The data of this study contain of students' worksheets, students' interview, teachers' interview, and textbooks used by students. Generally, we analysed these data using constant comparative methods [14]. Firstly, students' worksheets were analysed to find the categories of students' attempts when solving the problem and the difficulties faced by students. After that, we analysed students' interview to find possible explanation behind students' work. Finally, teachers' interview, and textbooks used by students were analysed to have an insight about learning process.

#### RESULT

It can clearly be seen from the pie chart (Fig. 1) that most students have a tendency to approach the question using algebraic operation i.e. doing operation in both of left and right hand side of the inequality simultaneously. In contrast, only few of them used non-algebraic approach.



**FIGURE 1.** Student's Approach on Solving 9x + 1 > 9x - 2 (left) and Solving 5x + 1 > 3x - 2 (right)

Going into more detail, Table 2 summarizes the category of students' answers on solving 9x + 1 > 9x - 2 found in the writing task done by student providing with the number and the percentage of students who done the category. The majority of students do operations to get the result but fail to do that correctly. Nevertheless, there are five students who do the operations correctly, but fail to get the conclusion that all real numbers satisfy the inequality. Besides, other students give blank solution, rewrite the inequality, translate the inequality into words, and provide whole-number solution.

It is interesting to notice, from Table 2 and Table 3, that in doing-operation-incorrectly and doing-operationcorrectly categories, there are considerable differences accounted for 11% and 22% respectively. These imply that there are fewer students who did operations incorrectly on solving 5x + 1 > 3x - 2. On top of that, we may ask what makes this difference, why students did well in one problem but failed in another and what step that can be highlighted according to this situation. The answer will be discussed later. By contrast, the other categories seem similar in both of the two problems with negligible differences.

No	Ans	swer Types	Private School N: 25 NTota	Public School N: 33 al: 58
1	Doing operations incorrectly		14 (56%)	20 (60%)
		Total	34 (5	<sup>5</sup> 9%)
2	Blank answer		2 (8%)	6 (18%)
2		Total	8 (14	4%)
2	A number or several numbers solution		6 (24%)	1 (3%)
3		Total	7 (12	2%)
4	Doing operation correctly and getting further	1 > -2 or $0 > -3$ , but not doing anything	0 (0%)	5 (15%)
		Total	5 (9	%)
5	Rewriting the question		3 (12%)	1 (3%)
5		Total	4 (7	′%)
6	Translating the inequality into words		3 (12%)	0 (0%)
0		Total	3 (5	5%)

**TABLE 2.** Student's Answer Types on Solving 9x + 1 > 9x - 2

No	Answer Types	Private School N: 25	Public School N: 33
		NTota	a: 58
1	Doing operations incorrectly	13 (52%)	9 (27%)
1	Total	22 (3	8%)
2	Blank answer	1 (4%)	4 (12%)
Z	Total	5 (9	9%)
2	A number or several numbers solution	7 (28%)	1 (3%)
3	Total	8 (1-	4%)
4	Doing operation correctly or/and getting $x > -\frac{3}{2}$ , but not doing anything further	0 (0%)	18 (55%)
•	Total	18 (3	1%)
5	Rewriting the question	3 (12%)	1 (3%)
3	Total	4 (7	′%)
6	Translating the inequality into words	1 (4%)	0 (0%)
0	Total	1 (2	2%)

**TABLE 3.** Student's Answer Types on Solving 5x + 1 > 3x - 2

## **Students' Answers**

To explain the students' typical answers on solving 9x + 1 > 9x - 2 problem and related inequalities in more detail, each of six students' answer types will be explained. It is provided with the example of students' works.

The first category is that students used algebraic operations incorrectly. Among fifty eight students, there are thirty four students performing this. This can be represented by Cahaya's work which can be seen in Fig. 2.

Jawab:				
Ju+1 >	94-2			
=7 021-920 >	-2 -1			
=7 20 >	-3			
		•		

#### FIGURE 2. Cahaya's Work

As it can be seen from the figure, Cahaya used algebraic manipulation correctly in his first step. He subtracted both sides of equation by 9x and 1 by writing down 9x - 9x > -2 - 1. In the next step, -2 - 1 is simplified into -3. Interestingly, he simplified 9x - 9x into x instead of 0.

The second category is that students left the answer sheet blank. Of course, it is due to their lacking of knowledge how to solve a linear inequality problem in this type.

The third category is that there are some students who stated a number or several numbers as the solution. For instance, it is illustrated by Fig. 3.

awab:	991	11 = 100	799-2=	97	
				-	

#### FIGURE 3. Gita's Work

It is clearly found that Gita replaced 9x with 99. Furthermore, the result of addition operation in the left-hand side was written in the same line as well as in the right-hand side. Yet, this complex mathematical sentence is accepted mathematically.

The fourth category is that some students did algebraic operations correctly and get 1 > -2 or 0 > -3. Still, they did not write anything further. To illustrate it, Fig. 4 is given.

Jawab:		
92+1792-2		
ラア - タン フー2-1		
0 7 - 3		

## FIGURE 4. Bayu's Work

Apart from his friends, Bayu did the inequality well and got the right final inequality which is 0 > -3. Notwithstanding, he did not state what kind of numbers that could replace the x so the inequality become a right sentence.

The fifth category is that some students rewrote the question. Similar with second category, it is because the students do not know how to solve it. Fig. 5 stands as an illustration.

lawab: 0	x + 1	79x.	-2			

#### FIGURE 5. Nadia's Work

It is clearly presented in the above figure that Nadia only write down the inequality again as same as in the question.

The last category is that students translated the inequality into words. As an example, Fig. 6 can be seen.

wab:					
9	(dsh	bosar	kari	2	

FIGURE 6. Satrio's Work

From the figure, it can be seen that Satrio tried to change the inequality into words. However, he did it incorrectly.

Taking all above categories into account, our findings indicate, from the majority, that to solve means to find the simplest form of an initial linear inequality problem. By "the simplest" we mean an algebraic form which contains only one variable standing in left-hand side (or right-hand side) of inequality with "1" as its coefficient, such as x < 3 and  $6 \ge x$ . There are only negligible numbers of students who seemingly think that solving linear inequality means to find a particular set containing all number satisfying the inequality.

	7. $9x + 1 > 9x - 2$
Jawab: Su + 1 > 3u - 2 (=7 5u - 3u > -2 - 1) (=7 2u > -3) (=7 u > -3)	Jawab: Su +1 > Su -2 1=7 OU - SU > -2 -1 1=7 U > -3
2 (=7 22 > -1,5	-

FIGURE 7. Cahaya's Work on Both Problems

To answer the question why there are more students who did operations correctly on solving 5x + 1 > 3x - 2 than those who did well on answering 9x + 1 > 9x - 2, we tried to compare students' answers on both questions carefully. It is revealed that most of the students in this situation performed well on simplifying 5x - 3x to 2x but wrongly did 9x - 9x simplified to x. To exemplify, Fig. 7 can be looked. In order to know how many students did this, Table 4 is provided.

Category	Public School	Private School	Total
Doing $9x - 9x$ simplified to x	14	4	18
Simplifying $5x - 3x$ to $2x$ and simplifying $9x - 9x$ to $x$	11 (79% of 14)	3 (75% of 4)	14 (78% of 18)

**TABLE 4.** Student's Answer Types on Solving 5x + 1 > 3x - 2

It is striking to be noticed that almost all of students who performed well on simplifying 5x - 3x to 2x, wrongly did 9x - 9x simplified to x. More specifically, there are 14 out of 18 or 78% of the total.

On the one hand, we found several difficulties faced by students from learning-difficulties perspective. The difficulty is not considered as a failure that a student did. Rather, it can be seen as a process of acquiring new understanding. Many students have related operations difficulty which means they made mistake when carrying out addition, subtraction, multiplication, or division of algebraic expression. It is found that many performed 9x - 9x simplified into x. Some faced lack of closure obstacle meaning they add or subtract algebraic terms and numbers to get an algebraic term. Another difficulty is lack of gestalt view which means that a student ignored local salience in an algebraic expression. Expected-answer-obstacle difficulty is also found that mean some expected to have numerical answer for an algebraic expression.

In addition, from all students' performance, it seems that they do not pay attention about what the solution of an inequality exactly means. Most students answer merely by mechanic style which is doing operations to get the simplest form of the inequality. No one stated in his work that the solution is a set of particular numbers (or a number).

Notwithstanding, these finding coincide with pervious study with sample of university student. Looking at detail, only 23% of the university student who were prospective mathematics teacher solved the problem correctly [3]. On the other hand, some pre-university students in their first-year class did operational mistakes, and did not arrive at correct answer [12].

## **Findings from Students' Interview**

This study is curious about how come students did simplify 9x - 9x into x. For this purpose, we interviewed two students from different school separately. One of them, not surprisingly, did the question carelessly while that x cannot be vanished is believed by another. To demonstrate the interview, Fig. 8 is provided.

7. $9x + 1 > 9x - 2$	R: Researcher S: Surya R: Well let us see how this can be like that (pointing $9x - 9x$ and $x)^2$ Do you
Jawab:	remember what did you think when you did this? Do you mean $9x - 9x$ and $x$ ? Do you
9x - 9x > 2 -1	S: Like what I said before that x must be in the left-hand side. When it is moved to another side, it becomes negative. So here I wrote $9x - 9x$ which becomes x.
~ > - 3 //	R: Okay, so you mean $9x - 9x$ becomes x.
7-3 x "	S: Yes
	R: Why it can be like that?
	S: Because, $x$ is fixed.
	R: So, do you mean that $x$ must be exist?
	S: Yes, that is the essence.
	FIGURE 8. Surya's Work and Interview

From the above, we clearly know that Surya think that x must not be vanished. It is plausible that it was due to the routine that we always solve linear equation and inequality by forcing x to be still existed in the left-hand side. Furthermore, students may think that it seem unreasonable that x is vanished at a time when they are trying to figure out what x is. This seems to be closely similar with previous finding that a student seemingly see symbol x as an unknown number that he try to figure out what it is [16].

## **Findings from Teachers' Interview**

From two teachers' interview, it is found that teachers think that the key of linear inequality learning is to strengthen students' understanding on linear equation. By this understanding, they think that students will do well in linear inequality problems. They think that what we do in solving linear inequality is quite similar in linear equation. This coincides with another study finding an intuitive belief that solving inequalities and equations are the same process [6]. Clearly, thinking that linear inequality has similar properties with linear equation is not misunderstanding. Despite that, this can lead to over simplifying by do not offer much attention to the linear inequality lesson. The teacher may miss to explain what the meaning of an inequality sign is, what the difference between linear inequality and linear equation is, and what the meaning of the solution of it is [6,7,12].

In addition, one of them said that we must explain how to deal with multiplying or dividing both side with a negative number, since it is the difference between a linear inequality and a linear equation. It reveals that the teacher focus on how to do algebraic expression well. Definitely, procedural fluency is necessary. But, the result shows how many students did not do algebraic operation well. This might be a consequence of students' lack of knowledge beyond the procedure. Therefore, as pervious research suggest, learning focus should be not only on how to do procedure, but also on understanding why the procedure works [2]. To exemplify, see the transcript of teacher's interview in figure 9.

Regarding the teacher's view of this concept, it is not surprised that most students face the difficulties. Certainly, teachers' knowledge can affect on the content that they deliver to the student and the way they build the class activities. This is in a line with what Paul Ernest said that mathematics teachers' beliefs have a powerful impact on the practice of teaching [17].

R: Researcher H: Hadi
R: Let us begin our interview. Firstly, I would like to ask you how to deliver linear inequality in one variable concept based on your experiences. This topic is after linear equation.
H: So, the core of teaching linear inequality in one variable is to give complete understanding on linear equation. It is believed that if someone has bad understanding on linear equation, he will fail on linear inequality. So, it is needed to make sure that the students know well hot to do operations such as subtraction, addition, multiplication, and division in both sides (of an equation) with same number. That is the essence. After this done, the only problem we will face is when dealing with dividing by a negative number.

FIGURE 9. Hadi's Interview

## **Findings from Text Books**

It is revealed from two books that there is no question that looks like 9x + 1 > 9x - 2. To exemplify, Fig. 9 can be seen. The questions are mostly in the form of "ax + b < c" and "ax + b < dx + e" with " $a, d \neq 0$ ", and " $a \neq d$ ." As a consequence, it is reasonable that students find the question very difficult to solve. Of course, to solve an unusual problem is difficult for most pupils.

On one hand, it is realised from these textbooks that there is no explanation about what the solution exactly means. Alternatively, it is explained about how to solve a linear inequality by doing some operations. This condition is believed as one of the causes of why some students stopped their work until they get 0 > -3 or equivalent expression and did not proceed anything further.



FIGURE 10. Linear Inequality Questions in The Text Book

Some problems in Fig. 10, even have closely similar properties with linear inequality, theoretically cannot be considered as linear inequality problems. Looking at detail, it can be clearly seen that the book contains  $\frac{5}{x} - \frac{3}{x} \le 2$ ,  $\frac{1}{x-3} > \frac{2}{x-4}$ , and  $\frac{5}{2n+4} > \frac{3}{n+2}$ . Definitely, these inequalities are rational inequalities containing linear algebraic forms. The techniques used to solve these problems may be seemingly same as those that are used for linear inequality problems. In spite of that, that the denominator should not be zero must be noticed.

Another point from Figure 10 is that the books show inconsistency of solution meaning. More specifically, problems number 14 to 24 which are in the figure indicate that solution is the simplest equivalent form of the initial inequality. It is due to the fact that the provided answers are x < 3,  $x \le -1$ , etc. On the other hand, that solution is a set of numbers satisfying the inequality is indicated by problem number 25.

#### DISCUSSION AND CONCLUSION

#### **Overview of the Result**

Overall, most students solve the problem by doing algebraic operations, while the other perform blank answer, rewriting the inequality, and expecting the problem to have numerical solution. Interestingly, many of those who approach the question algebraically did it wrong and simplified 9x - 9x into x. One of the reasons is that they forced x to be not eliminated. This condition may due to learning process which focuses on how to do algebraic operations well to get the simplest form. It is revealed from the teachers' interview and the text books.

#### Limitation

Nevertheless, there are some limitations in this study. First of all, it was conducted in second semester of VIII grade. This timing may made students having impediment to remember their lesson in VII grade. Secondly, the categories we made were just for only this sample. Still, another category can be found if another sample is included and it could be revised. Thirdly, this study dealt with the reason of why students encountered the difficulties only to a limited extent.

## Implication

Bearing the result in mind, this study leads to some implications for learning the linear inequality topic. First, the finding that many students fail on doing algebraic operations suggests why a procedure works. Furthermore, it can be done by bringing a contextual design as it is suggested by curriculum [1]. For instance, in the beginning of the study, a teacher gives a contextual story relating to linear inequality. To be more detail, there must be, as one the ways, a study concerning the learning design. Hopefully, this can help students to easily understand the concept since they find it having closed correlation with their daily activity.

In accordance with the above, the finding that no one who can state a correct solution which is the set of all number suggests solution of a linear inequality should be described clearly in class, apart from how to get it. Furthermore, pupils are expected to have a good conceptual understanding beside of procedural skill [18]. It is predicted that mathematics teacher or prospective mathematics teacher do not know the existence of such kind of solution, but, still, a study must be conducted to check that. We wonder if the result will be in the line.

The linear inequality, in line with the above, should not be put as a subordinate topic of the linear equation due to the findings. More specifically, that the teachers and the textbooks concern only little attention to what are behind the algebraic procedures and the definitions of linear inequality solution in both formal and informal way produced the bad impacts. In more general researchers also suggest that inequality must stand independently from equation [9,19].

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